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Rationale for *Vākyas* pertaining to the Sun in *Karaṇapaddhati*

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Abstract

In the *vākya* system of astronomy prevalent in south India, the true longitudes of the Sun, the Moon, the planets, and associated quantities can be directly found using *vākyas* or mnemonics. The set of *vākyas* for a specific physical variable presented at regular intervals is essentially a numerical table. The text *Karaṇapaddhati* of the Kerala astronomer Putumana Somayāji (ca. 1732 AD) describes methods to obtain the set of *vākyas*, based on the general principles of Indian astronomy. In particular, it presents the rationale for obtaining the various *vākyas* pertaining to the Sun, namely '*māsavākyas*', '*saṅkrāntivākyas'*, '*nakṣatrasaṅkramaṇavākyas*', and '*yogyādivākyas*'. In this article, we explain the procedures outlined in *Karaṇapaddhati* to obtain the sets of *vākyas* pertaining to the Sun.

Key words: *Karaṇapaddhati*, Mean longitude of Sun, *Māsavākya*, *Nakṣatra-saṅkramaṇavākya*, *Saṅkrāntivākya*, True longitude of Sun, *Vākya*, *Vākyakaraṇ*a, *Yogyādivākya*.

1. Introduction

Among the texts on Indian astronomy, the *siddhānta* texts lay down the procedures for the astronomical results, with detailed explanations in a theoretical framework, whereas the *tantra* texts merely express the results in the form of analytical formulae without much explanations. In contrast, the *karaṇa* texts have only direct computational algorithms, which are at times just arithmetical without even involving the trigonometrical functions, with a recent date as the epoch. The *vākya* texts like *Vākyakaraṇa* do not even have the algorithms, but just mnemonics or *vākyas* for finding the positions of celestial objects. Till recently, the *vākya*-based almanac in the Tamil areas of south India was based solely on the text *Vākyakaraṇa*, and the auxiliary tables for the longitude of the Moon (*candravākyas*) and the *kujādi-pañcagrahavākyas* (sentences for the five planets - Mars etc.).

The term *vākya* literally means a sentence consisting of one or more words. In the context of astronomy, the string of letters in which numerical values associated with some physical quantities are encoded. Usually *vākyas* are composed using the *kaṭapayādi* system (Subbarayappa and Sarma 1985, p. 47-48), which is one of the commonly employed systems to represent numbers in Indian astronomical works. Generally *vākyas* are composed in such a way that they not only represent numerical values, but form beautiful meaningful sentences that convey worldly wisdom or moral values.

In the normal method of finding the *sphuṭagraha* (true longitude) followed in most Indian astronomical works, the mean longitude of a planet is found first, and a few *saṃskāras* (corrections)[1] are applied to it to obtain the true position. In contrast, in the *vākya* method, the tables in the form of *vākyas* directly give the true

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[1] Only *mandasaṃskāra* in the case of the Sun and the Moon, with an additional *śīghrasaṃskāra* in the case of the other five planets.

longitudes at certain regular intervals.[2] The true longitudes at an arbitrary instant are to be found by using these *vākyas* along with interpolation techniques. The *vākya* method greatly facilitates the preparation of *pañcāṅgas* (almanacs),[3] because here we circumvent the normal procedure of arriving at the true longitudes by applying *saṃskāras*, which would be quite strenuous and time consuming.

In this paper, after giving a brief introduction to the text *Karaṇapaddhati* (*KP*) in section 2, we proceed to discuss the method for obtaining the *vākyas* pertaining to the Sun, as presented in *KP*. In these methods, an important ingredient is the determination of the mean longitude corresponding to a specified true longitude of the Sun, which is discussed in section 3. We explain the methods for obtaining the '*māsavākyas* (monthly sentences)', '*saṅkrāntivākyas* (transition sentences)' and '*nakṣatra-saṅkramaṇavākyas* (*nakṣatra*-transition sentences)' in section 4, and the method for obtaining the true longitude at any instant, using the '*yogyādivākyas*' in section 5. The appendix gives the details regarding the relations between the mean and true longitudes of the Sun in *KP*.

2. The text *Karaṇapaddhati*

KP of Putumana Somayāji composed around 1732 AD is one of the important texts of the Kerala School of astronomy (Sarma, 1972). The first edition of this text in *Devanāgarī* was brought out in 1937 (*Karaṇapaddhati*, 1937). Subsequently, it has also been edited with mathematical notes in Malayalam by Koru (*Karaṇapaddhati*, 1953), and there is another edition by Nayar with two commentaries in Malayalam whose authors are not known (*Karaṇapaddhati*, 1956). It is a unique work quite different from the category of *siddhānta*, *tantra* and *karaṇa* texts. Though the word *karaṇa* appears in the title of the text, this is not a usual *karaṇa* text. It is more like a manual for preparing the *karaṇa* and *vākya* texts, by giving the *paddhati* (the method) for them (Pai, 2011; Sriram and Pai, 2012; Sriram, 2015).

Karaṇapaddhati is divided into ten chapters. The first chapter is on the mean longitudes of the planets, their revolution numbers and epochal corrections. Chapter 2 is on the procedure for finding the small multipliers and divisors for the sidereal periods of the planets, using the method of continued fractions. The third and fourth chapters deal with the specific techniques for obtaining the longitude of the Moon, and with the development of the procedure for the more complicated problem of obtaining the longitudes of the actual planets, relevant for the *vākya* method. Chapter 5 deals with the corrections to the revolution numbers of the planets and the epochal values of the longitudes, when the traditional astronomical constants are not in agreement with observations. The sixth chapter deals with the exact relation between the circumference and the diameter of a circle, the series expansion for the sine and cosine functions, etc. Chapter 7 has a general discussion on the theoretical aspects of the computation of planetary longitudes, planetary distances, visibility etc., as well as on '*māsavākyas'* (month sentences), '*saṅkramaṇavākyas*' (transition sentences) and '*yogyādivākyas'* (sentences beginning with *yogya*). Chapters 8, 9 and 10 deal with the diurnal problems and topics related to the shadow .

3. The mean longitude of the Sun at the *saṅkramaṇas*

The word *saṅkramaṇa* or *saṅkrānti* refers to 'cross over' or 'transit' of an object from one space / division to another. According to the solar

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[2] The interval is usually one day for the Moon, and in the case of planets it varies longitudes at an arbitrary instant are to be widely and depends on several factors which include their own rate of motion with respect to their *mandocca* and *śīghrocca*.

[3] These are manuals that give the positions of planets for each day of the year.

calendrical system followed in India, a solar year is the time interval between successive transits of the Sun across the beginning point of the *Meṣarāśi* (First point of Aries). The solar year is divided into 12 solar months (*sauramāsas*). The duration of each month is the time spent by the Sun in a particular *rāśi* (zodiacal sign) among the twelve *rāśis*, namely *Meṣa* (Aries), *Vṛṣabha* (Taurus), *Mithuna* (Gemini) etc. In other words, it is the time interval between two successive *rāśi* transits (*rāśisaṅkramaṇa*), which occurs when the Sun just crosses the interstitial point between the two *rāśis*. For example, when the Sun is at the beginning of the *Siṃharāśi* (Leo), transiting from *Karkaṭaka* (Cancer) to *Siṃha* (Leo), it is *Siṃhasaṅkramaṇa*. Similarly, a *nakṣatra-saṅkramaṇa* (transition to the next *nakṣatra*) occurs when the Sun transits from one *nakṣatra* (27^{th} part of the zodiac, with the names *Aśvinī*, *Bharaṇī*, etc.) to the other.

The calculations related to *saṅkramaṇas* (transitions) are based on the true longitudes of the Sun. For instance, a *rāśi-saṅkramaṇa* (zodiacal transit) occurs when the true longitude is an integral multiple of 30^o. The true longitude of the Sun does not increase uniformly with time. However, the variation of the mean longitude is proportional to time. Conversely, the time-intervals are proportional to the difference in mean longitudes. As explained in the appendix the mean longitude of the Sun, θ_0 is obtained from true longitude θ, using the relation (see (9) in the Appendix)

$$\theta_0 - \theta = \sin^{-1}\left[\frac{3}{80}\sin(\theta - \theta_m)\frac{R}{R_v}\right]$$

where $\theta_{m,}$ is the longitude of the Sun's apogee (whose value is taken to be 78^o in the text), R is the *trijyā* (whose value is 3438'), and R_v is the *viparyāsakarṇa* (inverse hypotenuse) given by (see (7) in the Appendix)

$$R_v = \sqrt{\left(R - \frac{3}{80}R\cos(\theta - \theta_m)\right)^2 + \left(\frac{3}{80}\times\sin(\theta - \theta_m)\right)^2}$$

At the *saṅkramaṇa* (transit), the true longitudes of the Sun are multiples of 30. That is, $\theta_i = 30 \times i$, where i = 0, 1, ..., 11 for *Meṣa*, *Vṛṣabha*, ..., and *Mīna* respectively. We now illustrate the procedure for obtaining the mean longitude from the true longitude, by computing it using the formula stated in the text and explained in the Appendix, for two transits namely *Mithuna-saṅkramaṇa* (transition to Gemini, $\theta = 60^o$) and *Kanyā-saṅkramaṇa* (transition to Virgo, $\theta = 150^o$).

Example 1: *Mithuna-saṅkramaṇa* (transition to Gemini, $\theta = 60^o$)

$$R_v = \sqrt{\left(R - \frac{3}{80}R\cos(60-78)\right)^2 + \left(\frac{3}{80}\times\sin(60-78)\right)^2}$$

$$= 3321.52'$$

and $$\theta_0 - \theta = \sin^{-1}\left[\frac{3}{80}\sin(360-(78-60))\frac{R}{R_v}\right]$$

$$= -0.687^\circ$$

Therefore,

$$\begin{aligned}\theta_0 &= 60^o - 0.687^o\\ &= 59.313^o\\ &= 1^r 29^\circ 19'.\end{aligned}$$

Example 2: *Kanyā-saṅkramaṇa* (transition to Gemini, $\theta = 150^o$)

$$R_v = \sqrt{\left(R - \frac{3}{80}R\cos(150-78)\right)^2 + \left(\frac{3}{80}\times\sin(150-78)\right)^2}$$

$$= 3398.14'$$

and $$\theta_0 - \theta = \sin^{-1}\left[\frac{3}{80}\sin(150-78)\frac{R}{R_v}\right]$$

$$= +2.068^\circ.$$

Therefore,

$$\begin{aligned}\theta_0 &= 150^o + 2.068^o\\ &= 152.068^o\\ &= 5^r 02^\circ 04'.\end{aligned}$$

The mean longitudes at the transits known as '*saṅkramaṇārkamadhya'* are given as *vākyas* in one of the commentaries of *Karaṇapaddhati* (*Karaṇapaddhati,* 1956). These are listed in Table 1, and compared with the values computed as above. Here, the *Vṛṣabha* (Taurus) appears first, as the transit across its beginning point corresponds to the end of the first solar month, and the *Meṣa* (Aries) appears last as the transit across its beginning point marks the end of the twelfth solar month, and also the solar year itself. It may be noted that the two values differ only in three cases, and that too by 1' only.

4. Obtaining *Māsavākyas*, *Saṅkrānti-vākyas* and *Nakṣatravākyas*

Let d_i denote the number of days that have elapsed from the beginning of the year to the end of the particular solar month (corresponding to the i^{th} *rāśi*). Obviously, d_i need not be an integer. A *māsavākya* is the integer closest to d_i. The fractional part, in terms of *nāikās* can be found from the *saṅkrāntivākyas*, which give the remainders when d_i are divided by 7. A *nakṣatra-saṅkrāntivākya* is the equivalent of the [*rāśi*] *saṅkrāntivākya*, for the *nakṣatra* division of the zodiac. In this section, as well as the next, we provide some illustrative examples, where the computations for finding the true longitude from the mean, and vice versa are performed using the methods given in the text, and explained in the Appendix.

4.1 *Māsavākyas*

Verse 22 in chapter 7 of *Karaṇapaddhati* gives the procedure for obtaining the *māsavākyas* and *saṅkrāntivākyas*.

> *bhāgīkṛtāt tadanu saṅkramaṇārk-amadhyāt abdāntadoḥphala-yutāddharaṇīdi naghnāt*|
>
> *saurairdinairapahṛtaaṃ khalu māsavākyaṃ saṅkrāntivākyamiha tatsuhṛtāvaśicmam* ||
>
> After that, having obtained the mean longitude of the Sun in degrees at [the time of] transit (*saṅkrānti*) and adding the *doḥphala* (difference between the mean and the true Sun) at the end of the year (*abdānta*) to it, multiply it by the number of civil days (*bhūdina*) and divide by the number of solar days [in a *mahāyuga*].

Table 1. The mean longitudes of the Sun at *saṅkramaṇas* presented in the commentary II [*Karaṇapaddhati* (1956), p. 223) in the form of *vākyas*, compared with the computed values.

Name of the *rāśi* (Zodiacal sign)	*saṅkramaṇamadhyavākyas*				Computed		
	sign. (*r*)	deg. (°)	min. (′)	*vākyas* in *kamapayādi*	sign. (*r*)	deg. (°)	min. (′)
Vṛṣabha	0	28	22	*śreṣṭhaṃ hi ratnam*	0	28	22
Mithuna	1	29	19	*dhānyadharo'yam*	1	29	19
Karkaṭaka	3	00	27	*sukhī anilaḥ*	3	00	28
Siaṃha	4	01	29	*dharaṇyāṃ nabhaḥ*	4	01	29
Kanyā	5	02	4	*vānarā amī*	5	02	4
Tulā	6	02	5	*munīndro 'nantaḥ*	6	02	5
Vṛścika	7	01	33	*bālāhyo nāthaḥ*	7	01	33
Dhanus	8	00	38	*jale ninādaḥ*	8	00	38
Makara	8	29	35	*śūladharo hi*	8	29	34
Kumbha	9	28	37	*sāmbo hi pradhānaḥ*	9	28	36
Mīna	10	27	59	*dharmasukhaaṃ nityam*	10	27	59
Meṣa	11	27	53	*lakṣmī surapūjyā*	11	27	53

[The result obtained is] indeed the *māsavākya*. The remainders obtained by dividing those (*māsavākyas*) by 7 (su) are called *saṅkrāntivākyas*.

The true longitudes of the Sun at the end of each month are 30°, 60°, ..., 360°. At the end of the 12th month, which is the same as the beginning of the first month in the next year, the true longitude of the Sun is 360°. The mean longitude corresponding to the true longitude of 360° is found to be 357.883° = -2.117° = -2° 7' = 11^r27°53'. The difference between the true and the mean longitudes at the end of the year is termed the '*abdāntadoḥphala*' (the difference between the true and mean longitudes at the year-end), denoted by $\Delta\theta_0$ in the verse, whose value is 2° 7'.

The *madhyamabhoga* (difference in the mean longitudes) reckoned from the *Meṣa-saṅkramaṇa* to *iṣṭasaṅkramaṇa* (desired transition) is the difference in the mean longitudes at the desired zodiacal transit and the transit at *Meṣādi* of the true Sun. It is found by adding 2° 7' to the mean longitude at each transit. For example, the true longitude of the Sun at the *Siṃhasaṅkramaṇa* is 120°. The mean longitude corresponding to this is 121.479°. Adding $\Delta\theta_0$ to it, we obtain 123.596° as the *madhyamabhoga* from the *Meṣasaṅkrama* to the *Siaṃhasaṅkrama*.

A (mean) solar day is the time interval corresponding to an increase of 1° in the mean longitude. This is slightly longer than a civil day, and is given by $\frac{d_c}{d_s}$, where d_c and d_s represent the numbers of civil days and solar days in a *mahāyuga*. Note that the values given in the *Karaṇapaddhati* for d_c and d_s are 1577917500 and 1555200000 respectively. Let θ_{mb} represent the *madhyamabhoga*. Then,

$$d_i = \theta_{mb} \times \frac{d_c}{d_s}$$

For *Siṃhasaṅkramaṇa*, θ_{mb} = 123.596° and therefore

$$d_i = \frac{123.596 \times 1577917500}{1555200000} = 125.401 \qquad \ldots(1)$$

The *māsavākya* is the integral closest to d_i. Hence, 125 is the *māsavākya* at the *Siṃhasaṅkrama*. The *māsavākyas* corresponding to all the transits and also d_i's are listed in Table 2.

Table 2. The *māsavākyas* given in the textual commentary I [*Karaṇapaddhati* (1956), p. 225) compared with the computed values of d_i.

Name of the *rāśi* transited (*saṅkramaṇa*)	*māsavākyas* textual value of d_i in *kaṭapayādi*	in numerals	computed value of d_i
Vṛṣabha	*kulīna*	31	30.925
Mithuna	*rūkṣajīa*	62	62.326
Karkaṭaka	*vidhāna*	94	93 .933
Siṃha	*mātrayā*	125	125.401
Kanyā	*kṣaṇasya*	156	156.435
Tulā	*siṃhasya*	187	186.892
Vṛścika	*suputra*	217	216.795
Dhanus	*catvarām*	246	246.304
Makara	*tathādri*	276	275.654
Kumbha	*mīnāṅgi*	305	305.111
Mīna	*mṛgāṅgi*	335	334.919
Meṣa	*mātulaḥ*	365	365.258

By finding the difference between the successive *māsavākyas*, the number of civil days corresponding to each month can be calculated.

4.2 *Saṅkrāntivākyas* and *nakṣatravākyas*

The instant at which the *rāśisaṅkramaṇas* occur can be determined by dividing d_i by 7. The remainder obtained would give the *saṅkrāntivākyas*. For instance, in the previous example

$$\frac{125.401}{7} = 17 + \frac{6.401}{7}.$$

The remainder is 6.401. In this, the integral part represents the day and the fractional part multiplied by 60 would give the *nāḍikā*. Here the obtained day of the week corresponds to number 6 and *nāḍikā* is 24.1. The *vākya* for this is *marutaḥ*, which represents the day as 6 and *nāḍikā* as 25.

The *saṅkrāntivākyas* which are given in the commentary of the text for different transits are listed in Table 3, and compared with the computed values.

It is clear that the value of d_i corresponding to a *saṅkramaṇa* is obtained by adding a suitable multiple of 7, to the *saṅkrāntivākya*. For example, we have to add 91 to the day component of the *saṅkrāntivākya* for *Karkaṭaka* (2+91) to obtain d_3 whose value is 93 days 56 *nāḍikā*.

4.2.1 *Nakṣatravākyas*

nakṣatrāntasphuṭotpanna-
madhyārkādevameva ca |

nayennakṣatrasaṅkrānti-
vākyaṃ kaviṣu pūrvakam ||

In a similar manner, the *nakṣtravākyas* that commence with *kaviṣu* can be obtained by finding the mean longitudes of the Sun from its true longitudes at the end of the *nakṣatras*.

We know that the ecliptic (*rāśicakra*, 360°) is divided into 27 equal parts called *nakṣatras*, each part corresponding to 13° 20'. The basis of this division is the Moon's sidereal period (≈ 27 days). The term *nakṣatra* also refers to the time spent by the Moon in any of these divisions. In the same vein, the time spent by the Sun to traverse through these divisions are called *mahānakṣatras*. The true longitudes of the Sun at the end of the 27 *nakṣatras* are 13°20', 26°40', 40°, 53°20', ..., 360°. Converting these longitudes to the corresponding mean ones and adding 2°7' to them, we obtain the

Table 3. The *vākyas* in the commentary II (*Karaṇapaddhati*, 1956, p. 226) and the computed values of the *saṅkrāntivākyas*

Name of the *rāśi*	*saṅkrāntivākyas*				
	in *kaṭapayādi*	in numerals		computed values	
		day	*nāḍikā*	day	*nāḍikā*
Vṛṣabha	*timire*	2	56	2	55.5
Mithuna	*niratam*	6	20	6	19.5
Karkaṭaka	*camare*	2	56	2	56.0
Siṃha	*marutaḥ*	6	25	6	24.1
Kanyā	*surarām*	2	27	2	26.1
Tulā	*ghṛṇibhaḥ*	4	54	4	53.5
Vṛścika	*javato*	6	48	6	47.7
Dhanus	*dhaṭakaḥ*	1	19	1	18.2
Makara	*nṛvarāṭ*	2	40	2	39.3
Kumbha	*sanibhaḥ*	4	7	4	6.7
Mīna	*maṇimān*	5	55	5	55.2
Meṣa	*cayakā*	1	16	1	15.5

madhyamabhogas or the increase in the mean longitude of the Sun at the end of each *nakṣatra* starting from *Aśvinī*. The number of civil days corresponding to these *madhyamabhogas* can be calculated by multiplying them by the *bhūdinas* and dividing by the solar days in a *mahāyuga*. These values are presented in Table 4.

Table 4. No. of civil days elapsed at each *Nakṣatra-saṅkramaṇa*

Name of the *Nakṣatra*	**No. of civil days elapsed before the *Nakṣatra-saṅkramaṇa***
Bharaṇī	13.674
Krittikā	27.461
Rohiṇī	41.349
Mṛgaśirā	55.318
Ārdrā	69.343
Punarvasū	83.395
Puṣyā	97.442
Āśleṣā	111.454
Maghā	125.401
Pūrvaphālgunī	139.260
Uttarāphālgunī	153.015
Hasta	166.654
Citrā	180.175
Svātī	193.581
Viṣākhā	206.881
Anurādhā	220.090
Jyeṣṭhā	233.224
Mūla	246.304
Pūrvācāhā	259.352
Uttarācāhā	272.393
Śrāvaṇa	285.449
Dhaniṣṭhā	298.543
Śatabhicaj	311.697
Pūrvabhādrapadā	324.93 1
Uttarabhādrapadā	338.262
Revatī	351.702
Aśvinī	365.258

The instant at which the *nakṣatra-saṅkramaṇa* occurs can be obtained from the *nakṣatra-saṅkrāntivākyas*. When we divide the civil days at each transit by 7, the remainders obtained are the *nakṣatra- saṅkrāntivākyas*, similar in spirit to the *saṅkrāntivākyas* discussed earlier. The *nakṣatra-saṅkrāntivākyas* as given in the commentary are tabulated and compared with the computed values in Table 5.

5. The *Yogyādivākyas*

Unlike the *vākyas* discussed earlier, wherein the nomenclature was based upon a certain time interval or phenomenon, here the name *yogyādivākyas* stems from the fact that the set of 48 *vākyas* begins with the word *yogya*. These *vākyas* enable us to find the longitude of the Sun at any given instant. There are 4 *vākyas* corresponding to each solar month. Each month is divided into four parts with a maximum of 8 days per part. Now, the *sphuṭabhoga* of each part is the difference between the true longitudes of the Sun at the beginning and at the end of that part. The difference in minutes between the *sphuṭabhoga* of each part and 8^{o} are the *yogyādivākyas*. If the longitudinal difference is greater than 8^{o}, then it is to be taken as positive and negative otherwise.

The definition of the *yogyādivākyas* and the method of applying them to obtain the true longitude of the Sun at an interval of 8 days in a solar month, are given in verse 24, chapter 7 of *Karaṇapaddhati*:

> *māsādito 'ṣṭāṣṭadinotthasūrya-*
> *sphuṭāntarāṃśāṣṭadināntarāṇi* |
>
> *yogyādivākyāni dhanarṇataiṣāṃ*
> *dinālpatādhikyavaśādināptau* ||
>
> [First] the difference in the true longitudes of the Sun in degrees etc. at an interval of eight days from the beginning of the month [is found]. The difference between [this value] and eight constitutes the *yogyādivākyas*. These are [applied] positively or negatively, depending upon whether 8 is lesser or greater [than the difference in longitudes at each 8 days interval respectively], to obtain the [true] Sun [at any given instant].

Table 5. The *vākyas* in the commentary (*Karaṇapaddhati*, 1956, p. 228) and the computed values of the *nakṣatra-saṅkrāntivākyas*

***Nakṣtra* transit (*saṅkramaṇa*)**	***nakṣatra-saṅkrāntivākyas***				
	in *kaṭapayādi*	**in numerals**		**computed values**	
		day	***nāḍikā***	**day**	***nāḍikā***
Bharaṇī	*kaviṣu*	6	41	6	40.4
Krittikā	*hāriṣu*	6	28	6	27.7
Rohiṇī	*dīyata*	6	18	6	20.9
Mṛgaśirā	*dhīyate*	6	19	6	19.1
Ārdrā	*kariṣu*	6	21	6	20.6
Punarvasū	*māriṣu*	6	25	6	23.7
Puṣyā	*sāriṣu*	6	27	6	26.5
Āśleṣā	*dūrataḥ*	6	28	6	27.2
Maghā	*smarati*	6	25	6	24.0
Pūrvaphālgunī	*duṣyati*	6	18	6	15.6
Uttarāphālgunī	*yoniṣu*	6	1	6	0.9
Hasta	*parvaṇā*	5	41	5	39.2
Citrā	*trikaśa*	5	12	5	10.5
Svātī	*tāṇava*	4	36	4	34.9
Viṣākhā	*bhomṛga*	3	54	3	52.9
Anurādhā	*dhenugaḥ*	3	9	3	5.4
Jyeṣṭhā	*supura*	2	17	2	13.4
Mūla	*hāmaka*	1	18	1	18.2
Pūrvācāhā	*nīrana*	0	20	0	21.1
Uttarācāhā	*bhāratā*	6	24	6	23.6
Śrāvaṇa	*caraṇa*	5	26	5	26.9
Dhaniṣṭhā	*gālava*	4	33	4	32.6
Śatabhiṣaj	*viśvagu*	3	44	3	41.8
Pūrvabhādrapadā	*carmarām*	2	56	2	55.9
Uttarabhādrapadā	*cikura*	2	16	2	15.7
Revatī	*rāvaya*	1	42	1	42.1
Aśvinī	*markama*	1	15	1	15.5

5.1 How to obtain the *yogyādivākyas*?

The *yogyādivākyas* as given in the edited version of the commentary are listed in Table 6. Apart from the *vākyas* (here in the form of one word, which form part of meaningful sentences), the signs are also given in the commentary. Except in the case of *Tulā*, all the 4 *vākyas* corresponding to a partcular *raśi* have the same sign (+ or –) and indicated as such in the table. For *Tulā*, the sign for the first *vākya* is – and the signs for the other three are all +, as indicated in the table. The rationale behind these *yogyādivākyas* is best explained by taking up a couple of concrete examples.

Consider the solar month of *Mithuna*. The true longitude of the Sun is $\theta = 60^o$ at the beginning of the month. The mean longitude θ_0 can be determined using the method explained earlier and we find $\theta_0 = 59^o\ 18.7'$. Using the fact that the rate of motion of the mean longitude of the Sun is 59.136' per day, the mean longitude is $\theta_0 = 67^o\ 11.8'$ after 8 days in the month of *Mithuna*. The *mandaphala* ($\theta - \theta_0$) corresponding to this value of θ_0 is found to be 24.1'. Adding this to θ_0, we find the true longitude after 8 days to be 67° 11.8' + 24.1' = 67° 35.9'. Hence the increase in the true longitude after the first 8 days of the month is 7° 35.9'. As the longitudinal difference is less than 8°, the *yogyādivākya* is negative and is given by " (8° – 7° 35.9') = –24.1', compared with the value of –24' as given by the *vākya* '*vīraḥ*' in the commentary (*Karaṇapaddhati*, 1956).

After 16 days in the month of *Mithuna*, the mean longitude θ_0 = 59° 18.7' + 59.136' × 16 = 75° 4.8'. The true longitude corresponding to this is found to be θ = 75° 11.4'. Hence the difference between the true longitudes at the beginning and at the end of the second part is 75° 11.4' – 67° 35.9' = 7° 35.5'. Here again as the longitude difference is less than 8°, the *yogyādivākya* is negative and is given by – (8° – 7° 35.5') = –24.5', compared with the value of –25' as implied by the *vākya* '*śūraḥ*' in the commentary.

5.2 Finding the true longitude of the Sun from the *yogyādivākyas*

One can obtain the true longitude of the Sun on any day using the *yogyādivākyas*, and linear interpolation. For example, suppose we would like to find the true longitude of the Sun after the lapse of 18 days in the *Vṛṣabha* month. This comes in the third part (*khaṇḍa*). Therefore the approximate value of the true longitude of the Sun after 18 days elapsed would be

$$\theta' = 30^\circ + 18^\circ = 48^\circ.$$

Table 6. The 48 *yogyādivākyas* mentioned in the commentary (*Karaṇapaddhati*, 1956, p. 229).

Month name		***Yogyādivākyas* (in minutes)**							
Meṣa	–	*yogyo*	11 (11.2)	*vaidyaḥ*	14 (13.5)	*tapaḥ*	16 (15.7)	*satyam*	17 (17.7)
Vṛṣabha	–	*dhanyaḥ*	19 (19.3)	*putraḥ*	21 (20.9)	*kharo*	22 (22.3)	*varaḥ*	24 (23.3)
Mithuna	–	*vīraḥ*	24 (24.1)	*śūraḥ*	25 (24.5)	*śaro*	25 (24.6)	*vajrī*	24 (24.4)
Karkaṭaka	–	*bhadram*	24 (23.9)	*gotro*	23 (23.1)	*ruruḥ*	22 (21.9)	*karī*	21 (20.5)
Siṃha	–	*dhanyaḥ*	19 (18.9)	*sevyo*	17 (17.0)	*mayā*	15 (14.9)	*loke*	13 (12.7)
Kanyā	–	*kāyo*	11 (10.6)	*dīnaḥ*	8 (8.2)	*stanām*	6 (5.8)	*ganā*	3 (3.3)
Tulā		*yājño*	-1 (-1.5)	*yajñam*	+1 (+0.8)	*ganā*	+3 (+3.0)	*śūnā*	+5 (+4.9)
Vṛścika	+	*steno*	6 (6.2)	*dīno*	8 (7.7)	*dhunī*	9 (8.9)	*namaḥ*	10 (9.9)
Dhanus	+	*āpaḥ*	10 (10.3)	*pāpaḥ*	11 (10.7)	*payaḥ*	11 (10.8)	*pathyam*	11 (10.5)
Makara	+	*pūjyā*	11 (10.2)	*dhenuḥ*	9 (9.4)	*dine*	8 (8.2)	*rthinaḥ*	7 (6.8)
Kumbha	+	*tanuḥ*	6 (5.7)	*bhinnā*	4 (3.9)	*khanī*	2 (1.9)	*jñaanī*	0 (-0.3)
Mīna	–	*ratnaṃ*	2 (2.0)	*bhānuḥ*	4 (4.4)	*suniḥ*	7 (6.8)	*nayaḥ*	10 (9.3)

A correction which can be called *yogyādisaṃskāra* $\Delta\theta'$ has to be applied to θ' in order to obtain the true longitude θ.

Now, the correction for 8 days of the third *khaṇḍa* is given as 22' (*khara*). Hence the correction for 2 days is $\frac{22\times 2}{8}$ minutes. Adding this to the sum of the first two *vākyas* (*dhanya* and *putra*),

$$\Delta\theta' = 19 + 21 + \frac{22\times 2}{8} = 45.5'$$

These corrections are indicated as negative in the listing of the *vākyas* in the commentary. Hence applying this result negatively to θ' the true longitude of the Sun at the end of the 18th day of the solar month *Vṛṣabha* is given by

$$8 = 48^0 - 45.5' = 47°14.5'.$$

5.3 Some observations

It is clear from the examples given above, that this method can be used to determine the true longitude at any instant during the day using interpolation. In Table 6, our computed values for the difference between 8° and the actual angular distance covered by the Sun in 8 days (i.e., the difference between the true longitudes computed after a separation of 8 days) is given in the parenthesis below the *vākya* value. It is clear from these figures that the *yogyādivākyas* are very accurate.

More importantly, what is noteworthy here is the phenomenal simplification that has been achieved in computing the true longitudes of the Sun at any moment using the *yogyādivākyas*. The *yogyādivākyas* are given in the following verses:

> *yogyo vaidyaḥ tapaḥ satyaṃ dhanyaḥ putraḥ kharo varaḥ |*
> *vīraḥ śūraḥ śaro vajrī bhadraṃ gotro ruruḥ karī ||*
>
> *dhanyaḥ sevyo mayā loke kāyo dīnaḥ stanāṅganā |*
> *yājñī yajñāṅganā śūnā steno dīno dhunī namaḥ ||*
>
> *āpaḥ pāpaḥ payaḥ pathyaṃ pūjyā dhenurdine'rthinaḥ |*
> *tanurbhinnā khanī jñānī rat naṃ bhānuḥ sunirnayaḥ ||*

The literal translation of the above verse is:

> A qualified doctor; [Speaking] truth [by itself] is austerity; A blessed son; A donkey is better; A skilful warrior; Indra's arrow; This clan is safe; The antelope and elephant; In the world only the blessed are to be served by me; Pitiable is the state of the body; A lady with big breasts; The wife of the Yajamāna and performer if the sacrifice is swollen; The thief is miserable; The river is the dancer; The water is the culprit; Milk is good; Cow is to be worshiped during the day by those desirous of becoming wealthy; The body has been split; The wise is like a mine; The Sun is a pearl; The one who is completely unscrupulous.

By simply memorizing the above verses, one can find out the longitude of Sun on any given day at any given instant with reasonable accuracy. In fact, for all practical purposes, but for some crucial computations involved in eclipses wherein very high accuracies are required, the inaccuracies noted in Table 6 are negligible. This is a very small price paid for the enormous simplification and fun involved in computing the longitudes by simple arithmetic calculations.

Appendix

Relations between the mean and the true longitudes of the Sun

As per the standard procedure laid down in Indian astronomical works, once the mean longitude of the Sun, θ_0 is known, a correction known as *mandaphala* has to be applied to it to obtain the true longitude, θ of it. This essentially takes care of the eccentricity of the apparent orbit of the Sun around the earth. The equivalent of this

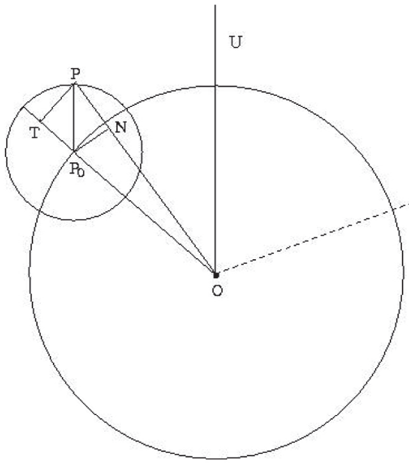

Fig. 1. Obtaining the *mandasphuṭa* in the epicycle model

in modern astronomy is the 'equation of centre'. Conversely, the mean longitude can be obtained from the true longitude by applying the '*mandaphala'* inversely to it.

The method given in *Karaṇapaddhati* for finding the *mandaphala* of any planet including the Sun can be explained with the help of an epicycle model represented in Fig. 1. The mean planet P_0 is assumed to be moving on a 'deferent' circle centered around O (centre of the earth), whose radius, $OP_0 = R$ (*trijyā*). $O\Gamma$ represents the direction of *Meṣādi* or the first point of Aries. OU is in the direction of the apogee or '*mandocca'*, whose longitude is given by $\Gamma\hat{O}U = \theta_m$. The longitude of the mean planet P_0, or the 'mean longitude' is given by $\Gamma\hat{O}P_0 = \theta_0$. Draw a circle of radius r around the mean planet, P_0. This is the epicycle. The true planet P is located on the epicycle such that $PP_0 = r$ is parallel to OU (the direction '*mandocca'*). The true longitude of the planet is given by $\Gamma\hat{O}P = \theta$. Join OP and draw PT perpendicular to the line OP_0, extended further. The difference between the mean longitude, θ_0 and the longitude of the *mandocca*, θ_m is given by

$$U\hat{O}P_0 = P\hat{P}_0T = \theta_0 - \theta_m,$$

and is known as *mandakendra*. Now, the *mandakarṇa* K is the distance between the planet and the center of the deferent circle. Clearly,

$$K = OP$$
$$= \sqrt{\left[OT^2 + PT^2\right]}$$
$$= \sqrt{\left[(R + r\cos(\theta_0 - \theta_m))^2 + (r\sin(\theta_0 - \theta_m))^2\right]}$$

Now, $P\hat{O}P_0 = \Gamma\hat{O}P_0 - \Gamma\hat{O}P = \theta_0 - \theta$. In the right triangle POT,

$$PT = OP\sin(P\hat{O}P_0) = K\sin(\theta_0 - \theta) \qquad \ldots(2)$$

Considering the right triangle PP_0T,

$$PT = PP_0\sin(P\hat{P}T) = r\sin(\theta_0 - \theta_m) \qquad \ldots(3)$$

Equating the above two expressions we have

$$K\sin(\theta_0 - \theta) = r\sin(\theta_0 - \theta_m) \qquad \ldots(4)$$

or

$$\sin(\theta_0 - \theta) = \frac{r}{K}\sin(\theta_0 - \theta_m) \qquad \ldots(5)$$

Obtaining the true longitude from the mean longitude

In Indian astronomy, particularly in the Kerala school, the radius of the epicycle, r was assumed to be varying in such a way that it is actually propotional to the *karṇa* K. In other words, the radius r satisfies the equation

$$\frac{r}{K} = \frac{r_0}{R},$$

where r_0 is the stated value of the radius of the epicycle in the text (*Tantrasaṅgraha,* 2011; *Gaṇita-yukti-bhāṣā*, 2008). Using this, equation (5) can be written as

$$R\sin(\theta_0 - \theta) = \frac{r_0}{R}R\sin(\theta_0 - \theta_m) \qquad \ldots(6)$$

Hence the *mandaphala*, which is the arc corresponding to the difference between the mean and true longitudes, that is, $R(\theta_0 - \theta)$, is given by

$$R(\theta_0 - \theta) = (R\sin)^{-1}\left[\frac{r_0}{R}R\sin(\theta_0 - \theta_m)\right] \qquad \ldots(7)$$

This is essentially the content of the verse 5 in chapter 7 of *Karaṇapaddhati* :

> *māndena sphuṭavṛttena*
> *nihatādicmadorguṇāt* |
>
> *nandāptaṃ cāpitaṃ māndam*
> *arkādīnāṃ bhujāphalam*||
>
> The Rsine [of the mandakendra, obtained by subtracting the apogee of the planet from its mean longitude][4] multiplied by the true/actual circumference (*sphuṭavṛtta*) and is to be divided by 80. The arc (Rsine-inverse) of the result [obtained] would be the *mandaphalas* of the planets beginning with the Sun.

Here, *sphuṭavṛtta* stands for the stated value of the circumference of the epicycle, when the circumference of the deferent circle is taken to be 80, that is,

$$\frac{r_0}{R} = \frac{\text{stated } sphuṭavṛtta}{80}$$

For the Sun, *sphuṭavṛtta* is stated to be 3. Hence,

$$\frac{r_0}{R} = \frac{3}{80}$$

The longitude of the apogee of the Sun, θ_m is given as 78° in the text. With the knowledge of the mean longitude θ_0, and the *mandaphala* $R(\theta_0 - \theta)$ from (7), we obtain the true longitude, θ.

Obtaining the mean longitude from the true longitude

The method which was used to find the true longitude from the mean longitude cannot be used to obtain the mean longitude θ_0 with the knowledge of the true longitude θ, as the expression for the *mandaphala* involves θ_0. However, it is possible to obtain θ_0 from θ by this method using an iterative procedure. This is not mentioned in *Karaṇapaddhati* but discussed in *Gaṇita-Yukti -bhāṣā* (*Gaṇita-Yukti -bhāṣā*, 2008, p. 501, 661). In the first step to obtain θ_0 from θ, θ_0 is replaced by θ in the RHS of (6), and $(\theta_0 - \theta)$ and thereby θ_0 is calculated. In the second step, this computed value of θ_0 is used in the RHS, and θ_0 is calculated again. In this manner, the iteration process is carried out till the successive values of θ_0 obtained are the same, to the desired accuracy.

Alternatively, the method can be modified to obtain an expression for the *mandaphala* in terms of the *sphuṭa-doḥphala*, $r_0 \sin(\theta - \theta_m)$ involving the *mandakendra* obtained by subtracting the apogee from the true longitude θ, and the concept of *viparyāsakarṇa* or *viparītakarṇa* or *vyastakarṇa* (inverse hypotenuse). The *viparyāsakarṇa* (inverse hypotenuse) is the radius of the deferent OP_0, in the measure of the *karṇa* K, that is, when K is set equal to the *trijyā*, R. It is denoted by R_v. Obviously,

$$\frac{K}{R} = \frac{R}{R_v}$$

or $$K = \frac{R}{R_v}.R$$

Also, $$r = \frac{r_0}{R}.K$$

$$= \frac{R}{R_v}.r_0$$

Verses 17 and 18 in chapter 7 of *Karaṇapaddhati* describe the procedure for finding the *viparyāsakarṇa*, R_v:

> *rāśyantabhānusphuṭato mṛdūccaṃ*
> *viśodhya doḥkomiguṇau gṛhītvā* |
>
> *trisaṅguṇau tāvatha nandabhaktau*
> *krameṇa dohkotiphale bhavetam* ||
>
> *koṭīphalaṃ karkamṛgādijātaṃ*
> *trimaurvikāyāṃ svamṛṇaṃ ca kṛtvā* |
>
> *tadvargato doḥphalavargayuktāt*
> *mūlaṃ viparyāsakṛto'tra karṇaḥ* ||

[4] In verse 3 of chapter 4, it is stated that the *mandakendra* is obtained by subtracting the apogee from the mean planet.

> The [longitude of the] *mandocca* has to be subtracted from the true longitude of the Sun at the end of the *rāśi*. Having obtained the Rsine and Rcosine of that [result], and multiplying it by 3 and dividing by 80, the *doḥphala* and the *koṭiphala* are obtained [respectively].
>
> The *koṭiphala* has to be added to or subtracted from the radius depending upon whether [the *kendra* is] *karkyādi* or *makarādi* respectively. The square root of the sum of the squares of the result thus obtained and of the *doḥphala* would be the *viparyāsakarṇa* here.

The term *mṛdūcca* appearing in the first line of the verse is a synonym for *mandocca*. The *sphuṭa* refers to the true longitude of the planet. Now θ is the true longitude and θ_m is the longitude of the *mandocca*. The [*sphuṭa*]-*doḥphala* and [*sphuṭa*]- *koṭi phal a* are given by

$$[sphuṭa] - doḥphala = \frac{r_0}{R} R\,|\sin(\theta - \theta_m)|$$

$$= \frac{3}{80} \times R\,|\sin(\theta - \theta_m)|$$

$$[sphuṭa] - koṭiphala = \frac{r_0}{R} R\,|\cos(\theta - \theta_m)|$$

$$= \frac{3}{80} \times R\,|\cos(\theta - \theta_m)|$$

Draw P_0N perpendicular to OP (see Fig. 1). Now $P_0\hat{P}N = P\hat{O}U = R\hat{O}P - R\hat{O}U = \theta - \theta_m$. Then, considering the right triangle P_0NP,

$$NP_0 = PP_0 \sin(P_0\hat{P}N)$$

$$= r\,|\sin(\theta - \theta_m)|$$

and $$NP = PP_0 \cos(P_0\hat{P}N)$$

$$= r\,|\cos(\theta - \theta_m)|\,.$$

Now,

$$OP_0 = R = \sqrt{ON^2 + NP_0^2}$$

$$= \sqrt{(OP - NP)^2 + NP_0^2}$$

$$= \sqrt{(K - r\,|\cos(\theta - \theta_m)|^2 + (r \sin(\theta - \theta_m))^2}$$

$$= \frac{R}{R_v} \sqrt{(R - r_0\,|\cos(\theta - \theta_m)|)^2 + (r_0 \sin(\theta - \theta_m))^2}$$

using the expression for K and r in terms of R and R_v. Hence,

$$R_v = \sqrt{\left(R - \frac{3}{80} \times |R\cos(\theta - \theta_m)|\right)^2 + \left(\frac{3}{80} \times R\sin(\theta - \theta_m)\right)^2}$$

The above expression for the *vyastakarṇa* is applicable when the *kendra* is *makarādi* (that is, lies in the first or fourth quadrant).[5] If the *kendra* is *karkyādi*, then the expression for R_v is given by

$$\sqrt{\left(R + \frac{3}{80} \times |R\cos(\theta - \theta_m)|\right)^2 + \left(\frac{3}{80} \times R\sin(\theta - \theta_m)\right)^2}$$

Both the relations can be combined in a single formula, and R_v is given by

$$\sqrt{\left(R - \frac{3}{80} \times |R\cos(\theta - \theta_m)|\right)^2 + \left(\frac{3}{80} \times R\sin(\theta - \theta_m)\right)^2}$$

Verse 19 gives the procedure for finding the mean longitude, θ_0 from the true longitude, θ:

> *trijyāhatād doḥphalato'munāptaṃ*
> *cāpīkṛtaṃ meṣatulāditastat* |
>
> *rāśyantabhānau svamṛṇaṃ ca kuryāt*
> *tadā bhavet saṅkramaṇārkamadhyam* ||
>
> The arc of the [quantity obtained by] multiplying the *doḥphala* by radius and dividing by this [*vyastakarṇa*] has to be added to or subtracted from the true longitude of the Sun when [the *kendra* is] *meṣādi* or *tulādi* respectively. The result would be the mean longitude of the Sun at the transit.

[5] Here, '*makarādi'* means half the ecliptic, from 270° to 360° (fourth quadrant) and 0° to 90° (first quadrant). '*Karkyādi*' means half the ecliptic from 90° to 270° (second and third quadrants).

The procedure outlined in the verse above can be understood by considering the right triangle ONP_0 (see Fig. 1). Here

$$NP_0 = OP_0 \sin(P_0\hat{O}N)$$
$$= OP_0 \sin(P_0\hat{O}P_0)$$
$$= \text{R} \sin(\theta_0 - \theta_m).$$

and $$NP_0 = \text{r} \sin(\theta - \theta_m) \mid .$$

Equating the above two expressions, we have

$$R \sin(\theta_0 - \theta) = r \sin(\theta_0 - \theta_m)$$

Now

$$r = r_0 . \frac{K}{R} = r_0 . \frac{R}{R_v}$$

Therefore,

$$R \sin(\theta_0 - \theta) = r_0 \sin(\theta - \theta_m) \frac{R}{R_v}$$

Hence,

$$R(\theta_0 - \theta) = (R \sin)^{-1} \left[r_0 \sin(\theta - \theta_m) \frac{R}{R_v} \right]$$

$$= (R \sin)^{-1} \left[\frac{3}{80} \times R \sin(\theta - \theta_m) \frac{R}{R_v} \right]$$

In the above expression, since θ is known, the mean planet θ_0 can be obtained by adding the above difference to the true planet θ. $(\theta_0 - \theta)$ is positive when the *kendra* (anomaly) $\theta - \theta_m$ is within the six signs beginning with *Meṣa*, viz., $0^o \leq \theta - \theta_m \leq 180^o$, and negative when the *kendra* is within the six signs beginning with *Tulā*, viz., $180^o \leq \theta - \theta_m \leq 360^o$, as implied in the verse.

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