SINE VALUES OF THE *VATEŚVARA SIDDHĀNTA*

R. N. RAI

Indian National Science Academy, Bahadur Shah Zafar Marg, New Delhi

Vateśvara has given $R \sin \theta$ values of ninety-six angles at intervals of 56' 15"
taking R to be equal to 3437' 44". The text of the *Vateśvara Siddhānta*,
published by the Indian Institute of Astronomical and Sanskrit Research,
is based on only one manuscript. Therefore the values given in the textual
portion of the *Vateśvara Siddhānta* in the published book are not correct.
In this paper an attempt has been made to give the actual values along with
the emended text of the portion concerned.

The Indian astronomers, with a few exceptions, have given the sine
values (actually *jyā* values, i.e. $R \sin \theta$, where R is the radius of the circle) of
twenty-four angles at intervals of 3° 45'. But Vateśvara[1] has given *jyā*
values of ninety-six angles at intervals of 56' 15". The value of R adopted
by Vateśvara is 3437' 44" though, a little earlier than he, Govindaswāmin, in
his commentary of the *Mahābhāskariya*[2] had adopted the value 3437' 44" 19'''
which is almost equal to 10,800/3·1416. The approximate value 3·1416 for π
given by Āryabhaṭa[3] is a little too high. Mādhava who knew the value of π
correct to 11 decimal places[4] has adopted $R = 3437'\ 44''\ 48'''$ for calculating
the *jyā* values.[5] However, the editors of the *Vateśvara Siddhānta* do not
seem to have examined carefully the concerned *jyā* values while editing the
text. If they had taken the trouble of calculating the *jyā* values by the
methods given by Varāhamihira in his *Pañcasiddhāntikā*,[6] the concerned
textual portion would have been undoubtedly accurate. The object of the
present paper is, to improve the text after calculating the *jyā* values with
$R = 3437'\ 44''$.

Method of calculating the jyā *values*

The relevant stanzas in the *Pañcasiddhāntikā* are:

1. *vyāsārddhakṛtidhruvasaṃjñikā Kṛtāṃśastataḥ sa meṣasya*
   dhruvakaraṇī meṣoṇā dvayostu rāśyoḥ padam jyāḥ syuḥ ||
2. *Śeṣeṣviṣṭeṣu dhanurdviguṇapadāyojyaśeṣaguṇahīnā |*
   trijyā tadardhavargo dviguṇajyārddhasya saṃyojyaḥ ||
3. *Tasya pado'bhimatajyā dhruvā tadūnāvaśeṣapiṇḍasya |*
   dhruvakaraṇidalamadhyarddhasaṃjñako'nyo'tra vidhiruktaḥ ||
4. *icchāṃśadviguṇonatribhajyayonā trayasya cāpajyā |*
   saṣṭiguṇā sa karaṇi tayā dhruvonāvaśeṣasya ||

The first of the above stanzas gives the values of the *jyā* of 30° and 60°. The second and the first half of the third give a method for calculating $R \sin \theta$ and $R \sin \left(\frac{\pi}{2} - \theta\right)$, knowing $R \sin 2\theta$ and $R \sin \left(\frac{\pi}{2} - 2\theta\right)$. The second half of the third stanza gives the method for calculating $R \sin \frac{\pi}{4}$. The last stanza gives another method of calculating $R \sin \theta$, knowing $R \sin \left(\frac{\pi}{2} - 2\theta\right)$. The exact translations are:

1. Take the square of the radius and call it the 'constant' (*dhruva*); the fourth part of it is (the square) of Aries (i.e. of the one sign). The 'constant' square is to be lessened by the square of the Aries (i.e. one sign). The square root of the two quantities (i.e. the square of the Aries and the 'constant' lessened by the square of the Aries) are the *jyā* values (viz. of 30° and 60° respectively).

2. To obtain the remaining desired (*jyā* values), take the double of the arc (whose *jyā* you wish to find) and deduct it from the quarter (of the circle), diminish the radius by the *jyā* of the remainder, and add to the square of half of that (viz. the radius so diminished) the square of half the *jyā* of double (the original arc).

3. The square root of the sum is the desired *jyā*. The 'constant' square lessened by that sum (is the square) of the remaining quantity (i.e. of the *jyā* of the complementary arc). Half of the 'constant' square is called (the square of) one and a half (viz. signs, i.e. 45°). Another method is also taught here.

4. Deduct from three signs (i.e. 90°) double the arc (of which the *jyā* is) desired, lessen the *cāpajyā* of three signs (i.e. the radius) by the *jyā* of the remainder (obtained after deducting from 90° double the arc) and multiply the remainder by sixty (i.e. half of radius); the result is the square (of the desired *jyā*). By deducting that square from the square of the radius you obtain the square of the remainder (the square of the *jyā* of the complementary arc).

These rules show that Varāhamihira knew the following rules:

$$jy\bar{a}^2\theta + jy\bar{a}^2 \left(\frac{\pi}{2} - \theta\right) = R^2,$$

or in modern terminology

$$\sin^2\theta + \cos^2\theta = 1. \quad .. \qquad .. \qquad .. \qquad .. \quad \text{(i)}$$

$$jy\bar{a}^2\ 2\theta + \left[R - jy\bar{a}\left(\frac{\pi}{2} - 2\theta\right)\right]^2 = 4\ jy\bar{a}^2\theta,$$

or $\qquad\qquad jy\bar{a}^2\ 2\theta + utkramajy\bar{a}^2\ 2\theta = 4\ jy\bar{a}^2\theta,$

or in modern terminology

$$\sin^2 2\theta + (1 - \cos 2\theta)^2 = 4 \sin^2\theta. \quad .. \qquad .. \qquad .. \quad \text{(ii)}$$

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The last rule can be deduced from the second and states that

$$\frac{R}{2}\left[R - jy\bar{a}\left(\frac{\pi}{2} - 2\theta\right)\right] = jy\bar{a}^2\theta,$$

or

$$\tfrac{1}{2}(1 - \cos 2\theta) = \sin^2\theta. \qquad .. \qquad .. \qquad .. \qquad .. \qquad \text{(iii)}$$

These results have also been stated very clearly by Bhāskara II in the *Golādhyāya*[7] of his *Siddhāntaśiromaṇi* and by Parmeśvara in his commentary on the *Āryabhaṭīyam*.[8] Indeed in deducing that[9]

$$jy\bar{a}\ 36° = R\left(\frac{5 - \sqrt{5}}{8}\right)^{\frac{1}{2}}$$

Bhāskara II must have used the result

$$jy\bar{a}\ \theta = jy\bar{a}\ (\pi - \theta),$$

and the equation which follows from it, that for $\theta = 36°$, $jy\bar{a}\ 2\theta = jy\bar{a}\ 3\theta$.

The method advocated by Āryabhaṭa in the *Āryabhaṭīyam Gaṇitapāda* is equivalent to the equations (i) and (ii) above.[10] The values of *jyā* and *koṭijyā* of the different angles can be accurately calculated with the help of equations (i) and (ii) from the knowledge of the *jyā* values of 30° and 45°. The other method given by Āryabhaṭa in the *Gaṇitapāda*[11] is only a rough method but has been used by later astronomers also. For instance, Nīlakaṇṭha Somasutvan in his *Tantrasaṃgraha*[12] says:

'The *jyā* of one-eighth of a sign is ten seconds less than the value in minutes of the arc. Subtract from the first *jyā* the quotient obtained by dividing the first *jyā* by two hundred thirty-three and a half and the remainder is the second *khaṇḍajyā*. Add it to the first *jyā* and sum is the second *jyā*. Now divide the second *jyā* by the same divider and subtract the quotient from the second *khaṇḍajyā* and the remainder is the third *khaṇḍajyā*. Add it to the second *jyā* and the sum is the third *jyā*. Similarly the fourth *jyā* etc. in that order.'

The commentator says that the first *jyā* is not 224′ 50″ but 224′ 50″ 22‴ and the correct divider is not two hundred thirty-three and a half but 233′ 32″. If

$x_n = $ nth *khaṇḍajyā* − (n+1)th *khaṇḍajyā*, we have

$x_n = 2R \sin n\beta(1 - \cos \beta)$,

where $R = 3437′\ 44″\ 48‴$ according to Mādhava, and

$\beta = 225′$ of arc

or

$$x_n = \frac{2R \sin n\beta\left[R - R \sin\left(\frac{\pi}{2} - \beta\right)\right]}{R}$$

$$= \frac{jy\bar{a}\ n\beta \times 2\ \text{last}\ khaṇḍajyā}{R} \qquad .. \qquad .. \qquad .. \qquad \text{(iv)}$$

This is the other method recommended by Nilakaṇṭha. The commentator then tells us that the last but one *jyā* is equal to the square root of the difference between the square of the radius and the square of the first *jyā*. This is only an application of equation (i). Substituting Mādhava's values of R and the last *khaṇḍajyā*, we have

$$x_n = \frac{jyā \ n\beta}{\dfrac{3437' \ 44'' \ 48'''}{2(7' \ 27'' \ 37''')}}$$

$$= \frac{jyā \ n\beta}{233 \cdot 5338}.$$

When written in the sexagesimal notation the denominator becomes 233; 32. This is the value given by the commentator.

The values of *jyā* θ and *utkramajyā* θ calculated by the use of the equations (i) and (ii) are shown in Table I.

TABLE I

| | A | | Jyā A | | Utkramajyā A | |
deg.	min.	sec.	min.	sec.	min.	sec.
0	56	15	56	15	0	28
1	52	30	112	29	1	50
2	48	45	168	41	4	8
3	45	0	224	50	7	22
4	41	15	280	56	11	30
5	37	30	336	57	16	33
6	33	45	392	53	22	31
7	30	0	448	43	29	25
8	26	15	504	25	37	12
9	22	30	559	59	45	55
10	18	45	615	25	55	32
11	15	0	670	40	66	3
12	11	15	725	45	77	29
13	7	30	780	38	89	48
14	3	45	835	18	103	1
15	0	0	889	45	117	8
15	56	15	943	58	132	8
16	52	30	997	55	148	2
17	48	45	1051	37	164	48
18	45	0	1105	1	182	26
19	41	15	1158	8	200	57
20	37	30	1210	56	220	20
21	33	45	1263	25	240	35
22	30	0	1315	34	261	41
23	26	15	1367	21	283	38
24	22	30	1418	47	306	26
25	18	45	1469	49	330	4

TABLE I—*contd.*

deg.	A min.	sec.	*Jyā* A min.	sec.	*Utkramajyā* A min.	sec.
26	15	0	1520	28	354	31
27	11	15	1570	43	379	49
28	7	30	1620	32	405	55
29	3	45	1669	55	432	51
30	0	0	1718	52	460	34
30	56	15	1767	21	489	5
31	52	30	1815	21	518	24
32	48	45	1862	53	548	30
33	45	0	1909	54	579	22
34	41	15	1956	25	611	0
35	37	30	2002	24	643	23
36	33	45	2047	51	676	31
37	30	0	2092	46	710	24
38	26	15	2137	6	745	0
39	22	30	2180	52	780	20
40	18	45	2224	4	816	22
41	15	0	2266	39	853	6
42	11	15	2308	38	890	32
43	7	30	2350	0	928	39
44	3	45	2390	45	967	26
45	0	0	2430	51	1006	53
45	56	15	2470	18	1046	59
46	52	30	2509	5	1087	44
47	48	45	2547	12	1129	6
48	45	0	2584	38	1171	5
49	41	15	2621	22	1213	40
50	37	30	2657	24	1256	52
51	33	45	2692	44	1300	38
52	30	0	2727	20	1344	58
53	26	15	2761	13	1389	53
54	22	30	2794	21	1435	20
55	18	45	2826	44	1481	19
56	15	0	2858	22	1527	50
57	11	15	2889	14	1574	51
58	7	30	2919	20	1622	23
59	3	45	2948	39	1670	23
60	0	0	2977	10	1718	52
60	56	15	3004	53	1767	49
61	52	30	3031	49	1817	12
62	48	45	3057	55	1867	1
63	45	0	3083	13	1917	16
64	41	15	3107	40	1967	55
65	37	30	3131	18	2018	57
66	33	45	3154	6	2070	23
67	30	0	3176	3	2122	10
68	26	15	3197	9	2174	19
69	22	30	3217	24	2226	48
70	18	45	3236	47	2279	36

TABLE I—*concld.*

| deg. | A | | Jyā A | | Utkramajyā A | |
	min.	sec.	min.	sec.	min.	sec.
71	15	0	3255	18	2332	43
72	11	15	3272	56	2386	7
73	7	30	3289	42	2439	49
74	3	45	3305	36	2493	46
75	0	0	3320	36	2547	59
75	56	15	3334	43	2602	26
76	52	30	3347	56	2657	6
77	48	45	3360	15	2711	59
78	45	0	3371	41	2767	4
79	41	15	3382	12	2822	19
80	37	30	3391	49	2877	45
81	33	45	3400	32	2933	19
82	30	0	3408	19	2989	1
83	26	15	3415	13	3044	51
84	22	30	3421	11	3100	47
85	18	45	3426	14	3156	48
86	15	0	3430	22	3212	54
87	11	15	3433	36	3269	3
88	7	30	3435	54	3325	15
89	3	45	3437	16	3381	29
90	0	0	3437	44	3437	44

The *utkramajyā* values have been included so that we can check the *jyā* and *utkramajyā* values of Vaṭeśvara against each other since we know that

$$jyā \ A + utkramajyā \ (90° - A) = R.$$

This fact was known to Āryabhaṭa who stated in one *āryā* only the *khaṇḍajyā* values which when read in the reverse order will give the *khaṇḍa-utkramajyā* values and progressively on addition the *utkramajyā* values of the different angles. This may be the reason why they were called *utkramajyā*.

The text of the Vaṭeśvara Siddhānta

We will now examine the text of the *Vaṭeśvara Siddhānta* and try to improve the corrupt text with the help of Table I and the *jyā* and *utkramajyā* values of the *Vaṭeśvara Siddhānta* itself. Vaṭeśvara has first enumerated all *jyā* values in minutes and then all the corresponding seconds. He has adopted the same method for stating the *utkramajyā* values also.

The text is:—

<div style="text-align:center">

56   112   168

ardhajyā rasabāṇāḥ karaśaśiśaśino gajāṅgacandramasaḥ

224   280   336

vedākṛtayo vyomastamberamabāhavo rasāgniguṇāḥ (13)

392   448   504

netranavahutabhujo gajajaladhikṛtāḥ kṛtanabhobāṇāḥ (14)

</div>

559   615   670

nandaśilīmukhabāṇāḥ śaraśaśyṛtavaḥ khaparvatāṅgāni

725   780   835   889

tattvāgāḥ khāṣṭanagāḥ śarāgnināgā navāṣṭapavanabhujāḥ (15)

943   997   1051

rāmābdhyaṅkā aganavanandāḥ kubāṇaśūnyahariṇāṅkāḥ (16)

1105   1158   1210

śarakhaśivāḥ stamberamatithibhuvaḥ khaśaśidviśaśāṅkāḥ (17)

. . . .

. . . .

. . . .

1767   1815   1862

saptarttusaptaśaśinastithidhṛtayo dvyaṅganāgahariṇadhṛtaḥ (18)

1909   1956   2002

navakhāṅkabhuvo rasaśaranavacandrāḥ karakhaśūnyakarāḥ (19)

2047   2092   2137

nagakṛtakhakarā dvinavavyomabhujāḥ saptaviśvanetrāṇi (20)

2180   2224   2266

khadhṛtiyamā vedabhujadvibhujā rasaṣaḍbhujākṣīṇi (21)

2308   2350   2390

vasukhāgniyamāḥ khaśaratribhujā ākāśanandaguṇayamalāḥ (22)

2430   2470   2509   2547

khaguṇajināḥ khāgajinā navābhratattvānyagābdhitattvāni

2584   2621   2657

vedāṣṭeṣuyamāḥ śaśidvyaṅgabhujā nageṣurasayamalāḥ

2692   2727   2761

dvinavarasayamāḥ saptadvinagabhujaścandraṣaṭnagākṣīṇi

2794   2826   2858

vedāṅkabhāni rasayamavasunetrānyaṣṭākṣavasuyamalāḥ (23)

2889   2919   2948

navavasvaṣṭabhujā navaśaśinandayamā gajābdhinavadasrāḥ

2977   3004   3031

nagasaptāṅkabhujāḥ kṛtakhakharāmāḥ śaśiguṇābhrahavyabhujaḥ

3057   3083   3107

saptaviśikhābhrarāmāstrināgakhaguṇā nagābhraśaśirāmāḥ (24)

3131   3154   3176

bhūguṇabhūguṇā abdhiśaraikguṇā rasadharādharaikguṇāḥ (25)

. . . .

3255   3272

viśikhaviśikhabāhvagnayo bāhudharitrīdharākṣihavyabhujaḥ (26)

3289

kramaparipāṭyā jivāśchidrastamberamadviguṇāḥ (27)

    3305   3320   3334   3347

śarakhasurā nakhadevā vedatrisurā nagābdhiguṇarāmāḥ

    3360   3371   3382

khāṅgatriguṇā bhūnaganākagṛhā netranāgaguṇarāmāḥ

    3391   3400

śaśinandāgniguṇāḥ khakhābdhiguṇāḥ..............(28)

                                    3426

.... *rasakarābdhihavyabhujaḥ*

    3430   3433   3435

khāgnisamudrahutāśistritryabdhiguṇāḥ śarāgniyugarāmāḥ (29)

    3437   3437

saptaguṇavedarāmā nagaguṇavedāgnayo liptāḥ

After this Vaṭeśvara gives the *vikalā* parts of the *jyā* values of these angles.

       15   29   41   50

āsāṃ vikalāstithayo nandabhujāḥ kvabdhayaḥ payodaśarāḥ (30)

   56   57   53   43   25

rasaviśikhāḥ saptaśarā agniśarāstrikṛtāḥ śarākṣiṇi

   59   25   40   45   38

navaviśikhāḥ pañcayamāḥ khakṛtāḥ pañcābdhayo dviradarāmāḥ

 18   45   58   55   37

dhṛtiriṣuvedā maṅgalaviśikhā akṣeṣavasturaṅgaguṇāḥ (31)

 1 5   56   25   34   21

bhūbānā(?) rasabāṇāstattvāni jalāgnayaḥ kubhujāḥ (32)

   47   49   28   43   32

nagavedā nandakṛtā vasunetrāṇyagnijaladhayo dantāḥ (33)

   55   52   21   22   53

viśikhasarā netraśarāḥ kubhujā dviyamā hutāsanākṣāḥ (34)

    54   25   24   52   46

vedeṣavo'kṣanetrānyabdhiyamā dviṣavo rasasamudrāḥ (35)

   6   53   4   39   39

aṅgānyagnipṛṣatkā vedā navavahnay'ṅkaguṇāḥ (36)

   1   45   51   18   5   12

rūpaṃ sāyakavedāḥ kuśarā gajabhūmayaḥ śarāḥ sūryāḥ

   38   22   25   44   21

gajarāmā netrayamāstattvāni kṛtābdhayaḥ kunetrāṇi

 13   21   45   23   15

viśve kubhujāḥ sāyakanigamā guṇabāhvastithayaḥ (37)

   20   39   10   53   49   55

khabhujā nandaguṇā daśa trisarā nandābdhayo'kṣaśarāḥ

 13   41   19   6   3   9   24

viśve kukṛtā atidhṛtiraṅgāni guṇā nandā abdhinetrāṇi (38)

- 47   18   57   43   36
saptābdhayo dhṛtirnagaviśikhā guṇasāgarāḥ rasaguṇāśca (39)
36   43   56   15   41
ṛturāmā rāmakṛtā raseṣavo vāsarāḥ kukṛtāḥ (40)
12   49   32   20   13
sūryā nandasamudrā radā nakhā vahnicandramasaḥ
11   14   23   36   54   16
īśā manavo'gnibhujā rasāgnayo vedasāyakā vikṛtiḥ (41)
44   0   1   4   7   11
vedakṛtā viyadiṣavaḥ (?) *khaṃ bhūrvedā nagā rudrāḥ* (42)
16   22   29   37   45
aṣṭirnetrabhujā navanetrānyagavahnayo viśikhavedāḥ
55   66   77   89   103
pañcaśarāḥ ṣaḍṛtavo nagamunayo nandakuñjarāstridaśa
117   132   148   164
nagarudrā radacandrā vasumanavo vedarasacandrāḥ
182   200   220   240
dvyaṣṭabhuvaḥ śūnyanakhāḥ khākṣibhujāḥ khābdhinetrāṇi
261   283   306   330
kūtkṛtayastryaṣṭabhujā rasakhaguṇā vyomagīrvāṇāḥ
354   379   405   432
vedeṣuguṇā navanagarāmāḥ śarakhābdhayo radasamudrāḥ (43)
460   489   518   548
khāṅgābdhayo'ṅkakuñjaravedā dhṛtisāyakā gajābdhiśarāḥ
579   610   643
navanagaviśikhā jaladharaśaśyṛtavo guṇakṛtā'ṅgāni
676   710   745
rasanagarasāḥ khaśaśadharanagāḥ pṛṣatkābhidharaṇidharāḥ (44)
780   816   853   890
khāṣṭanagā rasakugajāstriśaragajā jaladanandavasavaśca (45)
928   967   1006
vasubhujanandā nagarasāṅkāni rasakhābhrahariṇāṅkāḥ (46)
1046   1087   1129
ṛtvabdhidiśo nagāṣṭakhabhuvo'ṅkanetraśaśicandramasaḥ (47)
1171   1213   1256   1300
kunagaśivā viśvā'rkā rasatattvabhuvaḥ khakhāgnirūpāṇi
1344   1389   1435
vedakṛtāgniśaśāṅkā navāṣṭaviśve śarāgnikṛtacandrāḥ
1481   1527   1574   1622
kvaṣṭamanavo bhatithayo'bdhyagaśaracandrā dvibāhurasacandrāḥ
1670   1718   1767
khanagarasabhuvo vasubhūnagaśaśino nagarasāgacandramasaḥ (48)

1817   1867   1917

agaśaśidhṛtayo'garasadvipaśaśino'gaikanandarajanīśāḥ (49)

1967   2018   2070

saptāṅgāṅkabhuvo'ṣṭakukhabhujā vyomāgaśūnyanetrāṇi

2122   2174   2226

dvīnabhujāḥ kṛtanagaśaśinetrāṇyaṅgākṣibāhunetrāṇi

2279   2332   2386

aṅkāgākṣibhujā radarāmabhujā rasagajāgninayanāni (50)

2439   2493   2547

navarāmajinā guṇanavasiddhāḥ saptābdhitattvāni

2602   2657   2711

dvivyomotkṛtayaḥ parvataśarāṅganetrāṇi rudrabhānīha (51)

2767   2822   2877

saptāṅgabhāni yamayamanāgabhujā naganagāṣṭakarāḥ

2933   2989   3044

suranavabhujā navāṣṭachidrākṣīṇyabdhijaladhiśūnyaguṇāḥ

3100   56   32

khakhakuguṇā rasapañca . . . . bāhvagnayaḥ

3381

. . . . *candranāgaguṇarāmāḥ* (52)

3437

nagaguṇavedahutāśā vikalāḥ santi sthitāḥ pṛthak caiṣām (53)

8   21   30   33   31   24

vasavaḥ kubhujāḥ khaguṇāḥ surāḥ kurāmā jināḥ kharāmāśca(?) (54)

55   32   3   29   48

pañcaśarā netraguṇā rāmā navabāhavo dvipasamudrāḥ

1 8   8   1   47   26   57

bhūrvasavo'ṣṭau candrā nagavedāḥ ṣaḍbhujā acalabāṇāḥ

20   35   41   38   25

viṁśatiriṣuhavyabhujaḥ kukṛtā vasvagnayo'kṣabhujāḥ (55)

3   31   49   55   51

rāmāḥ kuguṇā vargasaptānāṁ pañcapañcaikaśarāḥ (56)

34   5   24   29   21

vedaguṇāśca pṛṣatkāḥ siddhā navabāhavaḥ kubhujāḥ

59   23   31   23

navaviśikhā rāmabhujā ilāgnayo vahninayanāni

0   19   22   6   32   39   27

khaṁ navacandrā dvibhujā rasā dviguṇā nandavahnayo'gabhujāḥ (57)

53   59   43   5   5

triśarā nandapṛṣatkā guṇābdhayaḥ sāyakā viśikhāḥ

40   51   38   58   52

khakṛtāḥ kuśarā maṅgalahavyabhujo vasuśara dviśarāḥ

20   19   50   51   22
vyomabhujā navacandrāḥ khaśarāḥ kuśarā dṛgakṣiṇi
    23   52   49   12   1
trikarā dviśarāśchidrapraṇimnageśa ināścandrāḥ (58)
  16   55   57   23   10   19
aṣṭiḥ pañcaśarā nagabāṇāgnibhujā diśo'ṅkabhuvaḥ
    48   36   43   7   49   46
aṣṭakṛtā rasarāmāstrikṛtā acalā aṅkābdhayo'ṅgakṛtāḥ (59)
    59   26   6   59   4   19
navaviśikhā rasanetrāṇyaṅgānyaṅkeṣavo'bdhayo'ṅkabhuvaḥ
    45
śaravedā .... (60)
          3   15   29   44
.... *havyabhujastithayo'ṅkabhujāḥ kṛtābadhayastrijyā* (61)
    3437   44
agaguṇavedahutāśāḥ kalikā vikalāḥ samudrajaladhayaḥ sapta
    11, 818, 007   35
jaladakhāṣṭaśaśidhṛtiśaśinaḥ kalikāḥ śarāgnayo vikalāḥ (62)
              1398   24
trijyākṛtiraṣṭanavatribhuvaḥ kathitā gaṇakairjināṃśajyāḥ

TABLE II

| Serial | Mādhava's values of *jyā* | | | Modern values of *jyā* | | | |
No.	min.	sec.	thirds	min.	sec.	thirds	fourths
1	224	50	22	224	50	21	48
2	448	42	58	448	42	57	32
3	670	40	16	670	40	15	58
4	889	45	15	889	45	15	31
5	1105	1	39	1105	1	38	49
6	1315	34	7	1315	34	7	18
7	1520	28	35	1520	28	35	54
8	1718	52	24	1718	52	24	0
9	1909	54	35	1909	54	34	59
10	2092	46	3	2092	46	3	16
11	2266	39	50	2266	39	49	58
12	2430	51	15	2430	51	14	20
13	2584	38	6	2584	38	5	15
14	2727	20	52	2727	20	52	5
15	2858	22	55	2858	22	54	48
16	2977	10	34	2977	10	33	24
17	3083	13	17	3083	13	16	36
18	3176	3	50	3176	3	49	37
19	3255	18	22	3255	18	21	14
20	3320	36	30	3320	36	29	48
21	3371	41	29	3371	41	28	47
22	3408	20	11	3408	20	10	38
23	3430	23	11	3430	23	10	16
24	3437	44	48	3437	44	48	0

It will be observed that the *jyā* values stated by Vaṭeśvara are the same as those given in Table I except that in a few cases there is a difference of one *vikalā*, Vaṭeśvara's value always being higher than those given in Table I. Further there is a slight error in the value of the square of the radius. The value given by Vaṭeśvara is 11, 818, 007′ 35″. But the actual calculation shows it to be 11, 818, 010′ 28″. Furthermore, the *jyā* of 24° as given by Vaṭeśvara is 1398′ while the modern value would work out to be 1398′ 15″.

However, there is a large difference in the *jyā* of 19° 41′ 15″. *Bāṇā* in the phrase *bhūbāṇārasabāṇāstattvāni* along with the corresponding *kalā* part would make Vaṭeśvara's value as 1158′ 5″. But the actual value is 1158′ 8″. It is very strange that in one case the error in Vaṭeśvara's value should be so large. It may be that the correct reading is probably *vyālā* and not *bāṇā*. This is supported by the fact that Vaṭeśvara states the *utkramajyā* of the complementary angle 70° 18′ 45″ as 2279′ 36″. The *vikalā* part of this value is the second word combination in *aṣṭakṛtā rasarāmāstrikṛtā*.

Though Vaṭeśvara has given the *jyā* values of ninety-six angles while Mādhava has given only twenty-four *jyā* values like most of the other astronomers, the values given by Mādhava are much more accurate as can be seen

TABLE III

Govindasvāmin's values						Modern values			
Khaṇḍajyā			Piṇḍajyā						
min.	sec.	thirds	min.	sec.	thirds	min.	sec.	thirds	fourths
224	50	23	224	50	23	224	50	19	54
223	52	30	448	42	53	448	42	53	45
221	57	18	670	40	11	670	40	10	20
219	4	57	889	45	8	889	45	8	0
215	16	22	1105	1	30	1105	1	29	30
210	32	26	1315	33	56	1315	33	56	12
204	54	26	1520	28	22	1520	28	22	28
198	23	48	1718	52	10	1718	52	9	30
191	2	9	1909	54	19	1909	54	18	52
182	51	27	2092	45	46	2092	45	45	37
173	52	58	2266	38	44	2266	39	30	51
164	12	10	2430	50	54	2430	50	53	50
153	46	49	2584	37	43	2854	37	43	27
142	42	46	2727	20	29	2727	20	29	5
131	2	2	2858	22	31	2858	22	30	41
118	47	38	2977	10	9	2977	10	8	17
106	2	42	3083	12	51	3083	12	50	35
92	50	32	3176	3	23	3176	3	22	50
79	14	31	3255	17	54	3255	17	53	46
65	18	8	3320	36	2	3320	36	1	50
51	4	59	3371	41	1	3371	41	0	20
36	38	41	3408	19	42	3408	19	41	49
22	3	0	3430	22	42	3430	22	41	20
7	21	37	3437	44	19	3437	44	19	0

in Table II which gives Mādhava's values along with the corresponding modern values.[5]

It will be noticed from the Table that on taking the approximation to the nearest *tatpara*, the value does not differ from the modern value by more than 1‴.

Though Govindasvāmin has also calculated the *jyā* values in *kalā*, *vikalā* and *tatpara*, his values are not as accurate as those of Mādhava.[2] This can be seen from an examination of Table III. The difference is especially large in the *jyā* values of 3° 45′ and of 41° 15′.

REFERENCES AND NOTES

[1] *Vaṭeśvara Siddhānta*, Vol. I, edited by R. S. Sharma and M. Mishra, published by the Indian Institute of Astronomical and Sanskrit Research, New Delhi, 1962, p. 309.

[2] *Mahābhāskarīya* of Bhāskarācārya (Bhāskara I) with the *Bhāṣya* (gloss) of Govindasvāmin and the supercommentary *Siddhāntadīpikā* of Parameśvara, edited by T. S. Kuppanna Sastri, Govt. Oriental Manuscripts Library, Madras, 1957, p. 200.

[3] *Āryabhaṭīyaṃ, Gaṇitapāda*, 10.

[4] *Āryabhaṭīyaṃ* with the *Bhāṣya* (gloss) of Gārgyakerala Nīlakaṇṭha Somasutvan, edited by K. Samvasiva Sastri, the Superintendent, Govt. Press, Trivandrum, 1930, I, p. 42.

[5] Quoted by Śankara Vāriar in his commentary on *Tantrasaṃgraha* of Gārgyakerala Nīlakaṇṭha Somasutvan, published by Honorary Director, University Manuscripts Library, Trivandrum, 1958, p. 19.

[6] *Pañcasiddhāntikā* of Varāhamihira, edited by G. Thibaut and S. Dvivedi, IV, 2–5.

[7] *Siddhāntaśiromaṇi, Golādhyāya*, V, 3–5.

[8] *Āryabhaṭīyaṃ* with the commentary *Bhaṭadīpikā* of Parmeśvara, edited by H. Kern, published by E. J. Brill, Leiden, 1874, p. 28.

[9] *Siddhāntaśiromaṇi, Golādhyāya*, XIV, 7.

[10] The *Khaṇḍakhādyaka* of Brahmagupta with the commentary of Bhaṭṭotpala, edited and translated by Mrs. Bina Chatterjee, Vol. I, 1970, p. 190. See also *Āryabhaṭīyaṃ* with the commentary *Bhaṭadīpikā* of Parameśvara, edited by H. Kern, pp. 27–30 and *Āryabhaṭīyaṃ* with the *Bhāṣya* of Gārgyakerala Nīlakaṇṭha Somasutvan, I, pp. 43–45.

[11] *Tantrasaṃgraha* by Gāryakerala Nīlakaṇṭha Somasutvan, p. 17.

[12] *Ibid*, p. 18.

[13] In this line *vedākratayo* is printed as *vedotkṛtyo* and *stamberamabāhavo* as *stambherama* and *bāhavo* is not joint to it.

[14] Here *netra* is printed separately from *navahutabhujo* and *bāṇāḥ* is printed separate from *Kṛtanabho*.

[15] In this line *navāṣṭa* and *pavanabhujaḥ* are printed separately.

[16] In this line *rāmābdhyankā* is printed as *rāmāgnyankā*, *aganavanandā* as *agagajanandā* and *Kubāṇaśūnyahariṇānkāḥ* as *Kuveda śūnya hariṇānkāḥ*. That the emended text gives the correct values is supported by the fact that the *utkramajyā* values of the complementary angles given by Vaṭeśvara are 2493 and 2439 for the first and second cases. But again in the third case the text gives the value 2356 which is obviously wrong as the *utkramajyā* of the previous angle is correctly stated as 2332.

[17] In this line *stamberamatithibhuvaḥ* is printed as *stambheramatithibhuvaḥ* and *khaśaśidviśaśānkāḥ* as *śaśidhṛti śaśānkāḥ*. After this three lines giving the minutes of ten *jyā* values are missing but the editors have given no indication of this fact. That the *jyā* value is 1210′ and a few seconds is supported by the fact that the *utkramajyā* of (69° 22′ 30″) is given by Vaṭeśvara as *angākṣibāhunetrāṇi*.

[18] *stithi* is printed as *sthiti*, and words which should be printed together have been printed separately.

19 Most of the words which should be printed together have been printed separately.

20 The word combinations have again not been printed together.

21 Here also the words which should be combined together have been printed separately and *vedabhujadvibhujā* has been printed as *vedabhujādvibhujā*.

22 In this line as well as in many other lines the word combinations indicating one number have not been printed together.

23 Here instead of '*aṣṭākṣa*' '*aṣṭa*' '*pakṣa*' has been printed.

24 In this line even *visikhābhra* has been printed separately as *visikhā* and *bhra*.

25 Here *abdhisaraikguṇā* has been printed as *aṣṭābdhyekguṇa* and the two parts of the *dharā-dhara* have been separated as *dharā* and *dhara*. Again one line pertaining to the minutes of the *jyā* values of the next three angles is missing. But the editors have given no indication of this fact.

26 In this line also *dharitrīdhara* has been separated into *dharitrī* and *dhara*.

27 In this line *stamberama* has been again printed as *stambherama*.

28 *khakhābdhiguṇā* is printed as *bhūkhābdhiguṇā*. That the correct values is *Khakhābdhiguṇā* is supported by the fact that Vaṭeśvara himself gives the *utkramajyā* of the complementary angle as 37'. Unfortunately the line giving the part of the value in seconds is missing. Again about one-third of this line and two-thirds of the next line giving the *Kalā* parts of the *jyā* values of three angles are missing.

29 After this there is one more line giving the value 3436' and 3437'. As the calculations show there is no place for these. There is no angle having the minute part of its *jyā* as 3436' and there are not three angles having the minute part of their *jyā* values as 3437'. Therefore this line has been deleted.

30 *nandabhujāḥ* has been printed as *nandabhujaḥ*.

31 *akṣeṣavasturangaguṇāḥ* is wrongly printed as *pakṣeṣavasturangaguṇāḥ*.

32 *kubhujāḥ* is printed as *Kṛbhujaḥ*.

33 *dantāḥ* is printed as *dandāḥ* which is meaningless.

34 *hutāśanākṣāḥ* has been printed as *hutāśanā* and *vede* which should go in the next time has been printed in this line.

35 *akṣa* has been printed as *ala*. *akṣa* denotes five, *ala* is meaningless.

36 *ankagunāḥ* has been printed as *ankāgunāḥ*. In this form they would denote two separate numbers, viz. nine and three. The combination denotes thirty-nine.

37 *ku* in *kubhujāḥ* has been left out.

38 *kukṛtā* is printed as *kudhṛtā* and *nandā* has been entirely left out and *guṇā* is printed as *guṇa*.

39 *saptābdhayo* is printed as *saptādhvaryo*.

40 In place of *ṛturāmā*, *dantā rāmā* have been printed and in place of *raseṣvo*, *rāmeṣavo* has been printed.

41 I have not been able to decide finally that it is *vikṛtiḥ* because this word stands for twenty-three. But it may stand for sixteen also. The actual word printed is *vidhṛtiḥ* which appears to be meaningless.

42 *viyadiṣavaḥ* seems to be unnecessary. From here *utkramajyā* values begin.

43 *śarakhābdhayo* is printed as *śarabdhayo* and *radasamudrāḥ* as *rasasamudrāḥ*.

44 *khaśaśadharanagāḥ* has been printed as *khaśaśadharanāgāḥ* and *dharaṇidharāḥ* as *dharaṇidharaḥ*.

45 *khāṣṭanagā* has been printed as *Khābdhināgā*.

46 *nagarasānkāni* is printed as *nagarasavilāni* which is meaningless.

47 *nagāṣṭa* is printed as *bhagāṣṭa*.

48 *vasubhū* is printed as *bhūbhū*, *nagarasāga* as *rasāganaga* and *khanagarasbhuvo* as *khanāgarasbhuvo*.

49 *agaśaśi* is printed as *bhagaśaśi*.

50 *ankāgākṣi* is printed as *angāgākṣi* and *rasagajāgni* as *rasapañcāgni*.

51 *dvibyomutkṛtayaḥ* is printed as *dvyabdhyutkṛtayaḥ*.

52 One line seems to have been left out here. *rasapañca* denotes the two figures of the number 3156. *bāhvagnayaḥ* may belong to 3212 or 3269. *candranāgaguṇarāmāḥ* has been printed as *candrarāmaguṇarāmāḥ*.

⁵³ The three words *sthitāḥ pṛthak caiṣāṃ* must express the *vikalā* parts of the *utkramajyā* of 56′ 15″ and 1° 52′ 30″ as *vasavaḥ* in the next line and the other word combinations following it give the *vikalā* parts of the *utkramajyā* values of 2° 48′ 45″ and other succeeding angles.

⁵⁴ *kharāmāḥ* is wrong.   It must be some word denoting twelve.

⁵⁵ *vasvagnayo* is printed as *vasavo'drayo*.

⁵⁶ *pañcapañcaikaśarāḥ* is printed as *pañca pañcaśarāḥ*.

⁵⁷ In place of *rasā dviguṇā* is printed *rasarasā* which is absurd.   Only in the case of *agabhujāḥ* the sum of this *utkramajyā* and the corresponding *jyā* of the complementary angle is not 3437′ 44″.   It may be that it is not *agabhujāḥ* but *rasabhujāḥ*.

⁵⁸ In place of *ināścandrāḥ* is printed *ināścandraḥ*.

⁵⁹ In place of *aṅkābdhayo'ṅgakṛtāḥ* is printed *khābdhayo'ṅkakṛtāḥ*.

⁶⁰ Again one line giving the *vikalā* parts of six *utkramajyā* values is missing.   But the editors have not indicated this.

⁶¹ At this point the statement of the *vikalā* parts ends.   The author now states the value of the radius, the square of the radius and *jyā* of 24°.

⁶² In this line *jaladakhāṣṭa* has been printed as *jalakhāṣṭa*.