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CORRECTIONS TO THE TERRESTRIAL LATITUDE IN *TANTRASANGRAHA*

K. RAMASUBRAMANIAN AND M.S. SRIRAM*

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The terrestrial latitude of an observer's location is equal to the zenith distance of the sun at noon on the equinoctial day. The effect of solar parallax on the zenith distance of the sun was known to the Indian astronomers right from Āryabhaṭa, but none of the astronomers prior to Nīlakaṇṭha Somayājī (c. 1500 AD) discussed its effect on the measurement of the observer's latitude. In his *Tantrasaṅgraha* Nīlakaṇṭha states not only about this correction but also explicity gives its magnitude. He also prescribes a correction due to the finite size of the sun. A detailed explanation of these corrections appears in *Yuktidīpikā*, a commentary on the *Tantrasaṅgraha* by Śaṅkara Vārier.

Key words : Equinoctial Parallax, *Śaṅku*, Semidiameter of the sun, Terrestrial latitude, Zenith distance.

EQUINOCTIAL NOON-SHADOW AND THE LATITUDE

The determination of the terrestrial latitude of an observer is an important problem in astronomy, as it plays a key role in the determination of the sunrise and sunset times, which in turn is important for civil, navigation and other purposes. In Indian astronomy, this is determined using a simple device called *śaṅku* (gnomon) which essentially consists of a stick of suitable height and thickness with a sharp edge at one of its ends. The procedure for finding the latitude of a place from the equinoctial noon-shadow of the gnomon is stated in almost all the Indian astronomical texts in the chapter on *Tripraśnādhikāra*. For instance, in his *Āryabhaṭīya*, Āryabhaṭa gives the procedure for obtaining the observer's latitude from the equinoctial shadow in the following verse implicitly[1]:

*Department of Theoretical Physics, University of Madras, Guindy Campus, Chennai 600 025, India; E-mail: tphysmu@satyam.net.in

madhyānnatabhāgajyā chāyā śaṅkostu tasyaiva |

"The Rsine of the sun's zenith distance (at noon) is the shadow of the same gnomon."

In his *Tantrasaṅgraha,* Nīlakaṇṭha states this explicitly as follows[2]:

chāyāṃ tāṃ trijyayā hatvā svakarṇena haret, phalam /

akṣajivā, tathā śaṅkuṃ kṛtvā lambakamānayet //

"The length of the shadow multiplied by *trijyā* and divided by *karṇa* gives R sine latitude. By repeating the same with the *śaṅku* (instead of the shadow) the R cosine of latitude may be obtained."

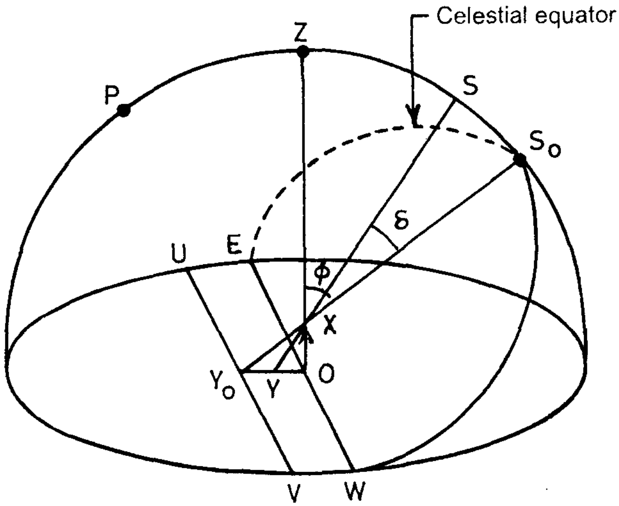

Fig. 1 : Noon shadow on an arbitrary day (OY) and the equinoctial day (OY_0).

The prescription given in these texts for the measurement of latitude may be understood with the help of Fig. 1. Here *OX* represents the *śaṅku* (gnomon). On the equinoctial day the diurnal motion of the sun is along the equator throughout the day, ignoring the small change in the declination during the day. Hence, the terrestial latitude (ϕ) is equal to the zenith distance of the sun as it crosses the prime meridian (at noon). On any other day, the zenith distance at noon would be $z = \phi - \delta$, where

δ is sun's declination.

Considering the triangle OXY, formed by *śaṅku* OX, its shadow OY and the hypotenuse XY, it can be easily seen that

$$OY = XY \times \sin(\phi - \delta).$$

On the equinoctial day, $\delta = 0$ and the tip of the shadow Y is Y_0. Hence, the above equation reduces to

$$\sin\phi = \frac{OY_0}{XY_0}.$$

Similarly,

$$\cos\phi = \frac{OX}{XY_0}.$$

Multiplying the above equations by *trijyā*, and using Indian astronomical terms for OX, OY_0 and XY_0, we have

$$ak\d{s}ajy\bar{a} = \frac{trijy\bar{a} \times ch\bar{a}ya}{kar\d{n}a}$$

and,

$$lambaka = \frac{trijy\bar{a} \times \acute{s}anku}{kar\d{n}a}$$

The above expressions can be found in almost all the *siddhānta* texts[3].

PARALLAX IN INDIAN ASTRONOMY

We first explain the concept of parallax and its effect on the measurement of the zenith distance of a celestial object using the modern notation. Then we proceed to discuss the effect of parallax as found in Indian astronomical texts. In Fig. 2, C represents the center of the Earth, S the celestial object which is chosen for observation, and O the observer. $R_e = OC$ is the radius of the Earth and d is the distance of the sun from the centre of the Earth. Z represents the geocentric zenith of the observer.

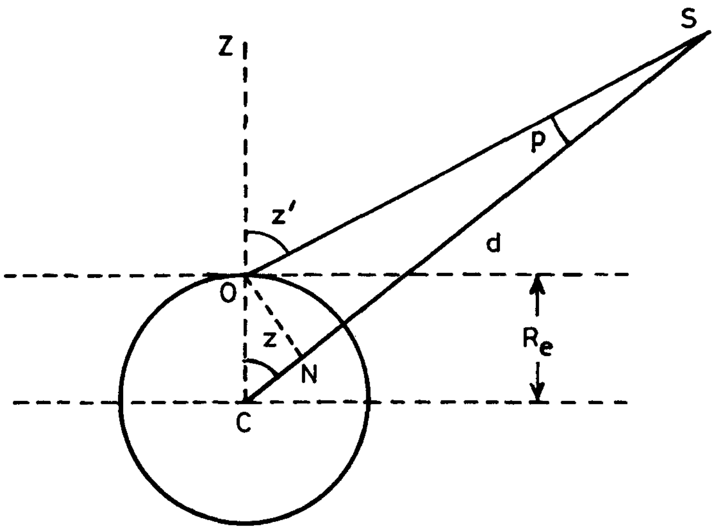

Fig. 2 : The effect of parallax on the measurement of latitude of the observer.

If z' and z represent the apparent and the actual zenith distances of the sun, then it is easily seen that

$$z = z' - p.$$

where $p = C\hat{S}O$, is the parallax of the celestial object, which is nothing but the angle subtended by the radius of the Earth at the centre of the sun. In other words, it is the angle between the direction of the object as seen by the observer O and the direction of the object as seen from the Earth's centre (which is the standard reference point for measuring the celestial co-ordinates).

From the plane triangle COS we have,

$$\sin p = \frac{R_e}{d} \sin z'.$$

Since $R_e \ll d$, p is small. Hence, the above equation may be written as

$$p = \frac{R_e}{d} \sin z' \qquad ...(1)$$

When $z' = 90°$, that is, the celestial object is on the observer's horizon, then the correction due to prallax is maximum and is called the horizontal parallax, and is given by

$$P = \frac{R_e}{d}.$$

Hence, Eq. (1) reduces to

$$p = P \sin z'. \qquad ...(2)$$

In Indian astronomical texts, the mean value of the horizontal parallax is taken to be one-fifteenth of the mean daily motion of the celestial object. This assumption is based on the fact that the mean value of the moon's horizontal parallax, is close to one-fifteenth of its mean daily motion. For instance, in *Sūryasiddhānta* R_e is given to be 800 *yojanas* and d as 15 × 3438 *yojanas* (as one minute of arc in moon's orbit amounts to 15 *yojanas* and 1 radian is 3438 minutes)[4]. Hence, the horizontal parallax of Moon, P_m is given by

$$P_m = \frac{800}{15}\ min. = 53'\ 20'',$$

which is close to the modern value of 57′, for the mean horizontal parallax of the moon. As moon's mean daily motion is given to be 790′35″, we have,

$$P_m \approx \frac{\textit{Daily motion of moon}}{15}$$

In *Siddhantaśiromaṇi*, the diameter of earth is given to be 1581 *yojanas* and the moon's distance is given to be 51566 *yojanas*[5]. Hence,

$$P_m = \frac{1581}{2} \times \frac{1}{51566} \times 3438\ min.$$

$$= \frac{790'30''}{15}$$

$$= \frac{\textit{Daily motion of moon}}{15}$$

In Indian astronomy, the linear velocities of all planets (including the sun and the moon) are taken to be the same. Hence, the horizontal parallax of the sun is also given by

$$P_s = \frac{\text{Daily motion of sun}}{15}$$

The sun's motion being approximately one degree per day, its horizontal parallax amounts to $\approx 4'$ per day. This value is quite large compared to the actual value of $\approx 9''$, and is due to the assumption of the same linear velocity for all planets.

The Effect of Parallax and Semidiameter on the Latitude

Nīlakaṇṭha explicitly states the correction due to the parallax and semidiameter to the observed value of the terrestial latitude in the following verses of *Tantrasaṅgraha*[6]:

akṣajyārkagatighnāptā khasvareṣveka-sāyakaiḥ[a] //
phalonamakṣacāpaiḥ syāt arkabimbārdhsaṅyutaṃ/
sphutaṃ tajjayākṣajīvāpi tasyāḥ koṭiśca lambakaḥ//

Note : *a*. In the printed text we find the word *khasvarādrokasāyakaiḥ*. The numeral represented by this word is 51770 in *bhūtasaṅkhyā-paddhati*. But the actual figure that occurs in the computation is 15 × 3438 = 51570. This number is given by the word *khasvareṣvekasāyakaiaḥ* in *bhūtasaṅkhyā*. In a similar context in Chapter V, verse 10, we find the number 51570 occuring again explicitly. There it is given by the word *khasvareṣvekabhuta*. Hence, we suppose that the text must be *khasvareṣvekasāyakaiḥ* and not *khasvarādrokasāyakaiḥ*.

"*Akṣajyā* multiplied by the true daily motion of the sun and divided by 51570 (is the correction factor p). This has to be subtracted from the *akṣa-cāpa* (latitude of the place). The semidiameter for the sun added to this is the true latitude. The R sine of this is the *akṣajyā* and its complement is the *lambaka*."

As seen in the previous section, Eq. (2), the solar parallax for a the zenith distance is given by

$$p = P_s \sin z'. \tag{3}$$

If D_{ms} represents the mean daily motion of the sun, the mean value of the parallax of the sun is given by

$$p_m = \frac{D_{ms}}{15} \times \sin z'.$$

Multiplying and dividing the above equation by *trijyā* and taking its value to be 3438', we have

$$p_m = \frac{D_{ms}}{51570} R \sin z'. \quad (4)$$

As the apparent motion of the sun around the earth is not along a circular orbit, the distance between them varies continuously; so does the value of the parallax. The value of the parallax given by the above equation refers to its mean value, since it is computed from the mean daily motion which in turn depends upon the mean distance of the sun from the Earth. The true value of the parallax (p) at a particular instant is be obtained by considering the actual distance of the sun from the earth at that instant, which in turn is related to the instantaneous angular velocity of the sun. Thus, true parallax is obtained by multiplying the mean value of the parallax by the instantaneous angular velocity (true daily motion) and dividing by the mean angular velocity (mean daily motion)

$$p = p_m \times \frac{\textit{true daily motion}}{\textit{mean daily motion}}$$

In D_{ts} represents the true daily motion of the sun, then substituting for p from Eq.(4), we have

$$p = \frac{D_{ts}}{51570} \times R \sin z'. \quad (5)$$

This equation is same as the expression given in the above verse for considering the effect of parallax on the measurement of the latitude at a given place. When the sun is on the prime-meridian on an equinoctial day, then the zenith distance of the sun is same as the latitude of the observer. Since all the measurements are made on the surface of the earth, one has to subtract the parallax from the observed value to get the true terrestrial latitude.

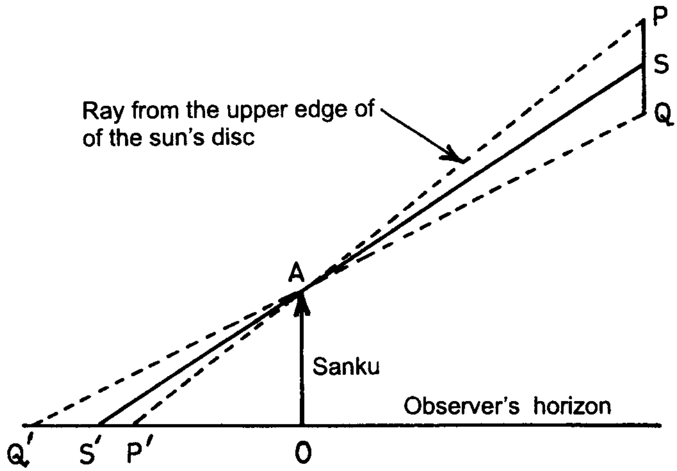

Fig. 3: Sectional view of the sun and the shadow of the *Śaṅku* generated by it.

The correction due to the finite size of the sun is explained in Fig. 3. Here *OA* represents the *śaṅku*. *PSQ* represents the sectional view of the sun, where *S* is its centre, and *P* and *Q* are the upper and the lower edges of the solar disc. Rays from S and P grazing the top edge of the *śaṅku* A, meet the plane of the observer's horizon at *S'* and *P'* respectively. If the sun were a point source of light, then the tip of the shadow of the *śaṅku* would be at *S'*. Then *OS'* would be the length of the shadow and the angle $S'\hat{A}O$ would be the terrestial latitude. However, due to the finite size of the sun, *P'* is the tip of the actual shadow and *OP'* its length. This is because the points beyond *P'* would be illuminated directly by the sun and do not fall inside the shadow. Hence, $P'\hat{A}O$ would be the observed latitude. It is obvious that $P'\hat{A}S'$ should be added to this to obtain the actual terrestial latitude $S'\hat{A}O$. $P'\hat{A}S'$ is equal to $P\hat{A}S$, which is the semi-diameter of the sun. Thus, the true latitude of the place is obtained by adding the semi-diameter of the sun to the observed value.

Rationale for the Correction in Yuktidīpikā

The *Yuktidīpikā* by Śaṅkara varier is a detailed commentary in verses on *Tantrasaṅgraha* wherein he discusses in detail the procedures for arriving at the different formulae used in Indian astronomical texts. It includes a discussion on the correction due to parallax. We also discuss the physical reasoning offered by him, which are quite interesting and novel. His explanations for the correction due to the finite diameter of the sun, are also unusual though convincing.

Formula for the correction due to parallax[7]

bhūvyāsārdhaṃ hi dṛgjyāyāṃ trijyāyāṃ lambayojanaṃ /
kiyat tadiṣṭadṛgjyāyāṃ svakakṣyāyāṃ bhavet tadā //
itthaṃ trairāśikāt kalpyaṃ dṛgjyālambanayojanaṃ /
madhyayojanakarṇena trijyātulyāḥ kalā yadi //
iṣṭayojanalambena kiyatyastatkalāssadā /
sphuṭabhuktyā hatātāśca madhyabhuktyā vibhājitāḥ //
sphuṭalambanaliptāḥ syuḥ iti trairaśikatrayāt /
hāro guṇaśca trijyaikā tayorādyadvitīyayoḥ //
madhyayojanakarṇaśca madhyabhuktiśca hārakau //
anyayoguṇakastavādye tadbhūvyāsārdhayojanaṃ //
guṇakārahṛte[a] *hāre hāra evātra no guṇaḥ /*
madhyayojana karṇaghna madhyabhuktestato hṛtaḥ[b] *//*
niyataireva lambārthaṃ bhūvyāsardhasya yojanaiḥ /
sa eva hārakaḥ khasvareṣvekeṣumito[c] *mataḥ //*
sphutabhuktirguṇo yābhyāṃ dṛgjyātaḥ sphuṭalambanam /

Note : *a*. In the original text, the reading found is *hate*, which means 'when multiplied'. We feel that it should be *hṛte* and not *hate*, since the procedure discussed in the later lines warrants division and not multiplication.

b. Here again, the context demands that the reading must be *hṛtaḥ* and not *hataḥ*. The prose order for this and the next two lines is : *tataḥ lambārthaṃ madhyayojanakarṇaghnamadhyabhukteḥ niyataireva yojanaiḥ hṛtaḥ (tena ca haranena yaḥ labdhaḥ, sa eva hārakaḥ (sa ca) khasvareṣvekesumito mataḥ /*

c. The reading found in the text is *khasvarādrokeṣumitaḥ*. For details regarding the change in the reading, the reader is referred to the variants given before in no. *a*.

"When the *dṛgjyā* is equal to *trijyā*, then the parallax in latitude is equal to the radius of the Earth (R_e) expressed in *yojanas*. For the desired *dṛgjyā* ($R \sin z'$) what will be the parallax in latitude in its own orbit ?

Thus the parallax in latitude in *yojanas* has to be obtained by using the rule of three. If the *madhyayojanakarṇa* (*d*, the sun's distance from the Earth's centre) is equated to *trijyā* (equal to 3438′), then what is the number of minutes corresponding to desired parallax in latitude (obtained in *yojanas*).

This value is multiplied by the true daily motion and divided by the mean daily motion. Thus we obtain the true value of the parallax in latitutde, in minutes, by applying the rule of three, three times successively.

Trijyā is the *hāra* (divisor) and *guṇa* (multiplier) respectively, in the first and the second rules of three. The *madhyayojanakarṇa(d)* and the mean daily motion (D_{ms}), are the divisors in the others (second and the third) rules of three. The radius of the earth in *yojanas* is the multiplier in the first rule of three.

If the *hāra* is divided by the *guṇa* (that is, divisor is divided by the multiplier), then, only the *hāra* remains and not the *guṇa*. Therefore, the product of *madhyayojanakarṇa* and the mean daily motion, divided by the fixed radius of the earth (R_e) becomes the divisor in obtaining the parallax in latitude. Its value is 51570. The true daily motion is the *guṇa* for them (second and third rule of three). From these and the *dṛgjyā*, the true value of the parallax in latitude (*p*) can be obtained."

The three rules of three given in the above verses are conventionally expressed as:

$$\begin{array}{llllllll} trijya & : & R_e & :: & iṣṭadyujya & : & p_{yoj}\ ? \\ d & : & R & :: & p_{yoj} & : & p_m\ ? \\ D_{ms} & : & p_m & :: & D_{ts} & : & p\ ? \end{array}$$

The above rules of three may be expressed in words as follows:

Rule 1:

This is to obtain the parallax (p_{yoj} = *ON* in *yojanas*) for a given zenith distance (*iṣṭadyujyā*), when the horizontal parallax is taken to be the radius of the earth (R_e) in *yojanas*.

$$p_{yoj} = ON = R_e \frac{R \sin z'}{R}$$

Rule 2 :

This is to convert the value of the parallax in *yojanas* obtained above into minutes. For this, p_{yoj} is first divided by the sun's mean distance from the earth (d), which gives the parallax in radians, and then multiplied by *trijyā* ($R = 3438'$) to obtain the value of the same in minutes.

$$p_m = p_{yoj} \frac{R}{d}$$

Rule 3 :

This is to obtain the true value of the parallax corresponding to the actual distance of the sun from the earth at the instant of measurement. For this, the parallax obtained in minutes is multiplied by the ratio of the sun's true and the mean daily motion.

$$p_t = p_m \frac{D_{ts}}{D_{ms}}$$

Combining these three rules, we have,

$$p = R_e \times \frac{R \sin z'}{R} \frac{R}{d} \frac{D_{ts}}{D_{ms}}$$

$$= D_{ts} \frac{R_e}{d \times D_{ms}} R \sin z' \qquad (6)$$

It is given that[8]

$$= \frac{d \times D_{ms}}{R_e} = 51570$$

Hence the true parallax is given by

$$p = \frac{D_{ts}}{51570} \times R \sin z'. \qquad (7)$$

The above equation is the same as Eq. (5).

Rationale behind the correction[9]

ghanabhūmadhyapārśvastham sarvatra kṣitijaṃ bhavet //
unnatajyā tataḥ śaṅkuḥ tadbhujā mahatī prabhā /
bhūpṛṣṭhagāminaḥ śaṅkoḥ chāyā syāllambitā tataḥ //
bhūvyāsārdhavihīno'sau śaṅkurbhūpṛṣṭhato'nyataḥ[a] */*
chāya ca vardhate trijyākarṇe koṭibhujatvataḥ //
tallambanavaśāt tasyāḥ yadādhikyaṃ bhavediha /
dṛksiddhayostyajet tattuḥ chāyā yena bhagolagā //
tattrijyāvargaviśleṣamulaṃ śaṅkuśca koṭikā /

Note : *a*. We feel that the reading should *śaṅkurbhūpṛṭhago'nyataḥ* instead of *śaṅkurbhū pṛṭhato'nyataḥ*.

"Everywhere (in all computations), the plane which lies in the neighbourhood of the centre of the earth, (*i.e.*, the one passing through the centre of the earth and parallel to the observer's horizon, which is the tangential plane drawn at the location of the observer) is taken to be the horizon. The cosine of the zenith distance is the *śaṅku*. The sine of it is the shadow, which is very long. The shadow of the *śaṅku* located on the surface of the earth (in the said horizon) will be very elongated. Hence the length of this *śaṅku* (CB in Fig. 5) (should be) is reduced by a measure of the radius of the earth. Otherwise, the shadow would increase. The hypotenuse taken to be *trijyā* is obtained from *koti* and *bhuja*. The increase in the shadow that is seen here (in the observer's horizon) is due to *lambana*, parallax in latitude. This (parallax in latitude) has to be subtracted from the observed value of the (angle corresponding to) shadow. Thus the shadow corresponding to the *bhagola* (the celestial sphere with the centre of the earth as its centre) is obtained. The square root of the square of it (shadow) subtracted from the square of *trijyā*, is the *śaṅku* and it is the cosine of the latitude".

We explain the content of the above verses with the help of the Fig. 4. Here, *O* refers to the observer. *OB* is the actual *śaṅku* and *OF* is the actual observed shadow. On the equinoctial day, z' is the observed latitude. *CB* and *CA* represent two hypothetical *śaṅkus* located at the centre of the earth. Since all the measurements are to be made taking the centre of the earth (C) as the standard reference point, the observed value of the latitude of the place must also be reduced to this reference point.

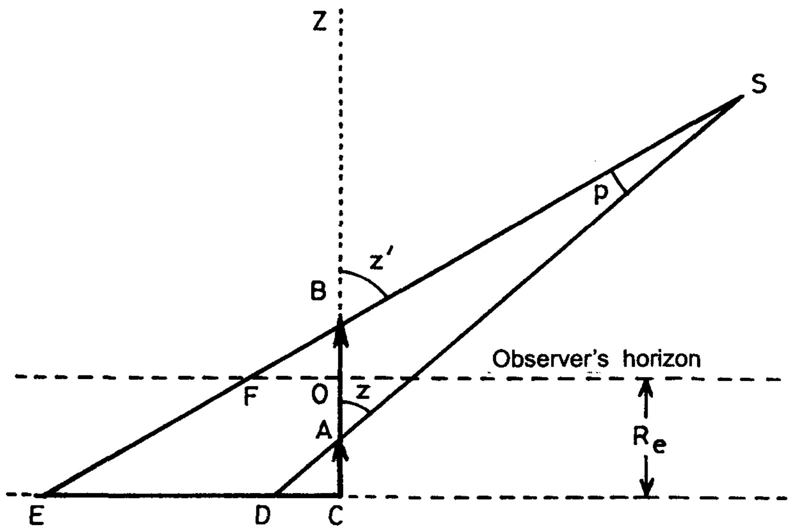

Fig. 4 : The shadow of the two *śaṅkus*, one imagined to be from the centre to the surface of the earth and the other at its centre.

The shadow cast by them are CE and CD respectively. The angle CAD gives the measure of the true latitude of the place on an equinoctial day. Obviously, this angle is obtained by subtracting the true value of the parallax from the observed zenith distance. Thus the exact latitude of the place is:

$$\phi = z = z' - p,$$

where p is given by Eq. (7).

Correction due to semi-diameter[10]

bimbordhvanemyāḥ prasṛtāḥ raśmayaḥ kṛśayanti'[a] *bhām//*
bimbavyāsārdhaniṣpañña śaṅkukhaṇḍena bhāsvataḥ/
vardhyanti ca doḥkhaṇḍā śaṅkuṃ bhūpṛṣṭhavartinaṃ//
bimbasya ghanamadhyānto bhavecchankuryato'paraḥ/
phalayorantaraṃ śaṅkuchāyayostadṛṇaṃ dhanaṃ/
pratyakṣasiddhayorbimbaghanamadhyagatau yataḥ//

Note : *a*. This is given as *kṛśayanti* in the text, edited by Prof. K.V. Sarma. We feel that it could be a typographical error as *kraśayanti* to our knowledge in Sanskrit does not convey any meaning.

"The rays that emerge from the upper part of the circumference (of the sun) make the shadow reduced (shortened). The rays emerging from the centre of the sun increase the length of the *śaṅku* on the surface of the earth by a measure determined by the semidiameter of the sun. This is because the *śaṅku* optained by the rays emerging from the centre, OB, is different (from the actual *śaṅku* OA). The difference in the observed values of the shadows of the *śaṅku* (OC-OD) which is negative has to be added (to the observed latitude) because these shadows correspond to the centre of the sun".

If the sun were a point object, then the length of the *śaṅku* will be OB corresponding to the length of the shadow (OC) observed (Fig. 5). Since it is not so, and the length of the actual *śaṅku* is OA, the effect of the finite size of the sun can be viewed as if the rays emerging from the centre of the sun have increased the length of the *śaṅku* by a measure AB, which is determined by the semi-diameter of the sun.

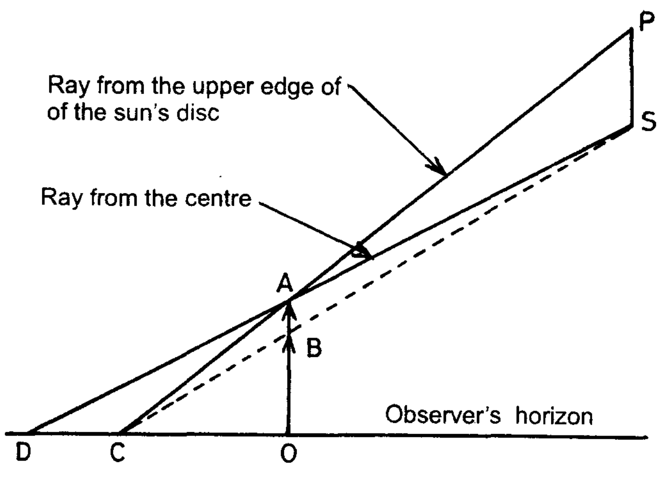

Fig. 5 : The two *śaṅkus*, OA (actual) and OB (imaginary) drawn to explain the effect of an extended source of light on the shadow generated by *śaṅku*.

CONCLUDING REMARKS

In this paper, we have described the correction to the terrestrial latitude of an observer, due to effect of solar parallax and the finite diameter of the sun, as discussed in Nīlakaṇṭha's *Tantrasaṅgraha* and its commentary *Yuktidīpikā*. The value of the solar parallax given by Nīlakaṇṭha is far too large compared to the actual value, as he too has adopted the traditional view point that the maximum value of the parallax is equal to one-fifteenth of the mean daily motion of the celestial object. However, it is indeed remarkable that the problem has been correctly formulated and the explanations given are quite sound and convincing.

The correction due to the finite size of the sun has also been correctly taken into account. In fact, this correction is far more significant as its magnitude is much more than the parallax correction. The explanations provided by Śankara Vārier in his *Yuktidīpikā* are quite novel and reflective of the methodology of the Kerala school of astronomers.

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7. *ibid.*, verses 45-52 (p. 191-92).

8. This is because, the horizontal parallax in minutes is

$$R_e/d \times R = 1/15 \times D_{ms}.$$

Hence $d \times D_{ms}/R_e = 15 \times R = 51570$, as $R = 3438'$.

9. *ibid.*, verses 52-56 (p. 192).

10. *ibid.*, verses 56-58 (pp. 192-93).