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EVALUATION OF THE ACCURACY OF MEASUREMENTS IN INDIAN ASTRONOMY – I : SĀMANTA CANDRAŚEKHARA

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The only measured data given in all the *siddhāntas* are the co-ordinates of the *yogatārās* of the *nakṣatras* and a few other bright stars. Evaluation of their accuracy is beset with a number of problems such as misidentification, uncertainly of the epoch and controversies regarding the origin of the coordinate system. In Sāmanta Candraśekhara's work, all these three parameters are well recorded and he used the traditional technology for his measurements. So this work has been used for evaluation of the accuracy of the traditional technology. It has been shown that, barring a few exceptions, the probable error is better than ± 0°.50'.

Key Words: *Accuracy of measurements, Identification, Indian system of celestial co-ordinates, Sāmanta Candraśekhara, Yogatārā.*

INTRODUCTION

Astronomical measurements were practised in India since very early times. The *Siddhāntas* describe various kinds of instruments for this purpose. Ohashi[1] has given descriptions and uses of these instruments. The only observational results quoted in almost all the *siddhāntas* are the co-ordinates of the *yogatārās* of the *nakṣatras* together with those for a few other bright stars like Sirius, Capella, β Tauri, Canopus etc. Many western scholars have complained that the values are out by several degrees and in some cases by as much as 10°-12°. However the details of their comparisons are not clear and in many cases the identification of the *yogatārās* is disputable. Thus the *yogatārā* of *asvini* has two claimants viz. α and β Ari., *Ārdrā* has two, viz α ori and γ Gem, and *Viṣākhā* has two viz α and i Lib. The origin of the co-ordinate system is under dispute. This point is called *meṣādya* (the first point of Aries). In modern astronomy, this point, denoted γ, is the point of intersection of the ecliptic with the equator and is moving westwards along the ecliptic at the rate 50.2" per year. This movement is called precession of equinoxes. Co-ordinates measured from this point as origin are called *sāyana* co-ordinates. However the data given in the siddhāntas follow the *nirayaṇa* system in which the *meṣādya* is an arbitrary fixed point on the ecliptic which coincided with the vernal equinox of the year 285 AD. Recently Abhyankar[2] has shown that about half the co-ordinates correlate better with *revatipakṣa*

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system where the *meṣādya* coincides with the vernal equinox of the year 575 AD.

THE CO-ORDINATE SYSTEM

The Indian astronomical literature gives the positions of celestial bodies in terms of spherical coordinates called *dhruva* and *vikṣepa* or *śara*. *Dhruva* is the angular distance along the ecliptic from *meṣādya* upto the point where the declination circle through the celestial body intersects the ecliptic. *Vikṣepa* or *Śara* is the angular distance of the celestial body from the ecliptic measured along the declination circle, and is designated north or south according to the position of the object with respect to the ecliptic. *Dhruva* is measured from west to east. It will be observed that the system is not orthogonal and both *dhruva* and *vikṣepa* would be affected by the precession of equinoxes. Modern astronomy uses the equatorial system of coordinates referred to a given epoch and their conversion equations to determine these coordinates for any other year.

The transformation of the equatorial system to the Indian system is straightforward and the necessary equations are derived in Note 1. The formulae for transformation of the equatorial coordinates from one epoch to another are also given in Note 1. The calculations for the latter are performed on an annual basis until the desired epoch is reached. Thus the parameters, viz. right ascension α, declination δ and the *ayanāṃsa* A have to be known at the time of measurement of U and V, before the necessary transformations as per equations (3) and (4) of Note 1, can be performed. Thus the *Karaṇābda* or epoch to which the U and V values refer are critical for any meaningful comparison with modern data. The *Karaṇābdas* of the principal *Siddhānta literature* are not known with any degree of certainity.

THE SIDDHĀNTA DARPAṆA OF SĀMANTA CANDRAŚEKHARA

Fortunately for us, the last of the great Indian traditional astronomers was active during the late nineteenth century. He has been aptly described by J.C. Ray[3] as the Indian Tycho Brahe for his prowess in making accurate measurements with home made instruments, with which he conducted most of his measurements, called the *mānayantra* and has been fully described by Nayak[4].

Sāmanta Candraśekhara accurately measured the coordinates of the *yogatārās* in 1870 AD and recorded them in his magnum opus, the *Siddhānta Darpaṇa*. He also identified the *yogatārās* and other stars by means of positional information and sketches from which Ray identified the respective stars in terms of modern nomenclature. Table I show this data. Table II gives the equatorial coordinates of the *yogatārās* for the epoch 2000 AD, alongwith the apparent visual magnitudes. Table III gives the computed equatorial coordinates of the *yogatārās* for the epoch 1870 AD,, the *Karaṇābda* for the *Siddhānta Darpaṇa*. Table IV gives the *Dhruva* (U) and the *Vikṣepa* (V) of the *yogatārās* from the data in table III by way of equations (3) and (4) of appendix 1. These data have been rounded up to the nearest arc minute. The table also gives the respective data from the *Siddhānta Darpana* and the differences (δU, δV) in arc minutes.

Table I

Sl.No.	*Nakṣatra*	No. of stars	Position of *Yogatārās*	Identification*
1	Aśvini	3	Northenmost	α Ari
2	Bharaṇī	3	Southernmost	41 Ari
3	Kṛttika	6	Middle	η Tau
4	Rohiṇī	5	Easternmost	α Tau
5	Mṛgaśira	3	Northernmost	λ Ori
6	Ārdrā	1	–	α Ori
7	Punarvasu	5	North eastern	β Gem
8	Puṣyā	3	Middle	M44
9	Aśleṣā	5	Easternmost	ζ Hya
10	Maghā	5	Southernmost	α Leo
11	Purva Phālguni	2	Northernmost	δ Leo
12	Uttara Phālguni	2	Southernmost	β Leo
13	Hastā	5	North eastern	δ Crv
14	Citrā	1	–	α Vir
15	Svāti	1	–	α Boo
16	Viśākhā	5	Northernmost	i Lib
17	Anurādhā	7	Middle	δ Sco
18	Jyeṣṭhā	3	Middle	α Sco
19	Mūlā	9	Northeastern	λ Sco
20	Purvāṣādhā	4	Northernmost	δ Sag
21	Uttarāṣādhā	4	Northernmost	Φ Sag
22	Abhijit	3	Westernmost	α Lyr
23	Śravanā	3	Middle	α Aql
24	Dhaniṣṭhā	5	Westernmost	α Del
25	Śatabhiṣā	100	Southernmost	λ Aqr
26	Purva Bhādrapada	2	Northernmost	β Peg
27	Uttara Bhādrapada	2	Northernmost	α And
28	Revatī	32	Southernmost	η Psc

Yogatārās of *Nakṣatras* from *Siddhānta Darpaṇa*

*Introcuction to *Siddhānta Darpaṇa* by J.C. Ray (1899)

Table II

Sl. No.	Star	App. vis. mag	Declination δ			Right ascension α		
			0	'	"	h	m	s
1	α Ari	2.00	123	27	45	2	7	10.3
2	4λ Ari	3.63	+27	15	38	2	49	58.9
3	η Tau	2.87	+24	6	18	3	47	29.0
4	α Tau	0.85	+16	30	33	4	35	55.2
5	λ Ori	3.39	+09	56	2	5	35	8.2
6	α Ori	Var	+07	24	26	5	55	10.2
7	β Gem	1.14	+28	1	34	7	45	18.9
8	M44	3.10	+19	59	0	8	55	23.6
9	ζ Hya	3.11	+05	56	44	8	55	10.2
10	α Leo	1.35	+11	58	2	10	8	22.2
11	δ Leo	2.56	+20	31	25	11	14	6.4
12	β Leo	2.14	+14	34	19	11	49	3.5
13	δ Crv	2.95	–16	30	55	12	29	51.8
14	α Vir	Var	–11	9	41	13	25	11.5
15	α Boo	–0.04	+19	10	57	14	15	39.6
16	i Lib	4.54	–19	47	30	15	12	13.2
17	δ Sco	2.32	–22	37	18	16	0	19.9
18	α Sco	Var	–26	25	55	16	29	24.3
19	λ Sco	1.63	–37	6	14	17	33	36.4
20	δ Sag	2.70	–29	49	42	18	20	59.5
21	Φ Sag	3.17	–26	59	27	18	45	39.2
22	α Lyr	0.03	+38	47	1	18	36	56.2
23	α Aql	0.77	+08	52	6	19	50	46.8
24	α Del	3.77	+15	54	43	20	39	38.1
25	λ Aqr	3.74	–07	34	47	22	52	36.6
26	β Peg	2.06	+28	4	58	23	3	46.3
27	α Anl	2.06	+29	5	26	0	8	23.2
28	η Psc	3.62	+15	20	45	1	31	28.9

Equatorial coordinates of Yogatārās in AD 2000

Source : Guinness Book of Astronomy, 3rd edition

Table III

Sl. No.	Star	Declination δ			Right ascension α		
		0	'	"	h	m	s
1	α Ari	+22	50	35	01	59	28
2	41 Ari	+26	43	14	02	42	14
3	η Tau	+23	42	04	03	40	07
4	α Tau	+16	14	25	04	28	57
5	λ Ori	09	50	44	05	28	25
6	α Ori	+07	22	54	05	48	30
7	β Gem	+28	20	15	07	39	20
8	M44	+20	26	24	08	34	07
9	ζ Hya	+06	26	16	08	48	57
10	α Leo	+12	36	00	10	02	16
11	δ Leo	+21	13	43	11	08	32
12	β Leo	+15	17	31	11	43	10
13	δ Crv	–15	47	55	12	22	22
14	α Vir	–10	29	07	13	18	02
15	α Boo	+19	47	10	14	09	52
16	i Lib	–19	18	04	15	04	52
17	δ Sco	–22	15	08	15	53	04
18	α Sco	–26	08	38	16	22	11
19	λ Sco	–37	00	37	17	26	40
20	δ Sag	–29	53	03	18	14	28
21	Φ Sag	–27	07	26	18	39	16
22	α Lyr	+38	40	43	18	29	57
23	α Aql	+08	32	33	19	43	56
24	α Del	+15	27	28	20	32	28
25	λ Aqr	–08	16	02	22	46	20
26	β Peg	+27	23	11	22	55	39
27	α Anl	+28	22	10	00	01	42
28	η Psc	+14	40	37	01	24	07

Computed Equatorial Co-ordinates for AD 1870

Source : Guinness Book of Astronomy, 3rd edition

Table IV

Sl. No.	Star	*Siddhānta Darpaṇa* Dhruva 0–	U '	Vikṣepa 0	V '	Modern data Dhruva 0	U '	Vikṣepa 0	V '	δU'	δV'
1	α Ari	09	45	+10	30	09	59	+10	39	–14	–9
2	41 Ari	21	00	+11	00	20	57	+10	58	13	12
3	η Tau	35	15	+04	15	35	16	+04	08	–1	+7
4	α Tau	46	30	–05	30	46	54	–05	33	–24	+3
5	λ Ori	60	15	–13	30	60	12	–13	24	–27	–6
6	α Ori	65	00	–15	40	65	19	–16	02	–19	+22
7	β Gem	90	15	+06	30	90	57	+06	51	–42	–21
8	M44	104	00	+01	15	104	06	+01	42	–6	–27
9	ζ Hya	108	00	–12	00	107	45	–11	22	+15	–38
10	α Leo	126	00	+00	25	126	21	+00	34	–21	–9
11	δ Leo	143	30	+15	00	143	58	+15	43	–28	–43
12	β Leo	153	00	+13	00	153	22	+13	28	–22	–28
13	δ Crv	165	00	–11	00	164	03	–13	23	+57	+143
14	α Vir	179	00	–02	10	179	04	–02	15	–4	+5
15	α Boo	193	00	+33	00	192	41	+32	53	+19	+7+
16	i Lib	207	00	–02	00	206	38	–01	55	+22	–5
17	δ Sco	218	30	–02	00	218	23	–02	00	+7	0
18	α Sco	225	30	–04	15	225	18	–04	36	+12	+21
19	λ Sco	240	40	–13	30	240	18	–13	47	+22	+17
20	δ Sag	250	00	–06	30	251	16	–06	29	–76	–1
21	Φ Sag	256	30	–03	40	256	58	–03	59	–28	+19
22	α Lyr	256	30	+62	00	251	49	+61	57	–19	+3
23	α Aql	273	00	+30	00	272	02	+29	50	+58	+10
24	α Del	285	30	+36	00	283	42	+34	18	+108	+102
25	λ Aqr	317	45	–00	20	318	00	–00	28	–15	+8
26	β Peg	322	00	+32	00	320	30	+34	14	+90	–134
27	α Anl	338	00	+28	00	337	59	+28	21	+1	–21
28	η Psc	00	00	+05	00	00	41	+05	50	–41	–50

$\overline{\delta U}$ = 0.96': σ_U = 38'.9 std error = 26', $\widehat{\delta V}$ = -0.82: σ_P = 45.9; std error = 30'.8, overall std error = 28'.5

Comparison of *Siddhānta Darpaṇa* data with modern data

Statistical analysis of the error data yield the following results :–

Average :– $\delta U = 0.96'$; $\delta V = -0.82'$

Std. dev :– $\sigma_u = 38'.9$; $\sigma v = 45'.9$

Std. error :– on U = 26'.0 and on V = 30'.8

In view of the magnitude of the errors the averages have no significance.

From the magnitudes of the standard error it would appear that both the sets belong to the same population and can be merged. The standard error for the merged data is 28'.5

CASES OF EXTREME ERROR

There are three cases where the error approaches or even exceeds the 3σ limits and the probability that they are due to chance errors is small. These are. δCrV, α Del, and β Peg. At first it may look like mistaken identification by Ray. In the first two cases there are other stars. viz η crV and β Del which will bring the computed coordinates somewhat nearer the *Siddhānta Darpaṇa* data. However these identifications millitate against the descriptive identification in Table 1. Thus the source of the errors in these cases is a bit mysterious.

Sāmanta Candraśekhara was the last of the great astronomers of the ancient school and used the traditional instrumentation technology. However he was sufficiently near our times so that positive identification of the stars in terms of modern nomenclature and star charts could be made. This is not always possible with the more ancient texts and varlous authorities have given different identifications to the *yogatārās*. The above discussion shows that comparison of coordinates is not always and infalliable method of identification in cases like this. On an average a probable error level of the order of $\pm 0.5°$ can be expected in the values of stellar coordinates quoted in the Siddhānta literature.

NOTE 1. COORDINATE TRANSFORMATION

γ = first point of Aries

S = star

M = Meṣādya

U = Dhruva

V = Vikṣepa

B = Point of intersection of ecliptic with the declination circle SC through the star.

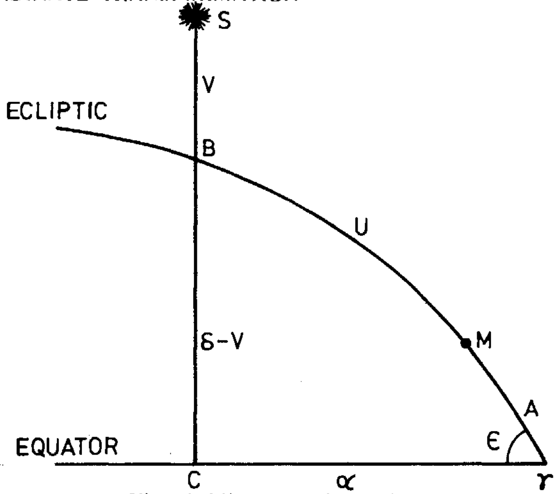

Fig. 1 Nirayana Coordinates

BM = U : Mγ – A – Ayanāṃsa; BS = V = Vikṣepa ; SC = δ = Declination

γ C = α = right ascension; $\angle C = 90°$; $\angle\gamma = \varepsilon$ = obliquity

In the spherical triangle γBC we have

by the sine rule :– $\dfrac{\text{Sin }(\delta - V)}{\text{Sin }\varepsilon} = \dfrac{\text{Sin }(U + A)}{\text{Sin }C} = \text{Sin }(U + A)$ ....(1)

and by the Cosine rule :- $\text{Cos}(U + A) = \text{Cos}\alpha\ \text{Cos}(\delta - V) + \text{Sin}\delta\ \text{Sin}(\delta - V)\text{Cos } C$

or $\text{Cos}(U + A) = \text{Cos}\alpha\ \text{Cos}(\delta - V)$ ....(2)

Squaring and adding equations (1) and (2) and simplifying

we get $\text{Sin }(\delta - V) = \dfrac{\text{Sin }\alpha\ \text{Sin }\varepsilon}{\sqrt{1 - \text{Cos}^2\alpha\ \text{Sin}^2\varepsilon}}$ ....(3)

and $\text{Sin }(U + A) = \dfrac{\text{Sin a}}{\sqrt{1 - \text{Cos}^2\alpha\ \text{Sin}^2\varepsilon}}$ ....(4)

Equations (3) and (4) transform equatorial coordinates (α, δ) into the Indian nirayaṇa coordinates (U, V).

Transform of equatorial coordinates from one epoch to another.

$$\delta_1 - \delta = \theta \text{ Sin}\varepsilon \text{ Cos}\alpha \; ; \; \alpha_1 - \alpha = \theta \{\text{Cos } \varepsilon + \text{Sin } \varepsilon \text{ Cos } \alpha \text{ Tan } \delta\} \quad \text{.......(5)}$$

Where, α, δ = right ascension and declination in any year

α_1, δ_1 = right ascension and declination after one year

ε, θ = obliquity and annual rate of precession

The calculation is repeated for each year until the desired epoch is reached.

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