

## DHRUVAKA-VIKṢĒPA SYSTEM OF ASTRONOMICAL COORDINATES

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It is shown that treatment of *Dhruvaka-Vikṣēpa* system of *Sūryasiddhānta* by M. N. Saha and N. C. Lahiri and K. Chandra Hari give almost identical results for the probable epoch of that system. It is pointed out that the midheaven phenomena i.e. meridian transits of stars directly give the Right Ascension and Declination of the star. They might have been called Polar longitude and Polar latitude during the time of Hipparchus in analogy with the terrestrial longitudes and latitudes. So translation of *Dhruvaka* as Polar longitude is incorrect. The purpose of this unique and novel system of *Sūryasiddhānta* is explained.

**Key words:** Epoch of *Dhruvakas* and *Vikṣēpas*, Hipparchus coordinates, Polar longitudes, *Sūryasiddhānta*.

### 1. INTRODUCTION

*Dhruvakas* and *Vikṣēpas* given in *Sūryasiddhānta* have been used for determining the zero point of the Hindu Zodiac. We shall use the notation  $(\Delta^*, \beta^*)$  for *Dhruvaka* and *Vikṣēpas* respectively, which is based on their translation as Polar longitudes and latitudes. There are two approaches for determining the epoch of *Dhruvaka-Vikṣēpas* system.

In Method I used by Saha and Lahiri<sup>1</sup>, one starts with  $(\lambda^*, \beta^*)$  given in *Sūryasiddhānta*, calculate the ecliptic coordinates  $(\lambda, \beta)$  from them and compare the results with the values of  $\lambda(1950)$ ,  $\beta(1950)$ . For this purpose they use the following transformation equations:

$$\sin(\lambda - \lambda^*) = \tan \beta \cot B \quad \dots(1)$$

$$\sin \beta = \sin \beta^* \sin B \quad \dots(2)$$

$$\cos B = \cos \lambda^* \tan \varepsilon \quad \dots(3)$$

Here, we use the usual symbol  $\varepsilon$  for inclination of the ecliptic.

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In Method II, used by K.Chandra Hari <sup>2</sup>, we start from the equatorial coordinates  $\alpha$  (500),  $\delta$  (500), calculate  $(\lambda^*, \beta^*)$  from them and compare the results with the values given in *Sūryasiddhānta*.

Here, we use the following transformation equations:

$$\tan \lambda^* = \tan \alpha / \cos \varepsilon \quad \dots\dots(4)$$

$$\beta^* = \delta - \sin^{-1} (\sin \lambda^* \sin \varepsilon) \quad \dots\dots(5)$$

In Table 1 we give the values of  $\Delta x = x(\text{epoch}) - x(\text{SS})$ , where  $x = \lambda, \beta, \lambda^*, \beta^*$  for the 27 *yogatārās* as obtained from Table 32 of Saha and Lahiri and those obtained by Method II as given in column 6 and 7 (vide Table I).

Here the epoch is 1950 AD for Saha and Lahiri and 500 AD for Method II, while SS stands for the epoch of *Sūryasiddhānta* list. There are large deviations in  $\Delta\lambda$  and  $\Delta\lambda^*$  for  $\alpha$  Librae and  $\alpha$  Andromedae in both treatments. We shall, therefore, exclude them from this discussion, while comparing the results of the two methods.

## 2. EPOCH OF DHRUVAKA-VIKṢEPAS SYSTEM

From Table 1 we see that:

(i)  $\Delta\beta$  and  $\Delta\beta^*$  in columns 5 and 7 are of the same order of magnitude. Further  $\Delta\lambda$  of Method I given in column 4 and  $\Delta\lambda^*$  of Method II given in column 6 have similar ranges of  $5^\circ.91$  (from  $16^\circ.97$  to  $22^\circ.88$ ), and  $5^\circ.98$  (from  $-2^\circ.81$  to  $+3^\circ.17$ ), respectively.

(ii) Taking averages we get  $\overline{\Delta\lambda} = +20^\circ.44$  and  $\overline{\Delta\lambda^*} = 0^\circ.43$  as given in the lower portion of Table 1. Then assuming an average precession of  $50''$  per year, or  $1^\circ$  in 72 years, we find the mean epochs of SS list to be  $(1950 - 20.44 \times 72) = 478$  AD for Method I, and  $(500 - 0.43 \times 72) = 469$  AD for Method II. The two values are quite close and the average epoch from the two treatments comes out to be  $474 \pm 5$  AD as shown in the lower portion of Table 1. This can be compared with the most probable value of 430 AD obtained by the author <sup>3</sup> by an entirely different method.

(iii) However average values of  $\Delta\lambda$  and  $\Delta\lambda^*$  do not indicate a unique epoch, because the average of absolute values of  $\Delta\lambda^*$  is  $1^\circ.8$ , which is too large for the error of observation. So the large ranges of  $\Delta\lambda$  and  $\Delta\lambda^*$  point to different epochs for different *yogatārās*.

**Table 1:** Comparison of results obtained by Saha & Lahiri and K. Chandra Hari

<i>Sūryasiddhānta</i>			Method I		Method II	
Yogatārā	<i>Dhruvaka</i> ( $\lambda^*$ )	<i>Viksepas</i> ( $\beta^*$ )	$\Delta\lambda(1950)$	$\Delta\beta(1950)$	$\Delta\lambda^*(500)$	$\Delta\beta^*(500)$
$\beta$ Ari	8	10	21.27	-0.68	1.45	-0.88
41Ari	20	12	22.88	0.63	3.17	0.83
$\eta$ Tau	37.5	5	20.17	0.67	0.34	1.11
$\alpha$ Tau	49.5	5	20.95	0.65	1.02	1.07
$\lambda$ Ori	63	10	21.98	3.57	2.4	4.13
$\alpha$ Ori	67.33	9	22.23	7.17	3.04	7.75
$\beta$ Gem	93	6	19.67	0.68	0.28	0.2
$\delta$ Cnc	106	0	22.02	0.08	1.78	0.33
$\alpha$ Cnc	109	7	22.95	1.85	2.85	1.38
$\alpha$ Leo	129	0	20.23	0.47	0.08	0.12
$\delta$ Leo	144	12	20.68	3.03	1.51	3.01
$\beta$ Leo	155	13	20.78	0.2	0.83	0.17
$\delta$ Crv	170	11	18.35	2.08	2.66	2.34
$\alpha$ Vir	180	2	22.35	0.22	2.14	0.13
$\alpha$ Boo	199	37	20.6	3.02	0.67	2.13
$\alpha$ Lib	213	1.5	10.87	1.73	8.49	2.21
$\delta$ Sco	224	3	16.97	0.88	2.81	1.37
$\alpha$ Sco	229	4	18.97	0.72	1.32	0.28
$\lambda$ Sco	241	9	21	4.98	0.17	4.55
$\delta$ Sag	254	5.5	19.23	1	1.07	0.47
$\sigma$ Sag	260	5	21.3	1.53	1.31	2.11
$\alpha$ Aql	280	30	18.57	0.6	1.45	0.22
$\beta$ Del	290	36	19.52	3.63	0.04	3.14
$\lambda$ Aqr	320	0.5	21.03	0.08	0.71	0.04
$\alpha$ Peg	326	24	18.15	3.08	0.73	3.06
$\alpha$ And	337	26	26.3	1.68	4.89	2.16
$\sigma$ Pis	359.83	0	19.35	0.22	0.78	0.26
	Mem Epoch of (1) & (2)	(1)	Epoch	(2)	Epoch	
Average	474 $\pm$ 5 Ad	+20.44	478	+0.43	469 AD	
GI (9)	360 $\pm$ 18 AD	+21.96	379	+2.18	343 AD	
GII (9)	481 $\pm$ 2 Ad	+20.43	479	+0.24	483 AD	
GIII (7)	615 $\pm$ 2 AD	+18.51	617 AD	- 1.58	614 AD	

(iv) We can divide  $\Delta x$  into three groups as follows:

GI (9) -  $\beta$  Ari,  $\alpha$  Ari,  $\lambda$  Ori,  $\alpha$  Ori,  $\delta$  Cnc,  $\alpha$  Cnc,  $\delta$  Leo,  $\alpha$  Vir and  $\delta$  Sag ( $\Delta\lambda^*$  = +1.31 to +3.17)

GII (9) -  $\eta$  Tau,  $\alpha$  Tau,  $\beta$  Gem,  $\alpha$  Leo,  $\beta$  Leo,  $\alpha$  Boo,  $\lambda$  Sco,  $\beta$  Del and  $\lambda$  Aqr. ( $\Delta\lambda^*$  = -0.67 to +1.02)

GIII (7) -  $\delta$  Crv,  $\delta$  Sco,  $\alpha$  Sco,  $\delta$  Sag,  $\alpha$  Aql,  $\alpha$  Peg,  $\zeta$  Pis. ( $\Delta\lambda^*$  = -0.72 to -2.81)

They have average values of  $\Delta x$  given in the lower portion of Table 1 and give almost identical epochs for each group in both treatments as shown therein. The mean epochs for the three groups from the two treatments are found to be  $360 \pm 18$  AD for group I,  $481 \pm 2$  AD for group II and  $615 \pm 2$  AD for group III. This is in conformity with the conclusion of Saha and Lahiri that there is no unique epoch for the zero point of Hindu Zodiac. The actual range would be given by the range of  $\Delta x = + 3^\circ$ , from which we get the range of epochs as  $474 \pm 216$  AD i.e. from 258 AD to 690 AD.

Here we would like to draw attention to the discussion about the epoch of traditional *Sūryasiddhānta* by S.B.Dixit<sup>4</sup>. He has shown that the epoch of *Pañcasiddhāntic Sūryasiddhānta* is 505 AD. But there existed an earlier *Sūryasiddhānta* of Lāṭadeva which preceded it by a few centuries. According to him the presently extant version of *Sūryasiddhānta* is a repeatedly revised version of Lāṭakṛt *Sūryasiddhānta*, the revision being carried out beyond Brahmagupta's period of 7th century. Thus the range of 258 AD to 690 AD seems reasonable.

We would like to conclude this discussion about the zero point of Indian *nirayana* system by stating that one has to put an end to this controversy now as all Indian *pañcāṅga* makers have adopted vernal equinox of 285 AD as the zero point on the recommendation of the Calendar Reform Committee. It is the practice among scientists to follow a mutually agreed convention once it is adopted by consensus. It does not invalidate the historical determination of the epochs of various *Siddhāntas*.

### 3. COORDINATES OF HIPPARCHUS' CATALOGUE

According to some authors the coordinates in the Hipparchus' Catalogue are *Dhruvakas* and *Vikṣepas* as understood in *Sūryasiddhānta*. And they were determined by observing the midheaven phenomena i.e the meridian tran-

sits of stars. It is also claimed that the *Dhruvakas* were obtained by noting the time interval between the transits of the sun and the star on an equinox day and converting it into degrees. Now, actually this time difference gives the difference in the right ascensions of the sun and the star on all days including the vernal equinox. Since the right ascension of the sun on that day is zero the above time interval will give the right ascension of the star and not its longitude. Hence the transit times on vernal equinox day give  $\alpha$  and not  $\lambda$ . It may be pointed out that the time of meridian transit is not affected by the rising times of the *rās'is* and the stars.

In column 3 of our Table 2 we give the transit times at Ujjain on the vernal equinox day of 522 AD provided by K. Chandrahari. They give the values of  $\alpha$  in column 4 after adding  $0^\circ.25$  which was  $\alpha = \lambda$  of the sun on that day. In column 5 of that Table we give the values of  $\lambda^*$  obtained from equation (4). It is seen that they are in agreement with the values of  $\lambda^*$  of Method II given in column 6.

It is thus clear that the time of meridian transit of a star, when compared with the meridian transit of the sun directly gives the right ascension of the star. Also the zenith distance of the star ( $z$ ) at that time gives the polar distance  $90^\circ - \phi - z$  and the corresponding equatorial distance  $\delta = \phi + z$ , where  $z$  is positive if the star is north of zenith and negative if it is south of zenith,  $\phi$  being the latitude of the place. So if the original Hipparchus' Catalogue was based on meridian observations, which is quite natural, his coordinates would be  $\alpha$  and  $\delta$  as claimed by Delambre<sup>4</sup> and Duke<sup>5</sup>. It is quite likely that  $\alpha$  and  $\delta$  were known as Polar longitude and Polar latitude in Hipparchus' time, in analogy with the terrestrial longitudes and latitudes.

It is easy to convert  $(\alpha, \delta)$  into  $(\lambda, \beta)$  by the use of armillary astrolabe or by trigonometric formulae. Someone might have done this to Hipparchus' observations and the transformed data would have been updated by Ptolemy by applying precession correction to  $\lambda$  to produce his ASC.

#### 4. PURPOSE OF DHRUVAKS-VIKṢĒPA SYSTEM

It should be obvious that it is wrong to translate *Dhruvakas* and *Vikṣepas* as Polar longitudes and latitudes and identify them with the Hipparchus' coordinates. They are the most unnatural coordinates, which were novel

**Table 2 : Dhruvakas from transit times at Ujjain on 21 March 522 AD**

Yogatārā	$\lambda^*$ (SS)	Transit time Ujjain 522 AD	$\alpha$	$\lambda^*$	$\lambda^*$ (II)	$\beta^*$ (I)
$\beta$ Ari	8.00	12 <sup>h</sup> 35 <sup>m</sup> 22	9.05	9.88	9.45	9.12
41 Ari	20.00	13 <sup>h</sup> 25 <sup>m</sup> 98	21.75	23.50	23.17	11.17
$\psi$ Tau	37.50	14 <sup>h</sup> 21 <sup>m</sup> 88	35.72	38.08	37.84	3.89
$\alpha$ Tau	49.50	15 <sup>h</sup> 12 <sup>m</sup> 12	48.28	50.70	50.52	-6.07
$\lambda$ Ori	63.00	16 <sup>h</sup> 13 <sup>m</sup> 65	43.66	65.56	65.40	-14.13
$\alpha$ Ori	67.33	16 <sup>h</sup> 34 <sup>m</sup> 73	68.95	70.46	70.37	-16.75
$\beta$ Gem	93.00	18 <sup>h</sup> 01 <sup>m</sup> 87	93.22	92.87	92.72	6.20
$\delta$ Cnc	106.00	19 <sup>h</sup> 17 <sup>m</sup> 08	109.52	108.05	107.78	- 0.33
$\alpha$ Cnc	109.00	19 <sup>h</sup> 34 <sup>m</sup> 40	113.85	112.08	111.85	- 5.62
$\alpha$ Leo	129.00	20 <sup>h</sup> 45 <sup>m</sup> 78	131.73	129.34	129.08	0.12
$\delta$ Leo	144.00	21 <sup>h</sup> 45 <sup>m</sup> 80	147.95	145.70	145.71	18.01
$\beta$ Leo	155.00	22 <sup>h</sup> 29 <sup>m</sup> 87	157.72	155.96	155.83	13.17
$\delta$ Crv	170.00	23 <sup>h</sup> 12 <sup>m</sup> 47	168.37	167.36	167.34	-13.34
$\alpha$ Vir	180.00	00 <sup>h</sup> 10 <sup>m</sup> 50	182.87	183.06	182.14	- 2.13
$\alpha$ Boo	199.00	01 <sup>h</sup> 09 <sup>m</sup> 75	197.69	199.13	198.33	34.87
$\alpha$ Lib	213.00	01 <sup>h</sup> 32 <sup>m</sup> 93	203.48	205.33	204.51	0.71
$\delta$ Sco	224.00	02 <sup>h</sup> 36 <sup>m</sup> 92	219.48	221.91	221.19	- 1.63
$\alpha$ Sco	229.00	03 <sup>h</sup> 02 <sup>m</sup> 73	225.93	228.33	227.68	- 4.21
$\lambda$ Sco	241.00	03 <sup>h</sup> 56 <sup>m</sup> 62	239.4	241.51	240.83	- 13.50
$\delta$ Sag	254.00	04 <sup>h</sup> 47 <sup>m</sup> 83	252.21	253.58	252.93	- 5.90
$\sigma$ Sag	260.00	05 <sup>h</sup> 24 <sup>m</sup> 05	261.25	261.96	261.31	- 2.81
$\alpha$ Aql	280.00	06 <sup>h</sup> 38 <sup>m</sup> 92	267.19	267.42	266.88	- 62.40
$\beta$ Del	290.00	07 <sup>h</sup> 26 <sup>m</sup> 48	292.37	290.97	290.04	38.80
$\lambda$ Aqr	320.00	09 <sup>h</sup> 34 <sup>m</sup> 12	323.78	321.59	320.71	29.40
$\alpha$ Peg	326.00	09 <sup>h</sup> 51 <sup>m</sup> 68	328.17	328.88	325.27	20.40
$\alpha$ And	337.00	10 <sup>h</sup> 54 <sup>m</sup> 32	348.83	342.45	341.89	28.10
$\zeta$ Pis	359.83	11 <sup>h</sup> 57 <sup>m</sup> 25	359.56	359.52	359.05	0.26

concepts of the author of *Sūryasiddhānta* introduced for some specific purpose. That *Dhruvakas* and *Vikṣepas* are mostly given for *yogatārās* gives us the clue.

From Equation (2) we see that  $\lambda - \lambda^* = 0$  for points on the ecliptic,  $\beta = 0$ . Since planets have  $\beta \approx 0$ , we have  $\lambda$  (planet) =  $\lambda^*$  (planet) =  $\lambda^*$  (*yogatārā*) when the planet and the *yogatārā* transit the meridian at the same time. So the meridian *yoga* of the planet with the *yogatārā* was observed to determine the longitude of the planet. The *yogatārā* is thus quite appropriate.

The simultaneous transit of the planet and the *yogatārā* also gives  $\beta$  (planet) as follows. We have

$$z(\text{star}) - z(\text{planet}) = \beta^*(\text{star}) - \beta^*(\text{planet}).$$

Therefore

$$\beta^*(\text{planet}) = \beta^*(\text{star}) - z(\text{star}) + z(\text{planet}).$$

Then since  $\beta^*(\text{planet})$  is small, Equation (2) gives

$$\beta(\text{planet}) = \beta^*(\text{planet}) \sin B$$

where  $\sin B = \sin \alpha / \sin \lambda^*$ , with  $\tan \alpha = \tan \lambda^* \cos \epsilon$ . It will give a better value of  $\lambda(\text{planet}) = \lambda^*(\text{planet}) + \beta \cot B$ .

Such observations can be used for comparing the computed and observed positions of planets.

## 5. CONCLUSION

We have shown that the Polar longitudes and latitudes of Hipparchus were nothing but Right ascension and Declination of later times. The *Dhruvaka-Vikṣepas* system is a unique and original concept of Siddhāntic astronomers which was used for determining the ecliptic longitude and latitude of planets by their meridian *yogas* (conjunctions) with the *yogatārās*. Also there is no unique epoch for them.

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