Indian Journal of History of Science, **24**(3) : 163–180 (1989)

SYSTEM THEORY IN JAINA SCHOOL OF MATHEMATICS–II

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(Received 29 September 1986 ; after revision 7 December 1987)

The first paper on the System Theory in Jaina School of Mathematics by the first author appeared in the *IJHS* (1979). It gave a brief survey of the inherent basic concepts and principles of the theory in ancient and working symbolism and terminology, related in texts on functional *(karma)* system. Now this system theory is being studied for its history and several aspects in details and is being reported in a series of papers.

In the present paper the same symbolism, abbreviations, notations, *etc.*, are followed as was the case in the first paper. Some additional symbolism is being given from some recent publications and certain old manuscripts. The operational symbolism, specifically found in old manuscripts, has been given in its original form which is not found in the printed texts in which some symbols have been deformed, perhaps for the sake of printing convenience.

Further, the place-value system, specially for subtraction, has been analysed for its history in view of certain new findings.

ADDITIONAL ABBREVIATIONS AND SYMBOLS

In what follows, we adopt the same system of abbreviations and ancient as well as working symbolism that was set out in the first paper.[1] However, some more abbreviations and symbols are needed in addition to those in the earlier paper, which are given below.

Recent Editions of Relevant Ancient Works

Period	*Contributor*	*Work*	*Abbreviation*
c. 473 A.D.	Yativṛṣabhācārya	i) *Tiloyapaṇṇattī*[2] vol.I	TP(1)
c. 850 A.D.	Mahāvīrācārya	*Gaṇitasāra Saṃgraha*[3]	GSS
c. 11th century A.D.	Nemicandra Siddhānta-Cakravartī	*i) Gommatasāra*[4] *(Jīvakāṇḍa)* vols. I and II	*GJK(1)*
		ii) Gommaṭasāra[5] *(Karmakāṇḍa)* vols. I and II	GKK(1)
		iii) Labdhisāra[6] *(Kṣapaṇāsāra garbhita)*	LS(1)
		iv) Labdhisāra-kṣapaṇāsāra[7]	LS(2)
		v) Trilokasāra[8]	TS(1)

Additional Ancient and Working Symbols[9]

Term	*Ancient symbol*	*Synonym*	*Working Symbol*
Ābādhā		Time-lag	1_t
Antaḥkotā koti Sagaras	आ अं को २ सा	Inter-crore-squared-sea	$((10^7)^2)_*C$
Asaṃkhyāta or *Asaṃkhyeya* (in different context)	रि	Innumerate or Innumerable	a'
Asaṃkhyāta Loka (in different context)	i) ९ ii) ७	Innumerate Universe Innumerate Universe	a" a"'

(Note : The symbol for *asaṃkhyeya* or *asaṃkhyāta* has been given in printed texts

from 1919 in the form whereas in various manuscripts of the relevant texts it appears as 

Āvali (in different contexts)	i) 	Trail	R'
	ii)		R"
Karma-sthiti		Functional-life-time	K
Saṃkhyāta or *Saṃkhyeya* (in different contexts)	i)	Numerate	s'
	ii)		s"

(*Note :* In printed texts s appears as whereas in manuscripts as).

OPERATIONAL SYMBOLISM

Jain (1967)[10] has already introduced the symbolism in the Jaina School of Mathematics. Its trace of development may be found in the *Tiloyapaṇṇattī* and very rarely in the Dhavalā texts in algebraic gauge *(artha)* form. The full development may be seen in the *Karṇāṭa Vṛtti,* Jīvatattvapradīpikā, and the Samyakjñānacandrikā commentaries of the *Gommaṭasāra.* Similarly, the same may be seen in the Sanskrit *Vṛtti* and the Samyakjñānacandrikā (bhāṣā ṭīkā) commentaries of the Labdhisāra, and the Samyakjñānacandrikā commentary of the Kṣapaṇāsāra. A good amount of symbolism may also be found in the Santakammapañjiyā given as an appendix of the Dhavalā Text, vol. xv. Undying credit goes to Ṭoḍaramala (*c.*1833-1881 Vikrama Samvat, or *c.*1719-1768 A.D.) who wrote gauge-symbolism (artha-saṃdṛṣṭi) chapters on the of the Gommaṭasāra and the Labdhisāra (inclusive of the Kṣapaṇāsāra) in Rājasthānī Ḍhūṇḍhārī language, a dialect of Hindi. He tried to explain practically every difficult result given in the earlier commentaries in ancient symbolism of mathematical display, without the help of any teacher.[11]

Here we give a sketch of the remaining symbolism after that given in the first paper. It is to be noted that sometimes the same symbol has been used to denote more than one term. Thus one has to be careful in assessing correctly the value, on the basis of the context alone.

The first press publication of the detailed commentaries of the Gommaṭasāra and the Labdhisāra came out of the Hari Bhāī Deokaraṇa Gāndhī, Granthamālā, publisher of Calcutta, in a holy way, making use of cloth cylinders, possibly owing to opposition in the community. This was about 1919 A.D. Some of the symbols were deformed, or rather shortened for the convenience of type casting. So also

had been the case of some types used in the Tiloyapaṇṇattī published as vol.I(1943) and vol.II(1951). The same convention was followed by the authors of and editors of later publications.[12]

As such in the following description of operational symbols we are giving such symbols separately from one of the manuscripts. The manuscript from which we are reproducing such symbols was found at the Pārśvanātha Digambara Jaina Khaṇḍelavāla Bīsā pantha Mandira, Maṇḍī Kī Nāla, Udaipur (Rajasthan). It seems to have been written in 1778 A.D., ten years after the death of Ṭoḍaramala of Jaipur. The name of the scribe is Sāhā Amaradāsa Jñānti Citorā. Pratapgarh. There was the reign of Maharana Bhim Singh at Udaipur.[13]

First of all there is a description of the Kaṭapayādi system in the following verse of the Arthasaṃdṛsti chapter of the Gommaṭasāra :[14]

"kaṭapayapurasthavarṇairnavanavapañcāṣṭakalpataiḥ kramaśaḥ /
svarañanaśūnyaṃsaṅkhyāmātroparimākṣaraṃ tyājyaṃ //"

This appears to be the same as the first variant of the system. The numbers like those of the vyavahāra palya, the developed human, the number of syllables in the scripture, *etc.* are given in this system.[15]

We shall refer to the pagination from the manuscript available at Hanumantal Ward, Jabalpur, Digambara Śrī Pārśva Nātha Jaina Baḍā Mandira, as it has been convenient to procure it for this purpose. However, it does not mention the name of the scribe nor the date of its writing. It has the same symbols as the former manuscript.[16]

In the following we brief the remaining symbolism from the Artha-saṃdṛṣṭi chapter of the Gommaṭasāra :

Addition (Saṅkalana) :[17]

The word saṅkalana denotes the addition of a set with another set. Here the word for set is rāśi in the manuscript. Whatever is to be added to a set is to be written above the set. Alternately, after having written the additively positive set above the original set (mūla rāśi), the addition is denoted by the shape of a tail (puñchadi).

For example, the symbol for a lac as increased by unity is  or 

The universe point set (loka pradeśa rāśi) as increased by two is 

The symbol for the infinite as increased by the universe-cubed point-set (ghana-loka-pradeśa-rāśi) is 

Note that the infinite here is the constructed proper infinite which is used to give the proper value-idea of the existential set.

Again, whenever two or more sets (rāśis) are to be added, the two or more sets are to be written above the original set.

For example, the original set may be taken as the set of all matter-particles (pudgala rāśi), denoted by १६ | ख

To this, when the time-fluent (kāla-dravya) whose measure is the same as that of the universe-point-set (loka pradeśa rāśi), as also the aether-fluent (dharma-dravya) and the anti-aether (adharma-dravya) sets are added one gets the following symbol

Further, a set which is slightly greater than the original value of its own, is denoted by placing a vertical bar above the set.

For example, the set slightly greater than its original value in case of infinite is denoted by

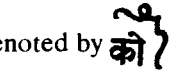
Ordinarily, whenever two or more sets are to be mixed, two or more vertical lines are placed above the original set. For example, when two sets are to be mixed with the numerate, the addition is denoted by 

In the Tiloyapaṇṇattī, however, the word dhaṇa has been used as a symbol for addition.[18]

In the printed books, the tail has been reduced to the size of a horizontal bar. For example, log of log of linear finger point set to the base two (aṅgula pradeśa rāśi varga śalākā rāśi) as increased by unity is denoted by[19]

Substraction (Vyavakalana) :

The word vyavakalana denotes the subtraction of a set. Whatever set (rāśi) is to be subtracted, is written above the original set and from it the tail (pūñchaḍī) is attached and a small circle (bindī) is placed above.

For example, the symbol for a crore (koṭi) as decreased by unity is denoted by 

The symbol for the infinite as decreased by unity is

The symbol for the universe cubed (ghana loka) as decreased by two is denoted by 

Sometimes the quantity to be subtracted from the original set is written below and a small circle (bindī) is placed in between the two sets. For example, the lac (lakṣa) as decreased by unity is denoted by ल
०
१

Further, the symbol for subtraction is also denoted by a horizontal bar placed in between the original set and the set to be subtracted from it. For example, a lac (lakṣa) as decreased by two is denoted by the expression ले-२ ल ० २

Sometimes a symbol ~ is written after the original set after which the set to be decreased is written. For example, a crore as decreased by two is denoted by

को ~ २ or को ~

The same expression is also denoted with the help of a small circle as follows

को ०
२

Further, whenever a small set is to be subtracted from an original set, it is usual to call the original set as slightly less, and the result is denoted by placing a horizontal bar after the original set. For example, slightly less than infinite is denoted by ख —

Whenever two or more sets are to be subtracted from an original set, two or more horizontal bars are placed after the set. For example, the set of the worldly souls as reduced by the two-three-four-sensed (vikalendriya) set as well as the five-sensed (sakalendriya) set, gives the one-sensed (ekendriya) set denoted by १३ =

Sometimes the crescent shape is also placed in between the sets as shown below. For example, a lac (lakṣa) as reduced by five is denoted by

ल ⌣ ५ or ल)
५

Further, the logarithm of pit (palya) to the base two as reduced by the log of log of log of pit (palya) to the base two is denoted by[20]

छे || व || छे
⌣

In the *Dhavalā* texts, the + symbol, called the foot of crow (kāka pada) appears for denoting subtraction.[21] This symbol also appears in the Bakhśālī Manuscript,[22] whereas yu appears for addition.[23]

In the Tiloyapaṇṇattī, ri or riṇa has been used to denote subtraction.[24] As the symbol + appears in the Dhavalā which was composed later, it is evident that this symbol could take this shape from the Brāhmī or later style of writing riṇa or its usual abbreviation, ri.[25]

The printed texts have removed the total tail symbol with a small circle above. They have adopted only a horizontal bar alongwith a small circle or small crescent above this bar.[26]

Apart from this, there is no total small circle above the bar. The tailed symbol can easily be correlated with the writing of riṇa and dhaṇa in the old script, reaching the period of the Nānāghāṭa inscriptions.[26(a)]

Multiplication :[27]

The multiplier[27] is written after the multiplicand. For example, lac multiplied by 5 is written as . *Palya* as multiplied by numerate (saṃkhyāta) is written as 

When the above is multiplied by the innumerate we write  When a trail (āvali) is multiplied by numerate we have This also denotes inter-muhūrta (antar-muhūrta). If it is again multiplied by numerate, one gets 

(Note that in the *MSJ*,  has been written in place of)

The finger-cubed (ghanāṅgula) as multiplied by numerate is given by 

(Note that the numerate, the innumerate, and the infinite are to be treated as variables.)

Universe as multiplied by innumerate is given by Here the universe denotes the point-set.

The set of living-beings (Jīva-rāśi) as multiplied by infinite is denoted as 

The term for multiplication is Guṇana and its various forms are used as guṇakara (multiplier), guṇya (multiple).

Division :[28]

Usually the word used for division is bhājana, or bhāga. Another word used is *apahṛta* through the rationale of subtraction. Viralana signifies this through distribution.

The dividend (bhājya) is placed above the divisor (bhāgahāra) which is placed below the former.

For example, a crore as divided by 5 is denoted by 

The pit (palya) as divided by numerate is denoted by 

The pit (palya) as divided by innumerate is denoted by 

The innumerate part of finger (aṅgula) is 

The innumerate part of trail is 

The innumerate part of finger-cubed (ghanāṅgula) is 

The innumerate part of cubic-universe (ghanaloka) is 

The numerate part of universe-line (jagaśreni) is 

(Note that there is mistake in the manuscript in writing numerate as innumerate.)

The infinitesimal part of the set of all living-beings is 

The infinitesimal part of the set of omniscience (Kevala Jñāna) is 

Similarly, the numerator and the denominator are written, the position of the former being above the latter. For example, one-third part is written as half the part is written as  three-fourth part is written as  .The combination of an integer and a fractional part are written both ways : For example, one and half is written as  and being the addition effected by the method of least common multiple of the denominators.[29]

Square :[30]

The operation of squaring is called vargita, or vaggita, meaning that a quantity is to be multiplied by itself. This is denoted by writing the term twice. For example, the square of five is written as ५ । ५ The square of vādāla is written as ४२ = ४२ =. The square of infinite is denoted by ख ख .

Similarly the square of a fraction is denoted. For example, the square of innumerate part of a finger (sūcyaṅgula) as a set of points is written as २ ।२ .
२।२

Note that *paṇṇaṭṭhi* is denoted as ६ ५ =, for 65536, and is obtained as a solution of

(4)
(2)
(2)

Similarly the vādāla is denoted by ४२ =, for the number obtained as a solution of

(6)
(2)
(2)

The ekaṭṭhī is denoted by १८ =, for the number obtained as a solution of

(6)
(2)
(2)

Here the symbol = stands for filling up the gap. It is also used for abbreviating an operational word say, jaghanya as denoted by the initial alphabet ja.[31]

Cube :[32]

The symbol for the operation of cubing is denoted by writing the number or set three times, one following the other in a row. For example, the cube of eight is written as ८।८।८ .

The cube of paṇṇaṭṭhī is denoted as ६ ५= ६ ५=६ ५= .

The cube of innumerate is written as २ २ २ .

The cube of numerate part of finger-cubed (ghanāṅgula) is written as ६।६।६ .
७।७।७

For square kṛti has also been used. Similarly cube is denoted by the word ghana. The world-line (jaga-śreni) is denoted by a single horizontal line—. The world-surface (jagapratara) is denoted by placing another horizontal bar just below the

first line or bar as = .This represents a geometrical figure denoting a set of points contained in a square of which one side is the measure of a world-line (jaga-śreṇi), which is also called seven rājūs in length.

Similarly the cube of the world-line (jaga-śreṇi) is denoted by placing one more line below the two lines or bars. This represents a set of points contained in the cube so formed geometrically. This is called cubic-universe (ghanaloka), cubing being represented geometrically, denoted by = and ≡ .

Square-root :[34]

The symbol for extraction of square-root is मू . This operation performed for one time is denoted by मू १ .The operation performed two-times is denoted by मू २ and called dvitīya-mūla. This may be carried on. The number of such operations of extracting square-root is a functional operation denoted by placing the number ahead of the symbol मू .

For such symbolization, the semantic representation in words may be illustrated by the following verse from the Ṣaṭkhaṇḍāgama[35] quoting the quarter-measure (kṣetra-pramāṇa) of the human illusive-visioned (mithyādṛṣṭi) bio-set (jīva rāśi) :

"khetteṇa seḍhīe asaṃkhejjadibhāgo/ tisse seḍhīe āyāmo
asaṃkhejjajoyaṇakoḍīo / maṇusamicchāiṭṭhīhi rūvā,
pakkhittaehi seḍhī avahiradi aṅgulavaggamūlam tadiya
vaggamūlaguṇideṇa //42// "

Translation :

"Relative to quarter-measure (kṣetra-pramāṇa) the human illusive-visioned bio-set (mithyādṛṣṭi jīvarāśi) is innumerate part of the world-line (jaga-śreṇi). The length of that śreṇi is innumerate crore yojanas. On multiplying the first square-root of the linear-finger (sūcyaṅgula) by its own third-square-root, the product is established through counting-rods (śalākās). Then the world-line (jaga-śreṇi) is found to be exhausted by the human-illusive-visioned bio-set (mithyā-dṛṣṭi jīva-rāśi) which is unity in excess of the thirteen control-stationed (guṇa-sthāna-vartī) bio-set."

Thus the geometrical representation of sets and the symbols for square-root and so on have a deep significance in the history of mathematics during the early centuries of the Christian era. The abbreviation through the initial accented alphabet of the sets (rāśis) had already begun to appear quite early, as may be seen in *Mahābandha* and *Tiloyapaṇṇattī*.

Cube-root :[36]

The meaning of cube-root (ghanamūla) is finding the number of which the given

number is the cube. Indians were already acquainted with this operation. Usually, factorization method may be seen for effecting this operation, for example, in the treatment of divergent sequences which were in dyadic form alongwith cube and non-cube forms.[37] The operation of cube-root thus appears to have been in use in India in the early centuries of the Christian era. Āryabhaṭa, Mahāvīrācārya and Ṭoḍaramala give its description, for finding the cube-root of a number.[38]

The above operations are called parikarmāṣṭaka.

Place-value-operators :

(a) Small circle has been called in Indian ancient relevant texts as śūnya, sunna, bindu, or bindī. Moreover, the symbol of the vowel i attached to a consonant has an important bearing for the symbol of riṇa (minus). Ojha[39] mentions about a small inscription of pre-Aśokan period found at a village Barali, Ajamera (Rajaputana Museum, Ajmer). It is a piece of an inscription from a pillar, in which in the first line there is written, "vira(i)ya bhagava(ta)", and in the next line there is written,

"caturāsitiva(sa)".

The eighty-fourth year of inscription seems to be related with the Jaina's *Vīra Nirvāṇa Saṃvat*, which may be 527 B.C. or still earlier. Ojha and Vidyabhusana, both have speculated the inscription year as 443 B.C. Its script appears to be earlier than that of Aśoka. It contains the accented alphabet *Vī* 

of *Vīrāya*. Thus the mixed symbol  has a deep significance in so far as writing of *vindu* as well as the negative sign is concerned.

In the *Mahābandha* we find all the seven volumes written in an unusual style of an abbreviation operator. A majority of words have been abbreviated with the use of small circles all through. For example,

Transcription :

52. *veuvviyamissa* o- *pañcaṇā* o *ṇavadaṃsa* o *micchatta* o *solasaka* o *bhayaduguṃ* o *orāliya-tejāka* o *vaṇṇa* o 4 *agu* o 4 *bādara-pajjatta-patteya* o - *ṇimiṇa-titthayara pañcanta* o *jahaṇṇu* o *anto* o / *sesāṇam sādādiṃari jaha* o *ega* o *ukka* o *anto* o /

Translation :

52. In the *vaikriyika* (transforming) mixed *(miśra)* body volition *(yoga)*, the minimal maximal bond-period of five knowledge-screening *(jñanāvaraṇa)*, nine vision-screening *(darśanavaraṇa)*, illusion *(mithyātva)*, sixteen affections *(kaṣāyas)*, fear *(bhaya)*, aversion *(jugupsā)*, macro-phosphorescent-karma-finishing body *(audārikataijasakārmāṇa śarīra)*, colour *(varṇa)* 4, non-

gravity-levity *(agurulaghu)* 4, gross *(bādara)*, developed *(paryāpta)*, every *(pratyeka)*, growth *(nirmāṇa)*, ford-founder *(tīrthaṅkara)*, and five interferences *(antarāyas)* is inter-muhūrta *(antarmuhūrta)*. The bond period of the remaining configurations *(prakṛtis)*, *sātā*, *etc.*, is an instant as minimal and an intermuhūrta *(antarmuhūrta)* as maximal.[40]

Jain has already contributed a paper giving many examples of the small circle as an operator for filling up gaps and for place-value notation as well as for operation of subtraction or negation.[41] The operation of the pole-set *(dhruva rāśi)* technique was used in the Jaina School of astronomy[42] where numbers or sets were kept fixed and other numbers or sets were rotated to find out the periodicity of astronomical events. It also seems that the pole-set system and the axis-propagation *(akṣa-sañcāra)* system of the permutation-combination *(bhaṅga-vicaya)* made the Jaina School to be well-versed also in the application of the place-value system for it was needed by it in the presentation of their functional *(karma)* theory quite long before the Christian era. This attempt may clearly be seen through an example[43] for the place-value operator in the process of subtracting numbers out of factors.

Here ल॥५॥४॥३ denote the product *la* ×5×4×3.

If ल or one lac is subtracted from the above, then the remaining quantity is written as ल ५ ४ ३

The quantity less five lacs is denoted as ल ५ ४ ३

The quantity less twenty lacs is denoted as ल ५ ४ ३

The quantity less three lacs is denoted as ल ५ ४ ३

The quantity less twelve lacs is denoted as ल ५ ४ ३

The quantity less fifteen lacs is denoted as ल ५ ४ ३

The quantity less thirty lacs is denoted as ल ५ ४ ३

Further ल॥५॥३ less twelve is denoted as ल ४ ३

It may be noted that in the printed texts the above symbols for the place-value operation of subtraction have been cut short in so far as the tail *(pūñchaḍī)* has been cut out and the use of two tails in the same expression is concerned, as given in the *MSJ*.

It is also important to note that according to Neugebauer[44] a strange sign  signifying 90 appears in Ptolemy's *Almagest.* Owing to its resemblance of shape with that of the earlier quoted *Vī* or  in an inverted form, there appears to be some relation between the symbols as 90 was called in India as *navve* or *ṇavasaṇa.*

There is one more point to be noted regarding the writing of numbers in the place-value notation through alphabets in Greece.[45] For example,  =11,  =12. Similarly, in the Jaina School the same style is followed and the various combinations of the 64 alphabets forming numbers of syllables of various topics of the scriptures *(śruta)* are quoted from the *Āgama* or Revelation.[46] The system followed is the *Kaṭapayādi* in the first and the fourth variant. for example,[47]

"talālīne madhugavimalaṃ dhūmasilāgāvicora bhayamerū /
tataharikhajhasā honti hu māṇusapajjattasaṃkhaṅkā //158//

denotes the number of developable human beings as

79 22 81 62 51 42 64 33 75 93 54 39 50 336.

The same number is denoted in other variant of the *Kaṭapayādi.*

"sādhūrarāja kīṭereṇāṅko bhāratī vilolaḥ samadhiḥ /
guṇavargga dharmmanigalitasaṃkhyāvanmānaveṣu varṇakramataḥ //"

This variant is called the *pūrvānupūrvī.*

The style of Ptolemy's *Almagest* may be seen in words in the *Kasāya Pāhuḍa* as follows :

"aṭṭhāvīsa cauvīsa sattarasa solaseva paṇṇarasā /
ede khalu mottūṇaṃ sesāṇaṃ saṅkamo hoi //27//"[48]

Similar style may be seen in the *Ṣaṭkhaṇḍāgama,* as follows :

"ukkasseṇa sāgarovam palidovamaṃ sādireyam ve satta dasa
coddasa solasa aṭṭhārasa sāgarovamāṇi sādireyāṇi //96//[49]

Translation of the above two verses is as follows :

"Twenty-eight, twenty-four, seventeen, sixteen and fifteen natural stations are unfit for transition as per rule, hence leaving these five non-transition stations there is transition of remaining twenty-three stations."//27//

"The maximal period is in excess over one *sāgaropama,* one *palyopama,* two *sāgaropamas,* seven *sāgaropamas,* ten *sāgaropamas,* fourteen *sāgaropamas,* sixteen *sāgaropamas,* and eighteen *sāgaropamas."//96//*

The examples show that the place-value system was a necessity in the Jaina School of mathematics, which they might have, in good probability, used alongwith

other methods originally invented in the School itself in the period a few centuries before the Christian era, at least in words, if not in numerals.

In the *Labdhisāra* one finds a number, numerate-thousand in place-value notation as showing that it represented a variable in thousands, a unique use of the system.[50]

Logarithmic Values Symbols :

Singh had already detailed the rules of logarithms in the Jaina School of Mathematics,[51] after Kapadia had recognized the *arddhacceda* as the logarithm of a number or set to the base two. He found them sufficient for the need. However, in the *Dhavalā*, they appear in semantical form. In the *Gommaṭasāra* and the *Labdhisāra*, symbolism appears in abruptly abundant form in their commentaries, as if burst forth from some hidden sources as yet unknown upto the period of the *Dhavalā*. We directly get the values of logarithms to the base two *(arddhacchedas)* and the logarithms of logarithm to the base two *(vargaśalākās)* of a few sets in terms of the logarithms of pit *(palya)* to the base two. These values can easily be calculated from the relations between the *palya* and the sets described below, from the *Tiloyapaṇṇattī*:[52]

logarithm of sets to the base two	ancient symbolism	working symbolism
logarithm of pit *(palya)*		$\log_2 P$
log of log of pit *(palya)*		$\log_2 \log_2 P$
log of sea *(sāgara)*		$\log_2 C$
	-actually it should be	
		
log of log of sea *(sāgara)*	stated as not ?	$\log_2 \log_2 C$
log of linear finger *(sūcyaṅgula)*		$\log_2 F$ or $\log_2 \log_2 P$
log of log of linear finger *(sūcyaṅgula)*		$\log_2 \log_2 F$ or $2 \log_2 \log_2 P$
log of finger-squared *(pratarāṅgula)*		$2 \log_2 F$ or $2 \log_2 P \log_2 P$
log of log of finger-squared *(pratarāṅgula)*		$\log_2 \log_2 (F)^2$ or $1+2 \log_2 P \log_2 P$

log of finger-cubed *(ghanāṅgula)* — छे छे छे ३ — $\log_2 (F)^3$ or $3 \log_2 P \log_2 P$

Note : It should be छे छे ३

log of log of finger-cubed *(ghanāṅgula)* — व २ — $\log_2 \log_2 (F)^3$ or $\log_2 3 + 2 \log_2 P \log_2 P$

log of world-line *(jaga śreṇi)* — छे छे छे ३ or वि छे छे ३ (g) — $\log_2 P \; 3 \log_2 P \log_2 P$ (a)

spread set *(viralana rāśi)* or broken up set — वि — b

set to be distributed *(deya rāśi)* — दे — d

Note : The spread set and the set to be distributed are usually equal, and the set to be operated upon so spread, is to be mutually multiplied into itself as many times as is the set so spread. This process is also called *vargita-saṁvargita.*[53]

log of log of world-line *(jaga śreni)* — व / १६ । २ / व २ — $\log_2 L$ or $\frac{\log_2 P \log_2 P}{2\ a^*} + \log_2 \log_2 (F)^3$

Note : Here a* and after what follows, this symbol will represent minimal-peripheral-innumerate *(jaghanya parīta asaṁkhyāta)*[54]

log of world-surface *(jaga-pratara)* — छे छे छे ६ or वि छे छे ६ (g) — $\log_2 (L)^2$ or $\frac{\log_2 P}{a} + 6 \log_2 P \log_2 P$

log of log of world-surface *(jaga-pratara)* — व / १६ । २ / व २ — $\log_2 \log_2 (L)^2$ or $1 + \frac{\log_2 \log_2 P}{2\ a^*} + \log_2 \log_2 (F)^3$

log of world-cubed *(ghanaloka)* — छे छे छे ६ or वि छे छे ६ (g) — $\log_2 (L)^3$ or $\frac{\log_2 P}{a} (9 \log_2 P \log_2 P)$

log of log of world-cubed *(ghanaloka)* — व / १६ । २ / व २ — $\log_2 \log_2 (L)^3$ or $\log_2 3 + \frac{\log_2 \log_2 P}{2a^*} + \log_2 \log_2 (F)^3$

Note that the symbol for the innumerate in *MSJ* is slightly different here from that of *MSU* in that the lower loop is not complete here. Further 'nine' is slightly differently written in *MSU.*

CONCLUDING REMARKS

There may have remained some more symbols which will be shown whenever and wherever needed. The above symbolism as also in the first paper may give an idea of how various types of symbols were used in the functional *(karma)* system theory in the *Digambara* Jaina School of Mathematics. This school worked with the least number of symbols as far as the context could allow it to be used for as many expressions as possible. The credit of differentiating them in his *Artha-Saṃdṛṣṭi* chapters goes to the last worker on symbolism, Ṭoḍaramala of Jaipur, of the 18th century A.D.

ACKNOWLEDGEMENT

The authors are grateful to Dr. A.K. Bag for taking interest in this research and guiding a systematic approach to materialize the programme financially supported by the Indian National Science Academy, New-Delhi, on the *Labdhisāra* of Nemicandra Siddhāntacakravartī. The first author is obliged to the INSA for the financial support. Thanks are due to Dr. S.C. Datta for providing research facilities, and to the referee for important suggestions to improve upon this paper. Co-operation is also acknowledged with obligations of Dr. R.E. Kalman.

REFERENCES AND NOTES

1. Jain, L.C., System Theory in Jaina School of Mathematics, *Indian Journal of History of Science*, **14**, 31-65, 1979. For an easy introduction, *cf.* Jain, L.C. and Jain, A., Philosopher Mathematicians, DJICR, Hastinapur, 1985.
2. Tiloyapaṇṇattī, vol. 1, commentary by Āryikā Viśuddhamatī, (ed.) C.P. Patni, Kota, 1984.
3. Gaṇitasāra Saṃgraha of Mahāvīrācārya, (ed.) L.C. Jain, Sholapur, 1963.
4. Gomatasara (Jīvakāṇḍa) with *Karṇāṭavṛtti* (abbr. *KVR*), Sanskrit Ṭīkā Jīvatattvapradīpikā (abbr. *JPT*) and Hindi Translation based on the Samyakjñānacandrikā Ṭīkā (abbr. *SJC*) of Ṭoḍaramala, by A.N. Upadhye and K.C. Shastri, New-Delhi, vol.1, 1978 ; vol.2, 1979.
5. Gommatasāra (Karmakāṇḍa), Upadhye and Shastri, New Delhi, vol.1, 1980 ; vol.2, 1981.
6. Labdhisāra (Kṣapaṇāsāra-garbhita) with *SJC* of Ṭoḍaramala, [abbr. *LS(1)*], (ed.) P.C. Siddhantashastri, Agas, 1980.
7. Labdhisāra-Kṣapaṇāsāra, edited with Hindi commentary and trans., by R.C. Mukhtar, Shri Mahaviraji(Raj), 1984, [abbr. *LS(2)*].
8. Trilokasāra with Sanskrit Ṭīkā of Mādhavacandra Traividya and Hindi Ṭīkā of Āryikā Viśuddhamatī, (ed.) R.C. Mukhtar, Shri Mahaviraji(Raj), 1975, [abbr. *TS(1)*].
9. Additional symbols have occured in the *GJK*, *GKK* and *LS*. In order that they may not be confused, separate working symbols have been given. *Cf.* also Trilokasāra with commentary of Ṭoḍaramala, (ed.) M.L. Shastri, Bombay, 1918, for some mathematical symbolism and processes.
10. Jain, L.C., On the Jaina School of Mathematics, Choṭelāla Smṛti Grantha, Calcutta, 1967, pp. 265-292.
11. Jain, L.C., Mathematical Contribution of Ṭoḍaramala of Jaipur, The Jaina Antiquary, vol.XXX, no.1, 1977, pp.10-23. Cf. also Jain, L.C., Elements of Operational Details in the Labdhisāra, (sketched by Ṭoḍaramala), The Jaina Antiquary, vol.30, no.1, 1984, pp.21-32.

[12]. *Cf.* the printed works *GJK, GJK(1), GKK, GKK(2), LS, LS(1)* with manuscripts detailed below, for symbolism, for example, those of addition, subtraction, asaṃkhyāta, *etc.*

[13]. This manuscript will be denoted by *MSU.* This is bound but paper has become very weak.

[14]. *Cf. AS-I,* p.2.

[15]. Datta, B.B. and Singh, A.N., History of Hindu Mathematics, parts i and ii, Bombay, 1962, pp.69-72. This may be compared with other three variants detailed by the authors. Cf. *GJK(1),* pp.286-287. The verse of *SK,* book 13/5,5,46 :

"saṃjogāvareṇaṭṭham causaṭṭhiṃ thāvae duve rāsiṃ /
annonnasamabbhāso rūvūṇamniddise gaṇidaṃ // 46 //

has been expressed in *GJK(1),* vol.2, pp.599,600 through the Kaṭapayādi system.

[16]. We shall denote this manuscript by *MSJ.*

[17]. *Vide MSJ,* p.711, *et seq.*

[18]. *Vide TP,* vol.1, p.231, *etc.* Addition of a fraction with a whole number is denoted by placing a vertical bar in between the two. Vide *ibid.,* vol.2, p.771, *et seq.*

[19]. Vid., *AS-I,* p.6, *et seq.*

[20]. Vid., *MSJ,* p.71ŀ, *et seq.*

[21]. Vid., *SK,* book 10, 4/2/4/32/,p.151. The symbol is stated thus :

". . . . . sojjhamāṇādo edisse riṇasaṇṇā /"

[22]. Datta, B.B., The Bakhśālī Mathematics, Bull. Cal. Math. Soc., **XXI,** 1-60, 1929.

[23]. *Cf. ibid.,* same pages.

[24]. *Cf. TP,* vol.1, p.20 ; vol.2, p.769, where the following is stated :

'"— 7 riṇa 1006ł riṇa daṇḍa 425 ri va 1 /

[25]. *Cf.* ref. 15, part ii, p.14. *Cf.*also ref.10. *Vide* also Bag, A.K., Mathematics in Ancient and Medieval India, Varanasi, 1979, p.85 for various remarks. Note that the tail (pūñchaḍī) and a small circle above was the form at the time of Ṭodaramala to denote this operation of subtraction. It was probably originated at the time of Nemicandra Siddhāntacakravartī, or still earlier, to replace the *riṇa* word by some convenient symbol, to separate out a negative set. This conclusion may be drawn, for the time being, until some earlier text with so extensive symbolism appears.

[26]. *Vide* editions of *GJK, GKK,* or *LS.*

[27]. *Vide MSJ,* leaf 712(i).

[28]. *Vide ibid.,* leaf 712(i) and 1(ii).

[29]. *Vide ibid.,* leaf 712.

[30]. *Vide ibid.,* leaf 712(ii).

[31]. *Vide ibid.,* leaf 709(i).

[32]. *Vide ibid.,* leaf 712(ii).

[33]. *Vide TP* vol.1, p.16, v.1/132. *TP* denotes the Tiloyapannattī, vol.1(1943), vol.2(1951), ed. A.N. Upadhye and H.L. Jain, Sholapur.

[34]. *Vide MSJ,* leaf 712(ii), and 713(i).

[35]. *Vide SK,* 1/2/42.

[36]. *Vide MSJ,* leaf 713(i).

[37]. *Vide TS,* vv.66-90.

[38]. *Cf.* ref. 3, p.18. *Cf.* also ref. 9, Pariśiṣṭa of the Trilokasāra, pp.9-10.

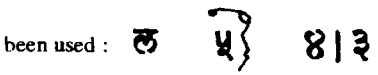
[39]. Ojha, G.H., *Bhāratīya Prācīna Lipimālā,* Delhi, 1959, pp.2, 47.

[40]. Vide MB. vol.1, v.52, p.55.

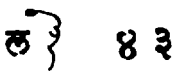
[41]. *Cf.* ref. 10, *loc. cit.*

[42]. Research paper accepted for publication in the *Proc. of IAU,* Colloquium 91, 1985.

[43]. *Vide MSJ*, leaf 716, and leaf 717(i). *Cf.* also *MSU*, the introductory prelude to the Artha-Saṃdṛṣṭi of the Gommaṭasāra. Note that in the *MSU* there is use of only one tail *(pūñchaḍī)*, and the small circle is quite clearly shown. For example, for subtracting twelve lacs from the product of

that is, factors, then the following symbolism has been used : 

Similarly when 12 is to be subtracted from the product the following symbolism has

been used : 

Further, if innumerate is to be subtracted from the product of the innumerate universe, then the following symbolism has been used : 

[44]. Neugebauer, O., The Exact Sciences in Antiquity, Providence, 1957, pp.10-11.

[45]. *Vide ibid.*, p.10.

[46]. *Vide GJK*, vv. 360-364. *Cf.* also *GJK*(1), vol.2, p.599, p.603 and p.604.

[47]. *Vide GJK(1)*, vol.1, pp.285, 287.

[48]. *Vide KP. CS*, v.27 of ch.5, p.260.

[49]. *Vide SK* (1965), v.96 of ch.5, p.143.

[50]. *Vide LS*(1), v.77, p.60.

[51]. Singh, A.N., Mathematics of Dhavalā-I. Ṣaṭkhaṇḍāgama, vol.4, Amaraoti (Amaravati), 1942, pp.v-xxi. *Cf.* also Singh, A.N., History of Mathematics in India from Jaina Sources, The Jaina Antiquary, vol.XV, no.2, 1949, pp.46-53.

[52]. *Vide MSJ*, leaf 710(ii), leaf 711(i). *Cf.* also *MSU* for variants in the symbolism. It is advisable to confer manuscripts of the commentaries.

[53]. *Vide* Jain, L.C., Tiloyapaṇṇattī kā Gaṇita, Intr. to the Jambūdīvapaṇṇattī Saṃgaho, Sholapur, 1958, pp.1-109. For a brief survey of mathematics as applied to the *karma* system theory, *cf.* the word Ganita in the Jainendra Siddhānta Kośa, by Jinendra Varni, part (ii), New-Delhi, 1971, pp.213-234, on relevant topics.

[54]. *Vide ibid*, for other types of the number (saṃkhyā) and the simile (upamā) measures (pramāṇas).