Indian Journal of History of Science, 49.3 (2014) 260-267

NEMICANDRA'S RULES FOR COMPUTING MULTIPLIER AND DIVISOR

DIPAK JADHAV*

(Received 23 September 2013, revised 27 July 2014)

Abstract

The paper makes a study of the two rules offered by Nemicandra (c. 981 AD), one for computing multiplier and the other for divisor. It finds that the term *viralita-rāśi* employed by the Jaina school of Indian mathematics is equivalent to the index of the power of a quantity. The fact that a logarithm is simply an index was not observed long after John Naiper (1550-1617 AD) who discovered theory of logarithms. On the basis of the illustration given by his pupil Mādhavacandra Traividya to the first of the above two rules, the paper also corroborates that the fact was known to the school.

Key words: *Ardhaccheda*, Divisor, Index of power, Jaina school of Indian mathematics, Logarithm, Multiplier, Nemicandra, *Viralita-rāśi*

1. INTRODUCTION

The Jaina schools played a prominent role in early and later Indian mathematics.[1] The canvas is vast and wide. However, it is divided into two classes the canonical and exclusive.[2] The canonical has dealt mainly with cosmological system and other the *karma* theory (the matter, exceptionally subtle, which actually does flow into the *jīva*, soul/bios). Mathematical materials found embedded in their works occurs in the form of rules and results and at some places in the functioning form. The *Bhagavatī Sūtra*[3] of Sudharma Svāmī (300 BC or earlier), the *Tattvārthādhigama Sūtra Bhāṣya*[4] of Umāsvāti (some period between 150 BC and 219 AD), the *Tiloyapaṇṇatti*[5] of Yativṛṣabha (some period between 176 AD and 609 AD), the *Dhavalā*, a commentary on the *Ṣaṭkhaṇḍāgama*[6] of Puṣpadanta and Bhūtabalī of some period between 87 AD and 156 AD, of Vīrasena (816 AD) and the *Samyakjñānacandrikā*[7], a solo commentary on the *Gommamṭasāra* and *Labdhisāra* of Nemicandra (c. 981 AD), of Ṭoḍaramala (1720-1767 AD) are some of the works of the canonical class. The authors of the exclusive class were originally mathematicians and contribute exclusively on mathematics. Some of the works of this class are the *Pāṭīgaṇita*[8] and *Triśatikā*[9] of Śrīdhara[10] (c. 799 AD), the *Gaṇitasārasaṅgraha*[11] of Mahāvīra (c. 850 AD) and the *Gaṇitasārakaumudī*[12] of Ṭhakkara Pherū (1265-1330 AD).

Nemicandra (c. 981 AD) belonged to the canonical class of the Jaina school of Indian mathematics. Cāmuṇḍarāya, his disciple and a celebrated commander-in-chief and wise minister of the *Gaṅga* dynasty during the period from 953 AD to 985 AD, erected the colossal image of Bāhubalī at Śravaṇabelagola in India. Nemicandra is said to have been associated with the first consecration ceremony of the image, held on March 13, 981 AD as it is well identified.[13]

Nemicandra appears to be the first mathematician to have set forth the laws of

* Lecturer in Mathematics, Govt. Boys Higher Secondary School, Pansemal, Distt. Barwani, 451770 (M.P.), India. Email: dipak_jadhav17@yahoo.com

logarithms but in terms of *ardhaccheda* (= $\log_2 x$ where x is some quantity) and *vargaśalākā* (= $\log_2 \log_2 x$ where x is some quantity).[14]

In this paper, <P> would indicate that P is a paraphrase supplied by the present author here to achieve comprehensiveness with clarity.

Ardhaccheda cannot be literally translated "half-divisor". Its actual meaning is "<the number of possible> divisions by two" as he himself refers to it to be equal to the number of times that a particular quantity is successively halved (or divided by 2) to get the quantity reduced to one.[15] Similarly, *vargaśalākā* should not be interpreted "square-stick" although *śalākā* literally stands for "stick". He refers to its two definitions. In one the *vargaśalākā* of a particular quantity is equated to the number of times that 2 is successively squared to get the quantity acquired, and in the other the *vargaśalākā* of a particular quantity is equated to the *ardhaccheda* of the quantity.[16]

Long before Nemicandra the Jaina school of Indian mathematics had been well acquainted with the concept of *ardhaccheda*. This can be easily traced in the works of the school such as in the *Tiloyapaṇṇatti*[17] of Yativṛṣabha (some period between 176 AD and 609 AD) and in the *Dhavalā*[18] of Vīrasena (816 AD). Like *ardhaccheda* the school also developed *trikaccheda* and *caturthaccheda*; they are equal to $\log_3 x$ and $\log_4 x$ respectively where is some quantity.[19] Logarithms of this sort were developed and used in only the canonical class of the school.

In Europe, theory of logarithms was discovered by John Naiper (1550-1617 AD), Baron of Merchiston (then near, now in Edinburgh), and Jobst Bürgi (1552-1632 AD), a court clock-maker by profession in Switzerland. However, their approaches were entirely different. The former had a geometric approach as he took two parallel lines, one infinite and the other finite, with moving particles while the latter used algebraic methodology as his perception was based directly in the relation between two progressions, one arithmetic and the other geometric.[20] It is Naiper who compounded the two ancient Greek terms *logos*, meaning ratio, and *arithmos*, meaning number, to coin the term logarithm, meaning ratio-number.[21] Logarithms made it possible to transform multiplications and divisions into additions and subtractions respectively. Facilities of these sorts were required in Naiper's time in many fields like observational astronomy and navigation.[22] He himself had written that "his logarithms will save calculators much time and free them from the slippery errors of calculations".[23]

If the *ardhaccheda* of a is n, then we can, denoting *ardhaccheda* by *AC*, write it $AC(a) = n$. A. N. Singh opined that *mediation*, an operation considered important in Egypt and Greece along with the *duplication*, was generalized into a theory of logarithms to the base 2, 3, 4, etc.[24] and does not allow us to deem that logarithms of this sort from the beginning of their conception were based on indices although $AC(a) = n$ is rightly transformed into $2^n = a$, $TC(a) = n$, into $3^n = a$, $CC(a) = n$ into $4^n = a$ where *TC* and *CC* are the abbreviations of *trikaccheda* and *caturthaccheda* respectively. But today we are able to say that a logarithm whether it was of *ardhaccheda* sort or approached through geometry was bound to be observed, sooner or later, to be an index.

The *Trilokasāra* ('An Essence of the Three Regions of the Universe') is Nemicandra's celebrated work in 1014 Prakrit verses, mainly on cosmology and cosmography. In it we find two rules, one for computing multiplier for a given difference between the indices of product and multiplicand and the other for computing divisor for a given difference between the indices of dividend and quotient.

Did the Jaina school of Indian mathematics use any general term for the index of the power of a quantity? The author raised this question in a paper published in *Arhat Vacana*.[25] That a alogarithm is simply an index was also referred to in the paper.[26,27]

The purpose of this paper is, therefore three fold namely to understand the two rules of Nemicandra on their own terms with modern impact and to corroborate that above fact was known to the school. This gives us an idea of the Jain historiographic tradition, where it lets us to know if a logarithm, discovered in ancient Indian culture-area, too was as an index.

For the reason that the school developed theory of indices in requisite structure by involving ideas such as ordinal succession and raising a quantity to its own power and using the particular terms such as *varga* (square) and *ghana* (cube)[28], the last two folds are essential to be dealt.

2. Multiplier and Divisor

Let P, Q and R be three quantities such that

$AC(P) = p$, $AC(Q) = q$ and $AC(R) = r$

or $P = 2^p$, $Q = 2^q$, and $R = 2^r$.

Further, suppose that P is operated by Q to yield R or in notation

$$P * Q = R \tag{1}$$

It happens to be

$$P \times Q = R$$

when $r > p$. And $(r - p)$ is said to be surplus to p with respect to r.

In this case Nemicandra gives the following rule to compute Q for a given surplus.

> *varalidarāsīdo puṇa jettiyamettāṇi ahiyarūvāṇi* |
>
> *tesiṃ aṇṇoṇṇahadī guṇagāro laddharāsisa*||[29]
>
> "The mutual product (i.e., the product obtained by mutual multiplications) of as many (q) of those <integers 2, 3, etc. (a)> as the unities (*rūva*) that are <placed> beyond the distributed quantity (*varalida-rāsī* , appropriate term: *viralida-rāsī*, Skt. *viralita-rāśi*, p) is the multiplier (*guṇagāra*, Skt. *guṇakāra*, a^q) of the quantity obtained (*laddha-rāsī*, Skt. *labdha-rāśi*, a^p) <by means of distribution and substitute>."

That is to say

$$\overbrace{\underbrace{11\ldots1}_{p}\,\underbrace{11\ldots1}_{q}}^{r} \qquad (a^p . a^q = a^r). \tag{2}$$

It may here be easily pointed out that *viralita-rāśi* and index are equivalent in mathematical sense. And what is beyond p is $(r - p)$. For $a = 2$, (2) can be written as

$$Q = 2^{r-p} \tag{3}$$

(1) happens to be

$$P \div Q = R$$

when $r < p$. And $(p - r)$is said to be deviation to p with respect to r.

In this case Nemicandra gives the following rule to compute for a given deviation.

> *viralidarāsīdo puṇa jettiyamettāṇi hīṇarūvāṇi* |
>
> *tesiṁ aṇṇoṇṇahadī hāro uppaṇṇarāsissa* ||[30]
>
> "The mutual product (i.e., the product obtained by mutual multiplications) of as many (q) of those <integers 2, 3, etc. (a)> as the unities (*rūva*) that are missing from the distributed quantity (*viralida-rāsī*, Skt. *viralita-rāśi*, p) is the divisor (*hāra*, a^q) of the quantity produced (*uppaṇṇa-rāsi*, Skt. *utpanna-rāśi*, a^p) <by means of distribution and substitute>."

That is to say

$$\overbrace{\underbrace{11\ldots1}_{r}\,\underbrace{11\ldots1}_{q}}^{p} \qquad (a^p \div a^q = a^r). \tag{4}$$

Here again we are able to point out that *viralita-rāśi* and index are equivalent in mathematical sense. And what is missing from p

is $(p - r)$. For $a = 2$, (4) can be written as

$$Q = 2^{p-r} \tag{5}$$

In order to make the interpretations drawn from the above two verses more convincing the explanation regarding the two terms, one *labdha-rāśi* and the other *utpanna-rāśi*, is the following. The term *labdha-rāśi* usually means 'the quotient-quantity' in Indian mathematics but in the first of the above two verses it has been taken in the sense of 'the multiplicand-quantity'. On the other hand, the term *labdha* has been employed in the verse 105 of the *Trilokasāra*[31] in the sense of 'product' while in the sense of 'quotient' in its verse 106.[32] In fact, the word *labdha-rāśi* or *labdha* has been engaged in all of these verses in the sense of 'quantity obtained' and the sense yields mathematical term according to the context. In the same spirit the word *utpanna-rāśi* has been inserted in the second of the above two verses and in the verses 107 and 108 of the *Trilokasāra*.[33]

The following is the context in which he offers the above two rules. In order to find P when $\log_2 P$ (i.e., p) is given, he refers to the rule, incorporated in the verse 75 of the *Trilokasāra*, which reads that "placing twos as many times as the *addhacheda* (Skt. *ardhaccheda*, $\log_2 P$) and mutually multiplying them, the quantity (*rāsī*, Skt. *rāśi*, P) is obtained"[34]. Following the same course of action what quantity is obtained when 'surplus to p' (*adhikaccheda*, full term: *adhikārdhaccheda*, $\log_2 R - \log_2 P$ i.e., $r - p$) is given?[35] This is what is stated in the preamble of the first of the above two rules. If there remains any doubt regarding when 'deviation to p' (*hīnaccheda*, full term: *hīnārdhaccheda*, $\log_2 P - \log_2 R$ i.e., $p - r$) is given, incidentally the rule for that purpose is below.[36] This is what is the meaning of the preamble of the second of the above two rules.

3. Reply to the Question

A product of n equal factors, $(a \times a \times a \times ... \times a) = a^n$, where a is the base and n is the index of the power, is called the n^{th} power of and reads 'a raised to the n^{th} power'.

Because of the algebraic power symbols used by Rene Descartes (1637 AD) we, today, easily express $a \times a \times a \times ... \times a$ (n factors) as a^n. Why was such a notation introduced? It is simply a matter of convenience. Surely it saves time and space if we write a^n instead of $a \times a \times a \times ... \times a$ (n factors). On the other hand in ancient time $a \times a \times a \times ... \times a$ (n factors) was to be taken as it is.[37]

For the multiplication of equal quantities Bhāskara I (c. 629 AD) employs a special term *gata*. According to him, the term *dvigata* means square, *trigata* means cube and so on. He illustrates that the *dvigata* of 4 is the product of 4 and 4 or 4^2; the *trigata* of 4 is the continued product of 4 and 4 and 4 or 4^3 and so on.[38] Following him, a^n will be expressed by saying the *n gata* of *a*. The same expression occurs in the *Brāhmasphuṭasiddhānta* (c. 628 AD) of Brahmagupta.[39]

Today, the term *ghātāṅka*, coined by compounding *ghāta* and *aṅka*, is used in Hindi Mathematics Education for the index of the power of a quantity. It is not known to the present author when and how it came into practice but it is certain that the term *ghāta* was accustomed either as multiplication or as product at least till the period of Nārāyaṇa Paṇḍita (c. 1356 AD).[40] Here it may be noted that the term 'index' was first used for n in 1586 AD by Schoner.[41] Before him, Michael Stifel had used the word 'exponent' for n.[42]

The question was posed to know if the Jaina school of Indian mathematics had any general term for the index of the power of a quantity.

Both of the rules contain a term *viralita-rāśi*. It has come definitely in the sense of the index of the power of a quantity. It seems to be formed by joining the two words: *viralita* (distributed) and *rāśi* (quantity). *Viralita* seems to be derived from *viralana* (distribution, abbreviated D) so that it can work as an adjective. The latter has been a noted operation in the school.[43] It means the separating of a given positive integer $n(>1)$ (say) ($n=1$ is also meaningful.) into its unities as shown below:

$$D(n) = 1 \quad 1 \quad 1 \ldots \quad n \text{ times.}$$

It is followed by another operation called *deya* (substitute, abbreviated *S*, original meaning: to be given) which means to put a given positive integer a (>1) (say) in place of everywhere in the above distribution as shown below:

$$S(a)_{D(n)} = a \quad a \quad a \ldots \quad n \text{ times.}$$

Then comes the turn of the act of multiplying (abbreviated *M*) together as shown below:

$$M[S(a)_{D(n)}] = a \times a \times a \times \ldots \times a \ (n \text{ factors})$$

or $$M[S(a)_{D(n)}] = a^n.$$

Following the above process, we can say that the index n is called *viralita-rāśi* ('distributed quantity') because its constituent parts (i.e., unities) are 'distributed' (i.e., put down with interstice).

For covering the case $n = 0$, we, today, define that $a^0 = 1$. In the above manner, there is no distribution when $n = 0$ and a is substituted nowhere. In such position, we shall have to define that a reduces to unity as a has lost, being no place to be substituted in the distribution, even its power of one time. Moreover, according to the Jaina school of Indian mathematics, unity is not a number but a collection of units is a number. Two, three etc are numbers.[44]

4. Corroboration of the Fact

Mādhavacandra Traividya was an immediate pupil[45] of Nemicandra. He wrote a commentary in Sanskrit on the *Trilokasāra*, which is available in published form along with the *Trilokasāra* itself. He gives an illustration to explain the first of the above two rules as follows:

> *viralitarāśiḥ pa* 16 *palyachedaḥ* 4 *tasmāda dhikarūpachedaḥ* 3 *tanmātradvikā-nyonyāhatau* 8 *labdhaḥ palyarāśeḥ* 16 *guṇakāro bhavati* |[46]

> "The distributed quantity (*viralitarāśi*, *p*), i.e., 'the <number of> divisions <into halves>' (<*ardhac*> *cheda*) of *playa* (*P*), *pa* 16 <in symbolic notation>, is 4. The <number of> divisions <into halves of the quantity to be obtained> from the unities that are <placed> beyond it (distributed quantity) is 3. What is obtained in mutual multiplications of as many two's as those (divisions), i.e. 8, is the multiplier (*guṇakāra*, *Q*) of the quantity of *playa*."

That is to say:

$$\left.\begin{array}{l} p = 4, \text{i.e.,} \\ \log_2 P = 4 \end{array}\right\} p = \log_2 P \tag{6}$$

and

$$Q = 2^{\log_2 R - \log_2 P} \tag{7}$$

when expressed in general terms.

What is *R* in the illustration? To make the above illustration fully clear, he further adds as follows:

> 16×8 *tayoḥ guṇyaguṇakārayorguṇane sāgaropamaḥ 128 syāt* /[47]

> "The *sāgaropama* (*R*, 128) is arrived at when the multiplicand (*guṇya*, *P*, 16) is multiplied by the multiplier (*guṇakāra*, *Q*,8)."

Before we analyze Traividya's above illustration it may be noted that *palya* and *sāgaropama* are simile measures founded, developed and applied in only the Jaina canonical texts.[48]

Equating the corresponding parts of the equations (3) and (7), we have (6) and

$$r = \log_2 R \,. \tag{8}$$

viralitarāśiḥ pa 16 *palyachedaḥ* 4

This is what the illustration begins with. This statement yields (6). Here works the fact with the base two. (8) too confirms the same. In her commentary on the *Trilokasāra* Āryikā Viśuddhamati (1929-2002 AD) interprets the statement, in Hindi, as follows:

> *yahaṁ viralana rāśi palya ke ardhaccheda haiṁ* |[49]
>
> "Here the distributed quantity (*viralana rāśi*, appropriate term: *viralita-rāśi*) is 'the <number of> divisions into halves' (*ardhaccheda*) of *palya*."

The above interpretation of hers supports our finding although it holds the fact in its converse form.

Sometimes the word *ardha* has been deleted by Nemicandra from *ardhaccheda* and there remains simply the term *cheda*.[50] This is why in the translation of the statement we have suggested that *cheda* should be replaced by the full term *ardhaccheda*. It is also supported by what is given in Viśuddhamati's interpretation. Otherwise, *cheda* would mean divisor. In the translation of her interpretation it has been suggested by us that the appropriate term for *viralana rāśi* is *viralita-rāśi*. It is evidently confirmed by the above original statement.

The context we have seen in the section two was a particular one. The following is the broad context. Nemicandra refers to fourteen sequences and their analysis in the 38 verses of the *Trilokasāra* extending from the verse 53 to the verse 90 with a purpose to realize the validity of *saṅkhyāta* (numerate), *asaṅkhyāta* (innumerate) and *ananta* (infinite),[51] three subclasses of natural numbers excluding one founded in the school prior to him for measure. Further, in the verse 91, he suggests to read the *Bṛhaddhārāparikarma* (Greater <Treatise> on the Logistics of Sequences) to know more about those sequences.[52] B. B. Datta reports that the treatise has been lost.[53] Here it can be easily inferred that material incorporated in those 38 verses was extracted from that treatise. Remarkable is that the verse 76 that contains the definitions of *ardhaccheda* and *vargaśalākā* and the verse 75 that contains the method for finding a quantity when its *ardhaccheda* is given are among those 38 verses. Those verses that, though they have something to do with *ardhaccheda* or *vargaśalākā* or the both, appear after the verse 91 seem to be his creations. Among them are the four verses extending from 105 to 108 that form the laws of logarithms but to the base two[54] and the above two verses that contain the rules for computing multiplier and divisor.

5. Conclusion

The term *viralita-rāśi* employed by the Jaina school of Indian mathematics is equivalent to the index of the power of a quantity. The fact that '<*ardhac*>*cheda*' was '*viralita-rāśi* <with base two>' was well known to the school.

6. Acknoledgements

Except for a few changes and a number of additions, this paper was first presented in *National Symposium on Sciences in India: From Early Times to Independence*, held during April 21-24, 2008 at University of Mysore, Mysore, India. The author is grateful to three unknown learned scholars in the field and the learned referee of this journal for their helpful suggestions and valuable comments.

7. Notes and References

1. The introductory articles on the Jaina school of Indian mathematics are the following two ones. Datta, B.B. The Jain School of Mathematics. *Bulletin of Calcutta Mathematical Society*, 21 (1929) 115-145. Jain, L. C. On the Jaina School of Mathematics in: *Chhotelal Memorial Volume*, Calcutta (now known as Kolkata), 1967, pp. 265-292.

2. For details see the following two ones. Jadhav, Dipak. Theories of A. P. and G. P. in Nemicandra's Works. *ArhatVacana*, 16.2 (2004) 35-40. Jadhav, Dipak. *Mensuration in India from Jaina Sources*, Ph. D. Thesis, Devi Ahilya University, Indore, 2013, pp. 34-52.

3. Sudharma Svāmī, *Vyākhyāprajñaptisūtra* (or *Bhagavatī Sūtra*), ed. and tr., Amar Muni and Shri Chandra Surana, 4 Vols., Śrī Āgama Prakāśana Samiti, Beawar (Rajsthan), 1991-1994. Also see: Deleu, Jozef. *Viyāhapaṇṇatti (Bhagavaī): Introduction, Critical Analysis, Commentary and Indexes*, University of Ghent, Ghent, 1970.

4. Umāsvāti, *Tattvārthādhigamasūtram* (with his own commentary), ed., Śrīmad Vijayarāmacandra Sūrīśvara, Navasari, 1994.

5. Yativṛṣabha, *Tiloyapaṇṇatti*, *Tiloyapaṇṇatti* (with Āryikā Viśuddhamati's Hindi commentary), ed. and tr., C. P. Patni, 3 Vols., Śrī 1008 Candraprabha Digambara Jaina Atiśayakṣetra, Dehra-Tijara, 1997.

6. Puṣpadanta and Bhūtabalī, *Ṣaṭkhaṇḍāgama* (with *Dhavalā* Commentary of Vīrasena), ed. and tr., H. L. Jain, et. al., 16 Vols.,Jaina Saṁskṛti Saṁrakṣaka Saṅgha, Amaraoti, 1940-58.

7. Jain, L. C. and Trivedi, R. K. Ṭoḍaramala of Jaipur (A Jaina Philosopher-Mathematician). IJHS, 22.4 (1987) 359-371.

8. Śrīdharācārya, *Pāṭīgaṇita* (with an Ancient Sanskrit Commentary), ed. and tr., Kripa Shankar Shukla, Department of Mathematics and Astronomy, Lucknow University, Lucknow, 1959.

9. Śrīdhara's, *Triśatikā*, ed., Sudhakara Dvivedi, Chandraprabha Press, Benares,1899

10. The reasons to regard Śrīdhara to be of Jaina faith may be seen in the followings. Sastri, N. C. Jain. Śrīdharācārya. *Jaina Siddhānta Bhāskara*, 14.1(1947) 31-42. Sastri, N. C. Jain. *Tīrthaṅkara Mahāvīra aura Unakīācārya Paramparā*, Vols. I and III, Bhāratavarṣīya Digambara Jaina Vidvatpariṣad, Sagar, 1974. pp. 187-195. Jain, Anupam and Jain, Jaychand. Kyā Śrīdhara Jaina the? *Arhat Vacana*, 1.2 (1988) 49-54.

11. Mahāvīra, *Gaṇitasārasaṅgraha*, ed. and tr., into English by M. Rangacharya, 1912 and into Kannada by Padmavathamma, 2000, Śrī Siddhārtakirti Granthamālā, Hombuja, 2000.

12. Ṭhakkara Pherū's, *Gaṇitasārakaumudī* (with introduction, translation, and mathematical commentary), ed. and tr., SaKHYa, Manohar Publishers and Distributors, New Delhi, 2009.

13. Sastri, op. sit.1974, p. 422; Jain, Jyoti Prasad. Bhagawana Gommatesh and Shravanbelgola. *The Jaina Antiquary*, 34.2 (1981) 1-18, Jadhav, Dipak. Why do I assign 981 A.D. to Nemicandra? *Arhat Vacana*, 18.1 (2006) 75-81.

14. Jadhav, Dipak. The Laws of Logarithms in India. *Historia Scientiarum* 11.3 (2002) 261-267.

15. Nemicandra, *Trilokasāra* (with Mādhavacandra Traividya's Sanskrit commentary and Āryikā Viśuddhamati's Hindi commentary), ed. and tr., R. C. Jain Mukhtara and C. P. Patni, Mahaviraji, 1975. Nemicandra, *Triloksara* op cit. 1975, the first three quarter of the verse 76, p. 69; and Jadhav, *op. sit.*2002, p. 262.

16. Jadhav, op. cit. 2002, p. 263.

17. Yativṛṣabha,*Tiloyapaṇṇatti*,op.cit. Vol. I, 1997, v. 1.131, p. 30.

18. Singh, A. N. Mathematics of Dhavalā. Published in: Puṣpadanta and Bhūtabalī, *Ṣaṭkhaṇḍāgama* (with *Dhavalā* Commentary of Vīrasena), ed. and tr., H. L. Jain, et. al., Book 4, 1942, 1-21 esp. p. 7.

19. Puṣpadanta and Bhūtabalī, op. cit. Vol. II and Book 3, 1940-58, p. 56; and Singh, op. cit., 1942, pp.7-8

20. Bose, A. C. John Naiper: His Life and Work. *Bulletin of Calcutta Mathematical Society*, 4 (1913-1914) 53-60. Bose, A. C. John Naiper: His Life and Work. *Bulletin of Calcutta Mathematical Society*, 6 (1914-1915) 13-31; Hooper, Alfred. *Makers of Mathematics*, Faber and Faber Limited, London, 1948, pp.169-193; Smith, D. E. *History of Mathematics*, Vol. II, Dover Publications, New York, 1958. pp. 513-523; and Clark, K. M. and Montelle, C. Logarithms: The Early History of a Familiar Function. *Loci* (Online http://dx.doi.org/10.4169/loci003495), June 2010, 1-11

21. Clark and Montelle,op.cit., June 2010, p. 4; and Mohanti, Subodh. John Napier the Inventor of Logarithms. *Dream 2047*, 10.10 (2008) 34-32.

22. Clark and Montelle, op. cit., June 2010, pp. 2-3.

23. Mohanti, op. cit. 2008, p. 33.

24. Singh, op. cit., 1942, p. 73, esp. pp 61-62. Also see: Jadhav, Dipak. Theories of Indices and Logarithms in India from Jaina Sources. *Arhat Vacana*, 15.4 (2003) 53-73.

25. Jadhav, Deepak. op. cit., 2003, p. 73.

26. Bose, op. cit. 1914-1915, p. 16; Hooper, op. cit., 1948, p.174; and Cajori, Florian. *A History of Mathematics*, McMillan Company, New York, 1958. p. 149.

27. Jadhav,op. cit., 2003, pp. 65, 67 and 73.

28. Jadhav, op. cit., 2003, pp. 54-59; and Jadhav, Dipak. On Raising a Number to its Own Power in Ancient India. *Gaṇita Bhāratī*, 30.2(2008):139-149.

29. Nemicandra, *Trilokasāra*,op. cit., 1975, v. 110, p. 106.

30. Nemicandra, *Trilokasāra*,op. cit., 1975, v. 111, p. 107.

31. Nemicandra, *Trilokasāra*,op. cit., 1975, p. 101; and Jadhav, op. cit., 2002, p. 263.

32. Nemicandra, *Trilokasāra*,op. cit., 1975, p. 101; and Jadhav, op. cit. 2002, p. 264.

33. Nemicandra, *Trilokasāra*,op. cit., 1975, p. 102; and Jadhav, op. cit., 2002, p. 265.

34. Nemicandra, *Trilokasāra*,op. cit., 1975, p. 67.

35. Nemicandra, *Trilokasāra*,op. cit., 1975, p. 106.

36. Nemicandra, *Trilokasāra*,op. cit., 1975, p. 107.

37. Jadhav, Dipak. Three Ancient Approaches to the Sum of a Geometric Progression. *Mathematics in School* 37.3 (2008) 26-29.

38. *Āryabhaṭa*, *Āryabhaṭīya* (with BhāskaraI's Commentary and Someśvara's Commentary), ed. and tr., K. S. Shukla, Indian National Science Academy, New Delhi, 1976., pp. 43-44.

39. Brahmagupta, *Brāhmasphuṭasiddhānta* (with Sanskrit and Hindi Commentaries), ed. and tr., Ram Swarup Sharma and others, Vol. II, Indian Institute of Astronomical and Sanskrit Research, New Delhi, 1966., vv. 18.41-42.

40. *Āryabhaṭa*, *Āryabhaṭīya*, op. cit., the commentary below the first hemistich of the verse 3, p. 49; BhāskaraII, *Bijagaṇita*, ed. and tr., Pt. Devachandra Jha, K. D. Academy, Varanasi, 1983, vv. 9.18-19, pp. 195-197; Bhāskara II, *Līlāvatī*, ed. and tr.,Patwardhan K.S., Naimpally S.A. and Singh S.L., Motilal Banarsidass, New Delhi, 2001, v. 20, p. 19 and v. 25; Nārāyaṇa, *Gaṇitakaumudī*, ed., P. Dvivedi, Part II, Benares, 1942, v. 67, p. 77 and v. 96, p. 113.

41. Smith, op. cit., Vol. II, 1958, p. 522.

42. Smith, op. cit., Vol. II, 1958, p. 522; and Bose, op. cit. 1914-1915, p. 16.

43. Singh, op. cit., 1942, p. 6; and Jadhav, op. cit.,2008, pp. 141-144.

44. Ganitanand. When there was no Unity in the Number-Land. *Gaṇita Bhāratī*, 8.1-4 (1986) 44-45.

45. Nemicandra, *Trilokasāra*,op. cit., 1975, pp. 4 and 768.

46. Nemicandra, *Trilokasāra*, op. cit., 1975, Traividya's comment below v. 110, p. 106.

47. Nemicandra, *Trilokasāra*, op. cit., 1975, Traividya's comment below v. 110, p. 106.

48. *Anuyogadvāra Sūtra*, ed. and tr., Miśrīmalajī Mahārāja Madhukara and others, Publication No. 28, Śrī Āgama Prakāśana Samiti, Beawar (Rajsthan), 1987, *sūtras* 369-382, pp. 291-297 and *sūtras* 392-398, pp. 322-326;Yativṛṣabha, *Tiloyapaṇṇatti*, op.cit., 1997, vv. 1.117-130, pp. 26-30; Nemicandra, *Trilokasāra*, op. cit., 1975, vv. 92-104, pp. 86-99.

49. Nemicandra, *Trilokasāra*, op. cit., 1975, Viśuddhamati's interpretation, p. 106.

50. Nemicandra, *Trilokasāra*,op. cit., 1975, the first hemistich of the verse 8, p. 12 and the last quarter of the verse 105, p. 101.

51. Nemicandra, *Trilokasāra*,op. cit. 1975, pp. 49-85; and Jain, L.C. Divergent Sequences Locating Transinfinite Sets in *Triloksāra*. *IJHS*, 12.1 (1977) 57-75.

52. Nemicandra, *Trilokasāra*, op. cit., 1975, p. 86.

53. Datta, B. B. Mathematics of Nemicandra. *The Jaina Antiquary*, 1.2 (1935) 25-44, esp. p. 32.

54. Nemicandra, *Trilokasāra*, op.cit., 1975, pp. 101-102; and Jadhav, op. cit., 2002, pp. 263-265.