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LUNI-SOLAR CALENDAR, *KALI AHARGAṆA* AND JULIAN DAYS

A K Bag*

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The paper computes the elements of luni-solar calendar, like *saura* months, lunar months, intercalary months, omitted *tithis*, and gives general rules for computing *ahargaṇa* from different epochs. The later *ahargaṇa* rules agree very well with that of the *Kali ahargaṇa* which started counting of days from the Kali epoch. The use of *Kali ahargaṇa* (KA) for recording time of important events was quite popular and became the part of India's history and culture. Since *ahargaṇa* is nothing but collection of civil days, it has corresponding agreement with number of Julian days, and as such the system of conversion from one to another has also been included and clarified with the help of tables and examples. This will show that the popularity of KA was based on a strong mathematical foundation, even though the major objective of KA rule was to calculate the mean planetary positions.

Key words : *Ahargaṇa* rules; Civil days; Epochs of–Bhāskara II, Brahmadeva, Brahmagupta, Gaṇesa, Kali, Varāhamihira; *Kali ahargaṇa* tradition; Julian days, Luni-solar elements; Weekdays.

INTRODUCTION

India originally had lunar calendar. During Indus Civilisation, the appearance of conspicuous stars or group of stars found in conjunction (*amāvasyā*) or opposition (*pūrṇimā*) with Moon or Sun were considered to be reliable guides for fixing agricultural and religious functions. The Mohenjo-daro Seal[1] (M. 2430) has depicted a New Year festival introducing the month of Mṛgaśirṣa (Agrahāyanī month) when the Moon is in conjunction with the star Viśākhe at the fag end of the night after the full Moon at Kṛttikā in the evening. Similar such festivals have also

*Indian National Science Academy, Bahadur Shah Zafar Marg, New Delhi-110002.

been found to be associated with the phases of the Moon in early Vedic and post-Vedic period and even in present times. During Vedic and post-Vedic period, Sun gradually assumed greater significance because of the emphasis on agriculture and seasons. The lunar months viz. Kārttika, Agrāhayaṇī (Mṛgaśirṣa), Pauṣa, Māgha etc had their origin from star names, Kṛttikā, Mṛgaśirṣa, Puṣyā, Maghā respectively in which full moon occurred. Hence the months were *pūrṇimānta* during Vedic times but it became *amānta* during Vedāṅga Jyautiṣa period. The *Nidāna Sūtra* and the *Lāṭyāyana Śrautasūtra* of the Sāmaveda, as explained by Shamasastry[2], considered lunar year to be of 354 days (6 months of 30 days and 6 months of 29 days) and also 13 months of 378 days. The lunar months were used more or less in a fixed fashion in reference to agricultural seasons, which undoubtedly contributed to the development of the luni-solar calendar in the post-Vedic period. The luni-solar calendar involved from time to time, the addition of an intercalary lunar month to the regular (civil) months of fixed length (of 30 days each). These inercalations were handled in a practical manner, whenever deemed necessary,to ensure that seasonal festivals and agricultural practices did not go out of step. There are references of intercalation of 21 days, which Shamasastry explains as a part of the four year cycles, the first three *sāvana* (civil) year is of 360 days each, and the fourth year is of 381 days. Such a period of 4 years contains 1461 days, average being 365¼ days. The *Vedāṅga Jyautiṣa*[3] conceived a cycle of 5 years (yuga) with Sun and Moon lying at the starting point of the nakṣatra, Dhaniṣṭhā. In this 5 years cycle, the number of revolution of the Sun, Moon (both sidereal and synodic), civil days, sidereal days, lunar days (*tithis*) are given. Naturally,three units of time measurement viz. the civil day,lunar month, and the solar year were involved. The type of adjustment was attempted through out, and the Siddhantic period ultimately fixed to a period of 43,20,000 years (*Mahāyuga* or simply *yuga*), and sometimes 432 x 10^7 years (*Kalpa*) in which planetary revolutions, number of intercalary months, civil days, *tithis*, number of omitted *tithis* (*kṣaya dina* or *avama dina*) were given. To give a logical meaning, the Siddhantic writers had replaced 27 nakṣatras (each naksatra is 13°20́) by a system of 12 zodiacal signs (each sign is 30°) as a frame of reference for ecliptic, and developed definitions of day, month and year, and conceived Kali and other epochs, *ahargaṇa* (collection of days from an epoch).In what follows, we will discuss different elements of luni-solar calendar, methods of calculating *ahargaṇa* from Kali epoch and other epochs. The *Kali ahargaṇa* tradition is then compared with Julian days (JD) to fix up Christian dates. This will show that the *Kali ahargaṇa,* though based on the astronomical hypothesis, was understood to be of great historical significance in establishing the dates of important events in subsequent years in India.

LUNI-SOLAR TIME UNITS IN SIDDHANTIC ASTRONOMY

The Siddhantic astronomy has used the technical words, *saura* (relating to Sun or Solar), *cāndra* (relating to Moon or Lunar), *sāvana* (relating to natural or civil). Obviously, a *saura* year (time the Sun takes to travel through 12 signs of the zodiac), *saura* month (time taken by the Sun to travel through one sign of the ecliptic), lunar month (time between two conjunctions of Sun and Moon or new moon), *saura* day (30th division of the *saura* month), lunar day (*tithi,* 30th division of the lunar month), civil day (*sāvana dina,* time interval from one sunrise to next sunrise), intercalary month (*adhimāsa,* extra month to be adjusted with the lunar month), omitted lunar days (*avama dina* or *kṣaya tithi* dropped for adjustment with civil days). A week was considered to be of 7 days named after the planets viz., Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday. The main problem arises from the fact that the *saura* year does not contain an integral number of lunar months since one lunar year of 12 months was considered to be of 354 days or 13 lunar months of 374 days. Further lunar month has 30 lunar days but 29.5 civil days, and the lunar months were pegged with the seasons. To adjust this problem the Siddhantic works conceived of a very big periods in which the planets more or less make complete revolutions. Accordingly, the texts have given parameters like revolution number of planets, number of intercalary months, omitted lunar days, civil days etc. in a period of 43,20,000 years or 432×10^7 years. It appears that the values for longer or shorter periods were calculated by the method of ratio-proportion. These parameters are tabulated below as per major Siddhantic works.

Texts[4]	Sun	Moon	Civil days
ĀBh; MBh; LBh	43,20,000	5,77,53,336	1,57,79,17,500
KK, SS (PS)	43,20,000	5,77,53,336	1,57,79,17,800
SS(Mod); Soma Si; Brahma Si.	43,20,000	5,77,53,336	1,57,79,17,828
VSi (Vaṭesvara)	43,20,000	5,77,53,336	1,57,79,17,500
Pai Si (Viṣṇu); Si Śi (Bhāskara II)	43,20,000	5,77,53,300	1,57,79,17,450

Obviously, in a yuga of 43,20,000 years, (as per ABh, MBh, LBh),

Saura months (Ms) = 43,20,000 x 12 = 5,18,40,000

Saura day (Ds) = 5,18,40,000 x 30 = 1,55,52,00,000

Lunar months (M_L) = lunar rev. – solar rev.

= 5,77,53,336 – 43,20,000 = 5,34,33,336,

Lunar days (D_L) = 5,34,33,336 x 30 = 1,60,30,00,080,

Intercalary months (M_I) = lunar months – saura months

= 5,34,33,336 – 5,18,40,000 = 15,93,336

Omitted *tithis* (Do) = lunar days – civil days

= 1,60,30,00,080 – 1,57,79,17,500

= 2,50,82,580

KALI AHARGAṆA

The *Kali ahargaṇa* or *Kalidina* (KA) is the collection of civil days between Kali epoch and specified date. The Kali epoch began on Friday, first Caitra (*caitra śuklādi*), – 3101 AD (i.e., 3179 years before the Śaka era)[5]. The day was originally counted by Āryabhaṭa I from midnight to midnight (*ardharātrika* system), but he subsequently changed it to sunrise to sunrise (*audayika* system). The number of civil days was accordingly modified. The well known Siddhantic astronomers, Varāhamihira, Brahmagupta, Brahmadeva, Bhāskara II, Gaṇeśa etc. had introduced epochs during their own times, which were in full agreement with the Kali epoch. Even modern *Sūryasiddhānta* introduced an epoch, on Sunday, 1st Caitra, 1,95,58,80,000 years before the Kali epoch which has been shown to be in agreement with the Kali epoch[6]. Here methods for finding *ahargaṇa* from epochs as prescribed in different texts are explained below with examples.

(A) Ahargaṇa from Kali epoch

Kali epoch started on Feb. 18, Friday, –3101 AD at UMT sunrise 6 AM.

Bhāskara I (628 AD), Brahmagupta (628 AD), Lalla (748 AD), Śripati (1039 AD), Bhāskara II (b. 1150 AD) have given the same method for calculating *ahargaṇa* from the Kali epoch.. Let me describe here the rule as described by Bhāskara I[7]:

"Add 3179 to the number of elapsed years of Śaka kings; then multiply (that sum) by 12; and then add the number of months elapsed (since the beginning of Caitra). Put down the result in two places. At one place multiply (that) by the number of intercalary months in a yuga and divide by the number of solar months in a yuga; and add the resulting intercalary months (omitting the fraction of a month) to the result put at the other place. Multiply that sum by 30 and then add the number of lunar days(*tithis*) elapsed (since the beginning of current month). Set down the result (i.e., the sum obtained) at two places. At one place multiply that by the number of omitted lunar days (neglecting the fraction of a day) from the result set down at the other place. The result (thus obtained) is the number of (mean) civil days elapsed since the beginning of Kaliyuga (at mean sunrise at Laṅkā on the given lunar day). These days are said to have commenced with Friday and at sunrise (at Laṅkā)".

This may be written in symbols as follows :

(*i*) m_s = *saura* months since the beginning of Kali epoch
= 12 (y + 3179) + m, where y = *saura* years elapsed in śaka, and m = number of months elapsed from 1st Caitra (*Caitraśuklādi*).

(*ii*) d_S = *saura* days =30 m_S + d, where d = number of days elapsed since the end of last *amāvasyā* i.e., 1st of *śukla pakṣa* (from *pratipad*).

(*iii*) m_I = intercalary months since the beginning of the epoch

$= m_S \,.\, M_I / M_S = d_S \,.\, M_I/D_S$ (since $M_I/m_I = M_S/m_S = D_S/d_S$).

(*iv*) d_L = lunar days since the epoch

$= d_S + 30.\, m_I = (30.\, m_S + d) + 30\, m_I.$

$= 30\,(m_S + m_I) + d$

(*v*) d_O = omitted lunar days since the epoch

$= d_L \,.\, D_O/D_L$

(*vi*) A = *Kali ahargaṇa* i.e., number of civil days from the Kali epoch to a desired date = $d_L - d_O$.

Note : (I) At Kali epoch, KA = 0, (2) The mean lunar day may differ from true lunar day by one. Obviously the *ahargaṇa* obtained by this process may sometimes be in excess or defect by one. The day of calculation may be tested, dividing the KA number by 7 and counting the remainder from Friday, since a week was taken to be of seven days. The defect or excess in the *ahargaṇa* number may be corrected accordingly by increasing or decreasing the number by one. Similarly, when a true intercalary month has occurred prior to the given lunar month or is about to occur thereafter, the true lunar month may differ from the mean lunar month by one, and accordingly the mean intercalary months obtained by this may be adjusted[8].

Example 1 : To find the *Kali ahargaṇa* corresponding to *aṣāḍa kṛṣṇa caturthī maṅgalavāra* of Śaka year 1923 (Tuesday 10 July 2001).

Solution : Here m = 2, d = 18, y = 1923 Śaka.

m_S = 12 (y + 3179) + m = 12 (1923 + 3179) + 2 = 61,226

$m_I = m_S \times M_I/M_S$ = 61,226 x 15,93,336/5,18,40,000 = 1882,

d_L = 30 (m_S + m_I) + d = 30 (61,226 + 1882) + 18 = 18,93,258,

d_O = d_L. D_O/D_L = 18,93,258 x 2,50,82,580/1,60,30,00,080 = 29,623,

(vii) A = d_L – d_O = 18,93,258 – 29,623 = 18,63,635.

Test : The *ahargaṇa* 18,63,635 when divided by 7 leaves a remainder 4. Counting from Friday the remainder 4 indicates, 0 – Fri., 1 – Sat., 2 – Sun., 3 – Mon., and 4 – Tues. It agrees with the weekday which is Tuesday.

Hence KA = 18,63,635 is the required solution.

(B) Ahargaṇa from Varāhamihira epoch

Varāhamihira epoch started on *caitra śukla pratipad maṅgalavāra śaka* 427 i.e.,Caitra 1, Tuesday Saka 427 (March 22, 505 AD); KA = 13,17,124 days.

Varāhamihira gives two rules,rule I is stated to be as per *Romakasiddhānta*[9], and the Rule 2 as per *Pauliśasiddhānta*[10].

Rule 1 : Using the same symbols,

(*i*) m_S = 12 (y – 427) + m,

(*ii*) m_I = m_S x 7/228,

(*iii*) d_L = 30 (m_S + m_I) + d,

(*iv*) d_O = (11 d_L + 514)/703, and

(*v*) A_I = Varāhamihira *ahargaṇa* = d_L – d_O

Rule 2 :

(*i*) m_S = 12 (y – 427) + m,

(*ii*) m_I = (m_S x 30 x 10 + 698)/9761,

(*iii*) d_L = 30 (m_S + m_I) + d,

(*iv*) d_O = (11 d_L + 444)/703, and

(*v*) A_I = Varāhamihira *ahargaṇa* = d_L – d_O

Note : (1) KA for Varāhamihira epoch = 13,17,124,

(2) Romaka gives 1050 intercalary months and 16,547 omitted lunar days in 2850 *saura* years.

Obviously in a yuga, Intercalary months (M_I)/*Saura* months (M_S) = 1050/(2850 x 12) = 7/228, and Omitted lunar days (D_O)/lunar days (D_L) = 16,547/30 (2850 x 12 + 1050) = 16,547/10,57,500 = 11/703.

(3) KA = *Kali ahargaṇa* at the Varāhamihira epoch +A_I.

Verification of Example 1 : To verify *ahargaṇa* for *aṣāḍa kṛṣṇa caturthī maṅgalavāra* Śaka 1923.

Rule 1 : Here m = 2, d = 18, and y = 1923.

m_S = 12 (1923 − 427) + 2 = 17,954,

m_I = m_S x 7/228 = 17,954 x 7/228 = 55,158/228= 552*,

d_L = 30 (m_S + m_I) + d = 30 (17,954 + 552) + 18 = 5,55,198.

d_O = (11 d_L + 514)/703 = 8687,

a_I = Varāhamihira *ahargaṇa* = d_L − d_o = 5,55,198 − 8,687 = 5,46,511.

Test : The *ahargaṇa* 5,46,511 which divided by 7 leaves the remainder 0. Counting from Tuesday, since the Varāhamihira epoch started on Tuesday, the required day is Tuesday, which agrees with example 1.

Rule 2 :

m_S = 17,954,

m_I = (m_S x 30 x 10 + 698)/9761 = (17,954 x 300 + 698)/9761

= 55,11,705/9761 = 552*

d_L = 30 (m_S + m_I) + d = 5,55,198.

d_O = (11 d_L + 444)/703 = 86,87, 661/703 = 8687,

A_I = Varāhamihira *ahargaṇa* = d_L − d_O = 5,55,198 − 8,687 = 5,46,511.

The Rule 1 and Rule 2 are same, only the calculation of m_I in both the Rules are based on more or less on the similar approximations.

Hence *Kali ahargaṇa* = 13,17,124 + 5,46,511 = 18,63,635. This gives the same number as in Example 1.

(C) Ahargaṇa from Khaṇḍakhādyaka epoch of Brahmagupta

The *Khaṇḍakhādyaka* epoch started on Caitra I Sunday, *śukla pratipad* Śaka 587 (March 23, 665 AD); KA = 13,75,565.

The Rule[11], as stated, may be summarized as follows :

(*i*) $m_S = 12\,(y - 587) + m$,

(*ii*) $m_I = (1/976)\,[(d_S + 5) - (d_S + 5)/14{,}945]$,

(*iii*) $d_L = d_S + 30\,m_I = 30\,(m_S + m_I) + d$,

(*iv*) $d_O = (1/703)\,[(11\,d_L + 497) - (11\,d_L + 497)/1{,}11{,}573]$,

(*v*) a_2 = *Khaṇḍakhādyaka* epoch = $d_L - d_O$.

Note : (1) KA for *Khaṇḍakhādyaka* epoch = 13,75,565.

(2) $M_I/D_S = 15{,}93{,}336/1{,}55{,}52{,}00{,}000$

$= 1 \div 1{,}55{,}52{,}00{,}000/15{,}93{,}336$

$= (1 \div 976)\,[1 - 1/14{,}945]$, and

$D_O/D_L = 2{,}50{,}82{,}580/1{,}60{,}30{,}00{,}080$

$= (11 \div 703)\,[1 - 1/1{,}11{,}573]$

$= (1 \div 64)\,[1 + 2/1403]$.

The quantities 5 and 497 in item (ii) and (iv) respectively are added since the *ahargaṇa* is computed from an arbitrary time i.e., from the year 587 Śaka and not from the Kali epoch. These quantities are called *kṣepa* quantities.

Verification of Example 1 : To veryfy *Kalidina* for *aṣāḍa kṛṣṇa caturthī maṅgalavāra* Śaka 1923 (Tuesday 10 July 2001).

Solution: Here m = 2, d = 18, and y = 1923.

$m_S = (y - 587) + m = 12\,(1923 - 587) + 2 = 16{,}034$.

$d_S = 30 \times m_S + d = 30 \times 16{,}034 + 2 = 4{,}81{,}022,$

$m_I = (1/976)\,[(d_S + 5) - (d_S + 5)/14{,}945],$

$= (1/976)\,[4{,}81{,}027 - 4{,}81{,}027/14{,}945]$

$= (1/976)(4{,}81{,}027 - 3) = 4{,}81{,}024/976$

$= 493.$

$d_L = 30\,(m_S + m_I) + d = 30\,(16{,}034 + 493) + 18 = 4{,}95{,}828,$

$d_O = (1/703)\,[(11\,d_L + 497) - (11\,d_L + 497)/1{,}11{,}573]$

$= (1/703)\,[54{,}54{,}605 - 48] - 54{,}54{,}557/703$

$= 7{,}759$

A_2 = *Khaṇḍakhādyaka ahargaṇa* = $d_L - d_O = 4{,}95{,}828 - 7{,}759$

$= 4{,}88{,}069.$

Test : The number 4,88,069 when divided by 7 leaves the remainder 1. Counting from Sunday, since the *Khaṇḍakhādyaka* epoch started on Sunday, the weekday is Monday. But the prescribed day is Tuesday, hence the *ahargaṇa* has to be increased by one i.e., A_2 = 4,88,070.

Obviously, KA = *Ahargaṇa* for *Khaṇḍakhādyaka* epoch + A_2

= 13,75,565 + 4,88,070 = 18,63,635.

The KA number agrees with KA number of Example 1.

(D) Ahargaṇa from Karaṇaprakāśa epoch of Brahmadeva

The *Karaṇaprakāśa*[12] epoch started on Caitra 1, Sunday Śaka 1014 (28 March, 1092 AD); KA = 15,31,532.

The Rule runs as follows :

(*i*) $m_S = 12\,(y - 1014) + m,$

(*ii*) $m_I = [(2\,m_S + 32)/65]\,[1 - 1/916],$

(*iii*) $d_L = 30\,(m_S + m_I) + d,$

(*iv*) $d_O = [(d_L + 62)/64]\,[1 - 2/1403]$, and

(*v*) $A_3 = d_L - d_O$.

Verification of Example 1 : Here, m = 2, d = 18, and y = 1014.

m_S = 10,910; m_I = 336 – 1 = 335; d_L = 3,37,368; d_O = 5272 – 7 = 5265; and

A_3 = 3,32,103.

Test: The *ahargaṇa* number divided by 7 leaves the remainder 2. Counting from Sunday, the weekday is Tuesday. Obviously, KA = 15,31,532 + 3,32,103 = 18,63,634. It agrees with results of Example 1.

(E) Ahargaṇa from Karaṇakutūhala epoch of Bhāskara II

The *Karaṇakutūhala* epoch started on Thursday, *caitra śukla pratipad,* mean sunrise Śaka 1105 (Feb. 24, 1183 AD); KA = 15,64,737.

The Rule[13] may be written as follows:

(*i*) $m_S = 12\,(y - 1105) + m$,

(*ii*) $m_I = (2\,m_S + 66) - (1/900)\,(2\,m_S + 66)$, only integer quotient is deducted.

(*iii*) $d_L = 30\,(m_S + 1/65\,m_I) + d$,

(*iv*) $d_O = (1/64)\,[(d_L + 3) + (1/703)\,(d_L + 3)]$, and

(*v*) $A_4 = d_L - d_O$.

Verification: Here, m = 2, d = 18, and y = 1923.

m_S = 12 (1923 – 1105) + 2 = 9,818; m_I = 19,702 – 21 = 19,681;

d_L = 30 (9818 + 303) + 18 = 3,03,648;

d_O = (1/64) [3,03,651 + 431] = 3,04,081/64 = 4750; and

A_4 = *Karaṇakutūhala ahargaṇa* = 3,03,648 – 4,750 = 2,98,898.

Test: The *ahargaṇa* 2,98,898 when divided by 7 leaves the remainder 5. Counting from Thursday, the weekday is Tuesday. Hence KA = 15,64,737 + 2,98,898 = 18,63,635. It agrees with Example 1.

(F) Ahargaṇa from Grahalāghava of Gaṇeśa Daivajña

The *Grahalāghava* epoch started on Monday, *caitra śukla pratipad Śaka* 1442 (March 19, 1520 AD) KA = 16,87,850.

The Rule[14] may be summarized as follows :

(*i*) C = Cakra = (1/11) (y – 1442); R = Remainder = (y – 1442) – 11 C;

(*ii*) m_S = 12 R + m = 12 [(y – 1442) – 11 C] + m

= [12(y – 1442) + m – 132 C;

(*iii*) m_I = (m_S + 2C + 10)/33;

(*iv*) d_L = 30 (m_S + m_I) + d;

(*v*) d'_L = d_L + C/6;

(*vi*) d_O = d'_L/64, and

(*vii*) A_5 = *Grahalāghava ahargaṇa* =d'_L – d_O.

Note: (1) *Cakra* is a period of 11 years = 4016 days elapsed.

(2) Number of civil days = 4016 C + A_5

Verification: Here, m = 2, d = 18, and y = 1923.

C = (1/11) (y – 1442) = (1/11) (1923 – 1442) = 481/11 = 43,

and, R = remainder = (y – 1442) – 11C = 481 – 473 = 8,

m_S = 12 R + m = 12 x 8 + 2 = 98,

m_I = (1/33) (m_S + 2C + 10) = (1/33) (98 + 86 + 10) = 194/33 = 6,

d_L = 30 (m_S + m_I) + d = 30 (98 + 6) + 18 = 3,138,

d'_L = d_L + C/6 = 3138 + 43/6 = 3138 + 7 = 3145,

d_O = d'_L/64 = 3145/64 = 49,

A_5 = d'_L – d_O = 3145 – 49 = 3096.

Test: 5C + A_5 = 5 x 43 + 3096 = 3311.

The number 3311 when divided by 7 leaves a remainder 0. Counting from Monday, the day is Monday. But the prescribed day is Tuesday. Hence the actual *ahargaṇa* number A_5 should be one more i.e., 3097. Hence no. of total civil days in A_5 = 4016 C + 3097

= 4016 x 43 + 3097

= 1,75,785

KA number = 16,87,850 + A_5 = 16,87,850 + 1,75,785 = 18,63,635. It agrees wth the results of Example 1.

Popularity of Kali Ahargaṇa in Indian Tradition

Use of *Kali ahargaṇa* or *Kalidina* in Kaṭapayādi notation (right to left system) in Indian tradition is very old. The Kaṭapayādi alphabetic system of notation was mostly known from early times and quite popular in Kerala among the poets and later astronomers. According to K.V. Sarma[15], "Whatever be the subject of a text, *Veda, Philosophy, Dharmaśāstra, Tantra, Śilpa, Vaidya, Vyākaraṇa, Saṅgīta* or *Kāvya*, the authors often indicated the Kali dates of completing their works in Kaṭapayādi and much more so the scribes of the manuscripts in their post-colophonic statements". The *Kali ahargaṇa* method of computing civil days became so popular that it began to be used to commemorate the date of birth, composition or completion of work or an important event. A few examples will be of interest:

(*i*) The great philosoper Śaṅkarācārya gave the beginning of *Kollam* era by a chronogram[16], *ācāryavāgabhedyā, i.e.,* 14,34,160 days (25 August, 824 AD).

(*ii*) The *Grahacāranibandhana Saṅgraha* has used the chronogram, *gorasaṃ rasavarya*[17], *i.e.,* 14,72,723 days (25 Mar 931 AD).

(*iii*) Parameśvara in his *Grahaṇamaṇḍana*[18] has used a chronogram, *svaratithibhujaṅ gasāgaraṣaḍvidhu* (word numerals) i.e., 1648157 *Kalidinas* (17 July 1411 AD) when he started his *karaṇa* calculation.

(*iv*) Mādhava of Saṅgamagrāma composed his *Sphuṭacandrāpti*[19] in *dinanamrānuśāsya* i.e., 15,02,008 Kali days (29 May 1011 A.D) plus 5180 anomalistic cycles of the Moon (390 years) i.e., 29 May 1401 AD.

(*v*) The date of Nīlakaṇṭha is given in the beginning verse of his *Tantrasaṃgraha*[20] by a chronogram, *he viṣṇo nihitam kṛtsnam* i.e., 16,80,548 Kali days (Mar 22, 1500 A.D). The date of his birth in his own commentary of his *Siddhānta Darpaṇa* by the chronogram[21], *tyasamyajñātam tarkaiḥ* i.e., 16,60,181 days (17 June, 1444 A.D).

(*vi*) Kerala poet, Nārāyaṇa Bhaṭṭa completed the writing of his work *Nārāyanīyam* in Kali days expressed by a chronogram, *āyurarogyasaukhyam*[22] i.e., 17,12,211 (9 Dec. 1586 AD).

WEEKDAY, JULIAN DAYS, AND KALI AHARGAṆAS

The system of counting civil days is more or less same for Julian Days (JD), *Kali ahargaṇa* (KA) and other systems, since a week was consisted of 7 days.

JD : The counting of JD started on mid-night (GMT) Monday, January 1, 4713 BC. At the beginning JD = 0.

To find the weekday from JD number, divide JD number by 7, count the remainder from Monday i.e., the remainder 0–Mon., 1–Tues., 2–Wed., 3–Thurs., 4–Fri., 5–Sat., 6–Sun.

KA : The counting started on – 3101 AD, Friday, 17 – 18 Feb. mean midnight (Ujjain). Under *audayika* system, this switched over to morning 6AM (Ujjain) on 18 Feb., – 3101 AD. To find the week day, divide KA number by 7, count it from Friday i.e., the remainder 0–Fri., 1–Sat., 2–Sun., 3–Mon., 4–Tues., 5–Wed., 6–Thurs.

Conversion: Three tables for conversion from JD to KA and vice versa for a Christian year are given here. These were given before in almost all the Indian ephemeris and by scholars like Sewell & Dikshit (London 1896; Delhi 1995), Rao (2000) etc.

Table 1: Christian year, JD, KA at 100 years interval

Table 2: *Ahargaṇa* or number of civil days for 0 to 99 years.

Table 3: *Ahargaṇa* or civil days in a year.

Examples for conversion:

Ex. 1. : To find Julian Days, *Kali ahargaṇas* and the weekday for 10 July 2001.

Soln:	JD	KA
Year 2000 (Table 1)	2451545	1863079
For 01 (Table 2)	365	365
For 10 July (Table 3)	191	191
Add :	2452101	1863635

For JD: Divide the JD number 2452101 by 7, it leaves the remainder 1. Counting from Mon., the day is Tues. The Julian day = 2452101.

For KA: Divide the KA number 1863635 by 7, it leaves the remainder 4. Counting from Fri., the day is Tues. The *Kali ahargaṇa* = 1863635.

Ex. 2: To find the JD and the Christian date for *Kali ahargaṇa* 1434160.

Soln:

Nearest number	**KA**	**JD**	**Christian year**
to 1434160 (from Table 1)	1424792	2013258	800 (J)
to 9368 (from Table 2) (diff. 1434160 and 1424792)	9131	9131	25 years
Rest : 237 (from Table 3)	237	237	25 Aug.
(diff. 9368 and 9131)	1434160	2022616	825 AD 25 Aug.

The Julian day = 2022616, and the Christian date = 25 Aug. 825 AD.

Table 1: Christian year, Julian Day(JD), *Kali ahargaṇa* (KA) at an interval of 100 years

Christian Year	Julian	Kali *ahargaṇa*	Christian Year	Julian	*Kali ahargaṇa*
- 3200 (J)	552258	-36208	- 300 (J)	1611483	1023017
- 3100 (J)	588783	317	- 200 (J)	1648008	1059542
- 3000 (J)	625308	36842	100 (J)	1684533	1096067
- 2900 (J)	661833	73367	0 (J)	1721058	1132592
- 2800 (J)	698358	109892	100 (J)	1757583	1169117
- 2700 (J)	734883	146417	200 (J)	1794108	1205642
- 2600 (J)	771408	182942	300 (J)	1830633	1242167
- 2500 (J)	807933	219467	400 (J)	1867158	1278692
- 2400 (J)	844458	255992	500 (J)	1903683	1315217
- 2300 (J)	880983	292517	600 (J)	1940208	1351742
- 2200 (J)	917508	329042	700 (J)	1976733	1388267
- 2100 (J)	954033	365567	800 (J)	2013258	1424792
- 2000 (J)`	990558	402092	900 (J)	2049783	1461317
- 1900 (J)	1027083	438617	1000 (J)	2086308	1497842
- 1800 (J)	1063608	475142	1100 (J)	2122833	1534367
- 1700 (J)	1100133	511667	1200 (J)	2159358	1570892
- 1600 (J)	1136658	548192	1300 (J)	2195883	1607417
- 1500 (J)	1173183	584717	1400 (J)	2232408	1643942
- 1400 (J)	1209708	621242	1500 (J)	2268933	1680467
- 1300 (J)	1246233	657767	1500 (G)	2268923	1680457
- 1200 (J)	1282758	694292	1600 (G)	2305448	1716982
- 1100 (J)	1319283	730817	1700 (G)	2341972	1753506
- 1000 (J)	1355808	767342	1800 (G)	2378496	1790030
- 900 (J)	1392333	803867	1900 (G)	2415020	1826554
- 800 (J)	1428858	840392	2000 (G)	2451545	1863079
- 700 (J)	1465383	876917	2100 (G)	2488069	1899603
- 600 (J)	1501908	913442	2200 (G)	2524593	1936127
- 500 (J)	1538433	949967			
- 400 (J)	1574958	986492			

J = Julian Calendrical System applicable upto Oct. 4, 1582 AD; 10 days were adjusted in 1582 without disturbing the weekday.

G = Gregorian System, applicable beyond prescribed in J.

Table 2: *Ahargaṇa* or Number of civil days for 0 to 99 years.

Year	Days	Year	Days	Year	Days
0	0	34	12418	67	24471
1	365	35	12783	68	24837
2	730	36	13149	69	25202
3	1095	37	13514	70	25567
4	1461	38	13879	71	25932
5	1826	39	14244	72	26298
6	2191	40	14610	73	26663
7	2556	41	14975	74	27028
8	2922	42	15340	75	27393
9	3287	43	15705	76	27759
10	3652	44	16071	77	28124
11	4017	45	16436	78	28489
12	4383	46	16801	79	28854
13	4748	47	17166	80	29220
14	5113	48	17532	81	29585
15	5478	49	17897	82	29950
16	5844	50	18262	83	30315
17	6209	51	18627	84	30681
18	6574	52	18993	85	31046
19	6939	53	19358	86	31411
20	7305	54	19723	87	31776
21	7670	55	20088	88	32142
22	8035	56	20454	89	32507
23	8400	57	20819	90	32872
24	8766	58	21184	91	33237
25	9131	59	21549	92	33603
26	9496	60	21915	93	33968
27	9861	61	22280	94	34333
28	10227	62	22645	95	34698
29	10592	63	23010	96	35064
30	10957	64	23376	97	35429
31	11322	65	23741	98	35794
32	11688	66	24106	99	36159
33	12053	—	—	—	—

Table 3: *Ahargaṇa*/Civil Days for a year

Dates A	Dates B	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0	1	0	31	—	—	—	—	—	—	—	—	—	—
1	2	1	32	60	91	121	152	182	213	244	274	305	335
2	3	2	33	61	92	122	153	183	214	245	275	306	336
3	4	3	34	62	93	123	154	184	215	246	276	307	337
4	5	4	35	63	94	124	155	185	216	247	277	308	338
5	6	5	36	64	95	125	156	186	217	248	278	309	339
6	7	6	37	65	96	126	157	187	218	249	279	310	340
7	8	7	38	66	97	127	158	188	219	250	280	311	341
8	9	8	39	67	98	128	159	189	220	251	281	312	342
9	10	9	40	68	99	129	160	190	221	252	282	313	343
10	11	10	41	69	100	130	161	191	222	253	283	314	344
11	12	11	42	70	101	131	162	192	223	254	284	315	345
12	13	12	43	71	102	132	163	193	224	255	285	316	346
13	14	13	44	72	103	133	164	194	225	256	286	317	347
14	15	14	45	73	104	134	165	195	226	257	287	318	348
15	16	15	46	74	105	135	166	196	227	258	288	319	349
16	17	16	47	75	106	136	167	197	228	259	289	320	350
17	18	17	48	76	107	137	168	198	229	260	290	321	351
18	19	18	49	77	108	138	169	199	230	261	291	322	352
19	20	19	50	78	109	139	170	200	231	262	292	323	353
20	21	20	51	79	110	140	171	201	232	263	293	324	354
21	22	21	52	80	111	141	172	202	233	264	294	325	355
22	23	22	53	81	112	142	173	203	234	265	295	326	356
23	24	23	54	82	113	143	174	204	235	266	296	327	357
24	25	24	55	83	114	144	175	205	236	267	297	328	358
25	26	25	56	84	115	145	176	206	237	268	298	329	359
26	27	26	57	85	116	146	177	207	238	269	299	330	360
27	28	27	58	86	117	147	178	208	239	270	300	331	361
28	29	28	59	87	118	148	179	209	240	271	301	332	362
29	30	29	—	88	119	149	180	210	241	272	302	333	363
30	31	30	—	89	120	150	181	211	242	273	303	334	364
31	—	31	—	90	—	151	—	212	243	—	304	—	365

A = Applicable for dates for all Non-Leap-years, and all Leap-years other than January & February

B = Applicable for dates for January & February only for Leap–years.

Notes and References

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2. Shamasastry, R., "The Vedic Calendar", *Indian Antiquary*, 18(1912). 26-32, 45-71.

3. *Vedāṅga Jyautiṣa* of Lagadha (in its *Ṛk* and *Yājuṣ* recensions) with Eng. Tr. and notes by T. S. Kuppanna Sastry and Sanskrit text ed. by K.V. Sarma, Indian National Science Academy, New Delhi, 1985, p. 50.

4. *Āryabhaṭīya* (ABh), iii. 5; *Mahābhāskarīya* (MBh) i. 4-6; *Laghubhāskarīya* (LBh), i. 48; *Sūryasiddhāta*-(SS, mod.) i. 30-38; *Siddhāntaśiromaṇi* (SiSi), i. 1-3; other symbols are: KK-*Khaṇḍakhādyaka*, VSi-*Vaṭeśvarasiddhānta* & so on.

5. *Āryabhaṭīya* of Āryabhaṭa, Gītikāpāda, v. 5.

6. Varāhamihira epoch: Tuesday, 1st Caitra, 427 Śaka, KA = 13,17,124 days (March 22,505 AD); Brahmagupta epoch: Sunday, 1st Caitra, 587 Śaka, KA = 13,75,565 days (March 23,665 AD), and *Sūryasiddhānta* epoch, See Bag, A.K., "Ahargaṇa and the Weekdays as per modern Sūryasiddhānta", *IJHS*, 36.1-2 (2001) 55-63.

7. *navādrirūpagniyutam mahibhujāṃ*
śakendranāmnām gatavarṣasaṃgraham/
dviṣaṭkanighnaṃ gatamāsasaṃyutaṃ
yugādhimāsairguṇayed dvirāśitam//
yugārkamasaptagatadhimāsakair
yutam tithighnam gatavāsarairyutam/
yugāvamaistad guṇayed dvirāśitam
niśākarahairvibhajet nityasaḥ//
tithiprānaśāptirato viśodhite
bhavatyathahnām nicayaḥ kālergataḥ/
vadanti varam ditisūnupujitāt
pravṛttimapyahurudañcato raveḥ//
(*Mahābhāskarīya*, i.4-6).

8. *Siddhāntaśiromaṇi*, i. 3.

9. *Pauliśasiddhānta*, i. 8–10.

10. *Pauliśasiddhānta*, i. 12.

11. *Brāhmasphuṭasiddhānta*, i. 29-30.
12. *Karaṇaprakāśa*, i. 2-3.
13. *Karaṇakutūhala*, i. 2-3.
14. *Grahalāghava*, i. 4-5.
15. Sarma, K.V. (2003), p. 43
16. Logan William (1989),pp. 155-156; Sarma, K.V., 'Kollam Era', IJHS, 31.1 (1996) 98-99.
17. Published as an Appendix to *Gracāranibandhana* of Haridatta.
18. *Grahaṇamaṇḍana*, Introduction, p. ix, and text, pp. 2-3.
19. *Sphuṭacandrāpti*, pp. 14, 26.
20. *Tantrasaṃgraha*, i. 1.
21. *Golasāra*, Introduction, p. xxiv.
22. Sarma, K.V. (2003), p. 41.

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