SOME EARLY CODES AND CIPHERS

SUBHASH C. KAK

Department of Electrical and Computer Engineering Louisiana State University Baton Rouge, LA 70803, USA

(Received 8 September 1987: after revision 7 September 1988)

Some early ciphers and codes used in India are reviewed. The significance to cryptology of the $Siva\ s\bar{u}tra$ and the Aryabhata and Katapayadi ciphers is described.

1

The numerology of the Rgveda, and its profuse use of symbols that make no sense to the uninitated, can be viewed as examples of encryption of ideas. The later literature is also rich in references to cryptography. The Arthaśastra, the Lalitavistara, and the $K\bar{a}mas\bar{u}tra$ refer to secret writing.

In a commentary on the Kāmasūtra, Yaśodhara describes two kinds of cryptographic schemes based on letter substitution. In the method called kautiliyam, the letter substitutions are based on phonetic relations so that certain vowels become consonants and vice versa. In mūladeviya the cipher transformation is the reciprocal mapping

with the remaining letters remaining unchanged. In the written form this cipher is called gūḍhalekhya. Of course, properly speaking, gūḍhalekhana means cryptography.

Amongst a variety of other cipher schemes known, one worthy of special mention is a finger language called *nirābhaṣa*, that can be used by deaf and dumb people. Here the phalanges stand for the consonants and the joints for the vowels. This method is sometimes used by traders in the market place with the hands under a piece of cloth.

The substitution ciphers of the gudhalekhana can be solved readily by means of statistical analysis. When the cipher words themselves are meaningful messages, which can be true only for short ciphertexts, the gudhalekhana method can be effective from the point of view of security.

Substitution ciphers have been used in other civilizations as well. But the ciphers described in the next paragraphs are unique in many respects.

II

In the Daśagītika chapter of $\bar{A}ryabhatīya$ a cipher is described that represents numbers by letter sequences or words. Aryabhata later uses this cipher to represent numbers by such words that allow a versification of his mathematics and astronomy. Since letters may be readily mapped into numbers, his cipher could be used in more general applications. It is conceivable that similar ciphers were used in military situations but we have no concrete evidence of that. As $\bar{A}ryabhat\bar{t}ya$ was an update of an earlier book of astronomy it cannot be established that $\bar{A}ryabhata$ himself discovered the cipher.

The cipher is generally known to historians of science, but it does not appear to have been examined by cryptologists. Its appeal lies not only in its novelty but also in the fact that it is an ingenious technique that can yield cipher words that appear meaningful.

The reading of the cipher mapping in Aryabhatiya is as follows:

Beginning with ka the varga letters (are to be used) in the varga places, and the avarga letters (are to be used) in the avarga places. Ya is equal to the sum of na and ma. The nine vowels each (are to be used) in two nines of varga and avarga places.

Since the cipher for the Sanskrit alphabet is well understood, we present a version for the Roman alphabet. This detracts the flexibility of the cipher transformation somewhat, since the Roman alphabet is only half as large as the Sanskrit alphabet.

Ш

The \bar{A} ryabhata cipher (AC) divides up the Roman alphabet into 3 classes. The letters with corresponding numeral equivalence are shown below.

Class 1 Consonants

letter: b c d f g h j k l m number: 1 2 3 4 5 6 7 8 9 10

Class 2 Consonants

letter: Z n S q 50 70 80 90 100 200 300 20 30 40 60 number:

Class 3 Vowels

letter: a e i o u number: 1 2 3 4 5

(exponents)

Letters from the first two classes ordinarily stand for the numbers written under each. However, when such a letter is followed by one from the Class Three it is multiplied by 10 to the power listed under it. In other words, the use of a letter from the Class Three shifts the digits generated by Class One or Class Two letters to positions of higher significance.

To consider the transformation between numbers and letters, consider the example of the number 89381. It may be mapped as *dekavilib*, because de is $3\times10^2=300$, ka is $8\times10^1=80$, vi is $80\times10^3=80,000$, b is 1 and li is $9\times10^3=9000$. This number may be alternatively mapped as any other permutation of de, ka, vi, b and li. One may break up 89381 in other ways to get different mappings. Thus breaking it up as 50000+30000+9000+3000+81 could also be represented as *gizewezkab*, as well as other variants.

In Āryabhaṭa's cipher for the Sanskrit alphabet,³ Class One has the 25 varga letters k to m numbered I to 25, Class Two has 8 avarga letters y to h numbered 30 to 100, and Class Three has 9 vowels, a, i, u, r, l, e, ai, o, au, representing multiplication by 10°=1 through 10¹6 in multiples of 10². Āryabhaṭa also requires the use of varga letters in odd places (varga places) counting from the right, and the use of avarga letters in even places (avarga places) counting from the right.

Āryabhata also uses the rule that if two letters from Classes One and Two are strung together and a letter from Class Three follows, then both these letters are followed by the letter from Class Three. Symbolically, $l_1 l_2 l_3$ stands for $l_1 l_2 l_3 l_2 l_3$ therefore. This does not lead to confusion because Āryabhata also insists that a letter from Classes One or Two should ordinarily be followed by a letter from Class Three. In other words, $l_1 l_2 l_3$ is just a shorthand notation for $l_1 l_3 l_2 l_3$. We shall not use this aspect of Āryabhata's notation due to the constraints placed by the smaller size of the Roman alphabet.

The longest numbers represented by Aryabhata in his treatise using his mapping run into 10 places. For example, cayagiyinusuchlr stands for 57,753,336. Aryabhata's formulation is slightly more restrictive than our exposition of it for the Roman alphabet. On the other hand, the larger Sanskrit alphabet provides a much richer set of substitutions so that it is easier to obtain cipher words that in themselves are valid words of the language.

A variant:

The Aryabhata cipher breaks up a number in any of the many additive components and constructs the cipher thereof. By defining other Classes of

letters that imply multiplication and subtraction one can increase the flexibility even further. Of course, when using a letter code for multiplication one would have to keep the code letters for the multiplicands separate.

Example of AC on text:

By mapping A to 01, B to 02, as so on and space to 27, one may obtain a cipher for text. An example is shown below:

Message

THE ARYABHATA CIPHER

THE - ARYABHATA - CIPHER

Numerical equivalent 2008052701182501020801200127030916080518

The encryption may be performed conveniently by grouping the numbers in sets of six.

200805 ↔ novag 270118 ↔ yitixbak 250102 ↔ cugoxc 080120 ↔ vemca 012703 ↔ bonejad 091608 ↔ wixasak 0518 ↔ rahak

The cipherwords do often have different lengths. These cipherwords could be written in groups separated by spacings. Adding a Class of letters for subtraction makes it easy to devise cipherwords that have fixed length.

Number of cipher transformations:

Clearly there are 26! ways the 26 letters can be assigned to the numbers listed in the transformation. Therefore.

Number of keys = 26!

Yet this figure does not give a fair idea of the effort of the cryptanalyst, since the large number of ways a given number can be broken up into additive factors, makes the effective cipher variation much greater.

Key size:

The key is the sequence of the letters in the transformation. It is 26 letters long.

IV

While we have mentioned the possibility of creating ciphertext that may be meaningful in itself, in practice this will work only for messages of very small length. For this reason this property can be exploited best when the objective is to hide numerical information that occurs in prose. For straight encryption of text one would not look for this characteristic in the ciphertext.

For a general setting to view the Āryabhaṭa cipher consider that the message and cipher alphabets are $(a_1, a_2, ..., a_n)$, and the code alphabet is $(b_1, ..., b_k)$ where n > k. The message block M is first converted into the code block R, which is then partitioned into the sum (with the choice of L left arbitrary):

$$R = R_1^2 + R_2^2 + ... + R_L$$

Each of R_i is encrypted by a one to many mapping $C_i = C(R_i)$ and a concatenation of the C_i 's in any order defines the cipherblock.

In a practical implementation, one may represent the elements of code digits themselves to be the partition. Now each b is mapped into one of the several candidate a's, the choice being greater if $k \ll n$. The cipher block letters could be chosen so as to follow the language statistics as closely as possible.

The Aryabhata encryption paradigm is likely to have uses in encryption of computer data and communications signals. Its main disadvantage is that the ciphertext is larger than the plaintext.

V

Even more versatile than the Āryabhaṭa cipher is the Kaṭapayādi system that is believed to have been devised by Vararuci (4th Century A.D.). It is also described in one of the extant manuscripts of the Laghu Bhāskarīya of Bhāskara I (629 A.D.), and in Grahacāranibandhana of Haridatta (683 A.D.). It was known in several variants and used to represent numbers as words. Again, as text could be converted into a number sequence one may thus talk about a Kaṭapayādi or Vararuci cipher.

In the Kaṭapayādi cipher the numerals are mapped into different letters. The conjoint vowels have no numerical significance, and in a conjoint consonant only the last one denotes a number. This provides a much greater flexibility in converting numbers into meaningful words than does the Āryabhaṭa cipher. The mapping table is shown below:

Numeral	Letters	
1	k, t, p, y	
2	kh, th, ph, r	
3	g, d, b, l	
4	gh, dh, bh, v	
5	n, n, m, ś	
6	c, t, ș	
7	ch, th, s	
8	j, d,	
9	jh. dh	
0	ñ, n, and vowels by themselves	

The words are written starting from the right. Thus the number 644 may be coded in many ways to give words such as *bhavati* (bha: 4, va: 4, ti: 6), *vibhata*, and so on. By choosing other numeral/letters mappings other *Kaṭapayādi* type ciphers are obtained.

VI

Amongst early codes the Śiva sūtras of Aṣṭādhyāyī are most remarkable. It has been argued that these sūtras were anterior to Pāṇini, but in any case their structure is closely linked to Pāṇini's grammar. The sūtras are listed in Figure 1.

Number	Speech-sound	Anubandha(end-marker)
1	aiu	ņ
2	ŗ ļ	k
3	e o	'n
4	ai au	c
5	hyvr	ţ
6	1	ņ
7	ñm'nņn	m
8	jh bh	ñ
9	gh dh dh	ş
10	j b g d d	ś
11	kh ph ch th th c t t	v
12	k p	y
13	śṣs	r
14	h	1

Figure 1. The Siva sutra

The letters a,i,u are, on the use of the Siva $s\bar{u}tra$ code, listed as a^{η} . The entire set of letters is represented as a^{l} , and so on. The Siva $s\bar{u}tra$ code is certainly a mnemonic device. The manner in which the letters are broken into the fourteen sets has important grammatical reasons. The $s\bar{u}tras$ are contraction/summation $(pratyah\bar{u}ra)$ rules.

A Siva sūtra type rule lends itself to use in conjunction with a cryptographic transformation for added security. The anubandha letters may be repeated to

distinguish them from the list letters. A subset of a given list can thus be represented very succinctly and further coded by a substitution, Āryabhata, or Vararuci cipher.

VII

The preceding sections show the great interest of Indians in codes and ciphers. It is not known whether they developed a systematic science of cryptography, however. It is hoped that this article would spur interest in the writing of a systematic account of the contributions of Indians to the subject of cryptology.

REFERENCES AND NOTES

- ¹ Kahn, D., *The Codebreakers*, New York, 1967 presents a brief outline of the history of cryptography in India. An authoritative account of this history is yet to be written, however.
- ² Kak, S.C., The Aryabhata Cipher, Cryptologia, 12, 113-117, 1988.
- ³ Datta, B. and Singh, A.N., History of Hindu Mathematics, Bombay, 1962; Shukla, K.S. and Sarma, K.V., Āryabhatīya of Āryabhata, New Delhi, 1976.
- ⁴ Datta and Singh, "History", p.71.
- ⁵ Sarma, K.V., A History of the Kerala School of Hindu Astronomy, Hoshiarpur, 1972.
- ⁶ Vasu, S.C., The Astādhyāyī of Pānini, Delhi, 1981.