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ACCURACY OF LUNAR ECLIPSE COMPUTATIONS OF THE *GRAHALĀGHAVAM*

An attempt has been made to compliment and contrast the results of the study on *Grahalāghavam* that appeared in the September 2003 issue of IJHS. It also throws light on the lucid work of Ramachandra Pandey on *Grahalāghavam*, which finds no mention in the study appearing in IJHS. The deficiency is rectified in the present work.

A study of the lunar eclipse computation of *Grahalāghavam* has recently appeared in IJHS 38.3 (2003)[1] which details the various arithmetical steps involved in deriving the *sparśa*, *mokṣa* and other times of a lunar eclipse. *Grahalāghava*'s reputation as a *karaṇa* work and the ingenuity of Gaṇeśa in achieving easy computation with accuracy is unparallel in the history of Indian Astronomy and the present note is an attempt to have a look at the accuracy achieved by Gaṇeśa through his ingenious *karaṇa*.

Methodology of Gaṇeśa

The salient features of the method employed can be summarized as follows:

1. Iṣṭa-Śaka of the *karaṇa* is 1441 and the epoch is the sunrise of Monday coinciding with *phālguṇa amāvasyā*, 19 March 1520 AD for which Gaṇeśa has provided the '*kṣepakas*' (K).
2. *Ahargaṇa* (A) or the elapsed days from the epoch is considered in terms of cycles of 4016 days, known as *Cakra*, which is less than 11 years of *Sūryasiddhānta* by 1.84625 days.
3. *Dhruva* (D) for each cycle is therefore defined using the orbital periods of the respective planets. In the case of Sun for example, *Dhruva* is [(4016/365.25875)*360 –360*11] = (-) 1°49′ 11″. For moon the sidereal period of 27.32168 gives D = (-)3°46′ 11″ and similarly other *Dhruvas* can be arrived at.

4. Longitude (λ) at any epoch having N *Cakras* is $\lambda = N*D + K + d\lambda_A$ where $d\lambda_A$ is the mean longitudinal increment over A days.

Efficacy of the method in giving correct positions

Criticism of the Gaṇeśa's arithmetical methods find a mention in the account of Gaṇeśa by SB Dikshit.[2] How good was the method in computing the planetary positions may be understood from working out the results and comparing the same with the modern astronomical values:

1. The epochal mean longitudes or *kṣepakas* given by Gaṇeśa is compared with modern mean longitudes in Table-1[3]. Also given are the mean longitudes as per *Grahalāghava* compared with the modern mean λ corrected for *ayanāṃśa* for 29.11.1974 at 0600 IST:

Table 1

Planet	*kṣepakas*	Mean λ*	Difference	29.11.74, 0600 IST		
				GL Mean λ	Modern Mean λ	Difference
Sun	349°41′	349°22′	+ 11′	223°47′	223°19′	+28′
Moon	349°06′	348°34′	+ 32′	39°51′	39°18′	+33′
Rāhu	27°38′	27°31′	+ 07′	226°47′	226°08′	+39′
Apogee-Moon	167°33′	165°18′	+ 135′	301°41′	298°12′	+209′
Mars	307°08′	306°06′	+ 62′	208°38′	208°24′	+14′
Mercury	269°33′	250°31′	+ 1142′	223°47′	163°33′	-----
Jupiter	312°16′	211°00′	+ 76′	331°03′	328°19′	+164′
Venus	230°09′	218°10′	+ 719′	223°47′	234°44′	----
Saturn	285°21′	283°38′	+ 103′	78°46′	78°53′	+11′

The following features are noteworthy in the above Table.

(a) The Sun, Moon and Rāhu are the most accurate values while Mercury, Venus and Moon's apogee are very much off the mark.

(b) *Dhruvas* given by Gaṇeśa for Sun was based on the *Sūryasiddhānta* year length of 365.25875 days and this introduces an error of one degree in 420 years. But the precession correction (*ayanāṃśa*) of Indian

* *Ayanāṃśa* used in converting the modern mean λ to the sidereal values comparable to *Grahalāghavam* is 16°38′. Gaṇeśa takes Śaka 444 or AD 522 as the zero year with the rate of 1 minute per year to fix the *ayanāṃśa*.

astronomy being 60″ in an year the above error fails to appear when contrasted with the modern mean λ.

(c) It is apparent from the data for 1974 that the *karaṇa* is giving reasonably good values even after 400 years.

2. Table -2 gives a comparison of the true positions of *Grahalāghava* with those of modern astronomy:

Table 2: Comparison of True λ

Graha	GL–λ	Mod.λ - 24°.2′	Modern λ	Error in GL
Sun	222°36′	222°12′	246°24′	+24′
Moon	35°13′	34°24′	58°36′	+49′
Mars	206°39′	207°29′	231°42′	-50′
Mercury	209°34′	210°45′	234°57′	-71′
Jupiter	317°13′	314°53′	339°05′	+130′
Venus	228°44′	227°46′	251°58′	+58′
Saturn	85°22′	83°13′	108°10′	+84′

It is apparent from the above the *karaṇa* of Gaṇeśa that it had its wide acceptance all over India in view of the easy techniques that gave reasonable accuracy irrespective of the lapse of time.

3. Data of IJHS September 2003. Rao et al gives the following data of *Grahalāghava* longitudes:

(a) 19th March 1520 AD, Sunrise at Ujjayini, 0600 LMT:
True Sun as per *Grahalāghava* = 34°35′, δλ = 57.5′ where as modern value (correctd for precession) is 34°06′ & δλ = 57.7′.
True Moon as per *Grahalāghava* = 205°56′, δλ = 736.25′, where as modern value (corrected for precession) is 205°31′, δλ = 734.17′.

Computed details of the lunar eclipse are contrasted with the modern results of the Skymap pro software in Table 3.

Table 3

GL		Modern		Remarks on GL values
True Sun	35-19-22	True Sun	34-50-34	+29'
True Moon	215-19-22	True Moon	214-50-34	+29'
Rāhu	25-15-39	True Rāhu	25-26-09	-11'
Full moon	$24^h\ 21^m\ 37^s$	Full moon	24-24-00	-2 min
Sparśa	23-29-39	Beginning of Umbral phase	23-31-41	-2 min
Madhya	24-21-37	Maximum	24-13-12	+8 min
Mokṣa	24-57-29	Moon leaves Umbra	24-54-52	+2 min
Duration	01-27-50	Duration	01-23-10	+4 min

(b) Another example given is the lunar eclipse of 16th July 2002
Authors begin computation on 16 July 2000 at 0529 IST

Table 4

GL		Modern		Remarks on GL values
True Sun	89-54-33	True Sun	89-08-11	+46'
True Moon	263-36-00	True Moon	262-50-09	+46'
Rāhu	90-38-00	True Rāhu	89-57-00	+41'
Parvānta	19^h -25^m -00	Full Moon	19-25 1ST	Exact
True Sun	0°57' -13"	True $\delta\lambda$ Sun	0°-57'-13.14"	Exact
True Moon	709'	True $\delta\lambda$ Moon	709'51"	Exact
Parvānta Sun	90° -27' - 46"	Sun λ opposition	89-41-24	+46'
Parvānta Moon	90° -27' - 36"	Moon λ opposition	89-41-24	+46'
Beginning	17^h -36^m 53^s IST	Enters Umbra	17-27-13	+9 min
Totality begins	18-40-04	Totality begins	18-32-02	+8 min
Middle	19-25-00	Middle	19-25-00	Exact
Totality ends	20-10-28	Totality ends	20 - 19 -03	-9 min
End	21-13-40	Moon leaves umbra	21-24-51	-11 min

It is very much remarkable that the opposition as per *Grahalāghava* and the modern astronomical methods perfectly match to the very second as above. Authors have also discussed the negligible error in the eclipse details by a comparison with the data of Indian astronomical ephemeris.

Remarkable work of Ramachandra Pandey

A study of *Grahalāghava* shall remain incomplete without a reference to the elucidation of the same by Ramachandra Pandey[4], which was published in 1994. Pandey has given a systematic work out of all computations and for the lunar eclise of 29 November 1974, sunrise 0623 LMT we find the following data in his work :

Table 5

Graha	GL-λ	Mod.λ-24^0.2	Difference
Sun	222^{0}36′	222^{0}36′	+22 ′
Moon	34^{0}52′	34^{0}24′	+28 ′
Rāhu	226^{0}47′	226^{0}08′	+39′
Apogee-Moon	301^{0}41′	298^{0}13′	+208′
Sun δλ	60′–53″–42	60-00-45	+0′53″
Moon δλ	800′–51″	822′–20″	+21′
Bhogya-ghaṭi	35gh-51vigh	34gh-35vigh	+34 vigh
Parvānta Sun	223-11-54	222-49-06	+22′
Parvānta Moon	53-11-54	52-49-06	+22′
Latitude of Moon	16′45″	17′58″	-1′13″
Sun diameter	31′11″	32′26″	-1′15′
Moon diameter	32′28″	32′16″	+0′12″
Beginning	1859LMT	1832 LMT	+27 min
Middle	2043LMT	2016 LMT	+27 min
End	2233 LMT	2201 LMT	+32 min
Begins Totality	2015 LMT	1938 LMT	+37 min
End of Totality	2117 LMT	2055 LMT	+22 min
Magnitude	1.249	1.295	-0.05

Full Moon occurs at 29 November Friday night 20:13 LMT as per modern astronomical computations. As derived using *Grahalāghava karaṇa* , the full moon is at 20:43 LMT.

Bhuja of *parvānta* Sun-Rāhu yields *bhuja* of 03-33-25, which is, less than 14^0 and therefore the eclipse is possible.

Śara or Latitude β is obtained as (7/11) times the above *bhuja* of 03-33-25 and β =05-35 *aṅgulas* = 16′45″. Modern astronomical value is 17′58″.

Sun's diameter = 30′ + (δλ -55)/5 = 31′11″; Pandey has added the footnote that Acārya Viśvanātha, the commentator of *Grahalāghava* did not accept this rule and as pointed out by Sudhakara Dvivedi the rule adopted by him was "2dλ/11 *aṅgulas*" which in this case yields the value 11.07 *aṅgulas* = 33′13″. This value is little more than the modern value. It may be noted that the average of both the methods leads to the correct value.

Moon's diameter is δλ/74 *aṅgulas* = 10.82 *aṅgulas* = 32′28″ against a modern value of 32′16″.

Beginning of the eclipse (*sparśa*) = 31*gh* 31 *vigh* = 1859 LMT
Middle = 35 *gh* 51 *vigh* = 2043 LMT.
End of the eclipse (*mokṣa*) = 40 *gh* 25 *vigh* = 2233 LMT.
Beginning of Totality = 34 *gh* 41 *vigh* = 2015 LMT
End of totality = 37 *gh* 15 *vigh* = 2117 LMT.
Magnitude = *Grāsa*/Moon's dia. = 13.52 *aṅgulas*/10.82 = 40′33″/32 = 1.249.

Above data is illustrative of the efficiency of *Grahalāghava karaṇa* and the present author was extremetly delighted to go through the example worked out by Ramachandra Pandey and to note its convergence with the modern values.

Comments on the example of Rao et al.

Balachandra Rao's example of computation of lunar eclipse for 16 July 2000 shows a complete agreement of the modern computed derived values for time of opposition and the δλs of Sun and Moon. This is quite unnatural and the time used 0529 IST also does not match with the sunrise at Ujjain which is 0523 LMT or 0550 IST.

Parvānta	19^h-25^m-00	Full Moon	19-25 IST	Exact
True δλ Sun	0^0-57′-13″	True δλ Sun	0^0-57′-13.14″	Exact
True δλ Moon	709′	True δλ Moon	709′51″	Exact

Against the above exactness, the longitudes given by Rao has a greater variation than in the example worked out by Ramachandra Pandey. Days elapsed from the epoch of Gaṇeśa is 175426 = 43 cycles and 2738 days that gives a longitude of $90^0 01'$ for Sun of 0550 IST for manual computation as per *Grahalāghava*. *Parvānta* Sun given by Rao at 1925 IST is 90-27-46 where as the actual value is 90-33-23 for the $\delta\lambda$ given by Rao. Authors of the paper under reference may give some clarification as to where the manual computation has gone wrong and how the above exactness can be reconciled with the variation that we see in the example worked out by Ramachandra Pandey. Exact coincidence of the middle of the eclipse - opposition - in respect of *Grahalāghava* based program and modern astronomy is quite unlikely and may be due to some error in the computation program.

In conclusion, it may not be out of place to mention that, a thorough treatment of the eclipse computation in terms of the *karaṇa* arithmetic of *Grahalāghava* is given by Rāmachandra Pandey. The credit for elucidating the methods of *Grahalāghava* in familiar arithmetical forms in recent times must certainly go to him. *Grahalāghava* results of the paper in *IJHS* and those of Ramachandra Pandey in the case of lunar eclipse computation have been contrasted with the corresponding modern values to illustrate the efficiency of the *karaṇa* work in yielding the correct values. Paper also gives a brief outline of the method adopted by Gaṇeśa in this work using cycles of 4016 days.

References

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