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Gaṇitapañcaviṃśi

English Translation with Introduction

Introduction

The Sanskrit text of the *Gaṇitapañcaviṃśi* was first edited by Professor David Pingree and published in 1979 in Ludwich Sternbach Felicitation Volume of the Journal *Ṛtam* of the Akhila Bharatiya Sanskrit Parishad, Lucknow. The single manuscript used for this edition was discovered in London. It was incomplete as a number of verses about the middle of it which dealt with inverse proportion, barter of commodities, separation of mixtures, simple interest, partnership, alligation and mensuration of triangle and quadrilateral were missing from it. Instead there were some interpolatory verses, of which $8\frac{1}{2}$ belonged to the *Līlāvatī* of Bhāskara II (1150 CE) and $\frac{1}{2}$ to the *Gaṇitatilaka* of Śrīpati (1039 CE). There was also a rule ascribed to Nārmada (ca. 1375 CE) and an example in illustration of it.

The present edition of the *Pañcaviṃśatikā* was prepared at the request of late Shri G C Sinha, Secretary, Akhila Bharatiya Sanskrit Parishad, Lucknow, who wanted revision of Professor Pingree's edition with the addition of notes and comments where necessary or an altogether a new edition of it on the basis of complete manuscripts if possible. Fortunately a complete manuscript of the *Pañcaviṃśatikā* was discovered in the library of Mithila Sanskrit Research Institute, Darbhanga, Bihar. A zerox of this manuscript was acquired and was used for the present edition, besides Professor Pingree's edited text (which has been referred to as P).

The manuscript acquired from the Mithila Research Institute Accession No. 3444 may be described as follows:

Extent 6 folios, 9 lines to a page and 40 letters to – a line, approximately 97 lines or 121 *granthas* in all; character–modern devanāgarī; script-good, pagination on both margins, on one side of each folio as usual; the beginning and end as follows:

Beginning: ।। श्रीगणेशाय नमः।। ।। हरि ऊँ३म्।। शिवप्रणम्य etc.

End: वेदांगोदधिम् १४६४ तूपे शुभकृन्नाम्रिसंस्थिक ळ।। आयंनचोत्तंरे जात चैतेपक्षे च बाहुले।।१।। ।।छ।। ।। श्रीः।। इतिश्रीपंचविंशतिका श्रीधराचार्य विरचिता समाप्ता।। ।। ग्रंथसंख्या १२१।।

The text exhibited by the manuscript contains $53\frac{1}{2}$ verses, excluding three verses at the end, of which two are in appreciation of the work and praise of its author and the third, the last one, which is due to the scribe, mentions the time of writing the manuscript. Of these $53\frac{1}{2}$ verses, $45\frac{1}{2}$ are in *anuṣṭubh* metre, 2 in *āryā*, $\frac{1}{2}$ in *indravajrā*, 2 in *upajāti* (*indravajrā* plus *upendravajrā*), $1\frac{1}{2}$ in *upendravajrā*, 1 in *dodhaka*, and 1 in *bhujaṅgaprayāta*. These verses are distributed as follows:

Introduction and rules: 25 verses

Definitions and examples: $28\frac{1}{2}$ verses

The name *Pañcaviṃśatikā* or *Pañcaviṃśi* was given to the work evidently on the basis of the 25 verses which constitute the introductory verse and the verses embodying the rules. These indeed form the kernel or nucleus of the work.

The subject matter of the work is divided as usual into two major sections, one containing the arithmetical operations (*parikarma*) and the other the determinations (*vyavahāra*), although such division is not actually made and is only implied. The operations dealt with are addition, subtraction, multiplication, division, squaring , cubing, square-root, and cube-root, both for whole numbers and fractions. Four classes of fractions viz. a/b ± c/d, a/b of c/d, a ± b/c and a/b ± c/d of a/b as also operations with zero are also dealt with. Besides this, rules for certain specific problems involving arithmetical operation resulting in a number called the "visible" (*dṛśya*), and those for direct and inverse proportion and barter of commodities are also given. The determination dealt with are those pertaining to mixtures (*miśraka*), arithmetic series (*śreḍhī*) plane figures (*kṣetra*), excavations (*khāta*), piles of bricks (*citi*), sawing of wood (*krakaca*), heaps of grain (*rāśi*), and brief, concise & quite suitable for the beginners for whom the book was written.

In the opening verse and the colophon at the end we are told that this work is the composition of the well-known Indian mathematician Śridharācārya and that it constitutes the essence of his *Pāṭīgaṇita*. This, however, is not substantiated by the contents of the work. To us it seems that this work is based on the *Līlāvatī* of Bhāskara II rather than on the *Pāṭīgaṇita* of Śridharācārya. The unit of gold, grain, money and land measures defined in the beginning are exactly the same as given in the *Līlāvatī*. Of these measures, some are either different from those defined and used in the *Pāṭīgaṇita* or quite new. The rules too are in most cases based on those given in *Līlāvatī* rather than those given in the *Pāṭīgaṇita*. It is noteworthy that whereas the *Pāṭīgaṇita* and the *Pāṭīgaṇita*–sāra (or *Triśatikā*) of Śridharācārya both make use of $\pi = \sqrt{10}$, or $\pi = 3.16$, the present work following the *Līlāvatī* uses $\pi = 22/7$. Another noteworthy point is that whereas Śridharācārya in his *Pāṭīgaṇita* finds fault with, and in his *Triśatikā* ignores, the rule area of triangle or quadrilateral = ½(a+c) × ½(b+d), the author of the *Pañcaviṃśatikā* prescribes it and sets an example on it.

Exploitation of the name of Śridharācārya is evidently made to gain popularity for the work, for Śridharācārya and his writings were held in the highest esteem throughout the length and breadth of India. Survival of manuscripts of the *Pañcaviṃśatikā* or *Pañcaviṃśi* clearly shows that this work was indeed popular. But we do not hear of this work before the fourteenth century CE. The earliest quotations from this work have been discovered so far in the works of Padmanābha (ca. 1400 CE), son of Nārmada (ca. 1375 CE), and Gaṇeśa (1520 CE), the commentaries of the *Līlāvatī* and his father Keśava.

We are therefore inclined to believe that the author of the *Pañcaviṃśatikā* lived posterior to Bhāskara II (1150 CE) and composed this work on the basis of the *Līlāvatī* which was then in more popular use. Mention of the name of Nārmada in one of the rules in Pingree's edition seems to suggest some relationship of that writer with this work.

A commentary on the *Pañcaviṃśatikā*, entitled *Bālabodha*, written by Sambhūnātha is known to exist in the Central Library of the Oriental Institute, Baroda. We wished to consult this manuscript and the present secretary of the Akhila Bharatiya Sanskrit Parishad, Dr J P Sinha, requested the authorities of that library to supply us a zerox of that manuscript but no reply was received. A reminder was then sent but it also fell on deaf ears.

I am grateful to Dr J P Sinha, Secretary, Akhila Bharatiya Sanskrit Parishad, Lucknow for his keen interest in the publication of the present work.

Homage and Introduction

1. Having paid obeisance to God Śiva, Śrīdhara gives the essence of the *Pāṭī* (*gaṇita*)[1] composed by himself, in twenty-five verses with clear meaning.

Definitions

2-3 (a-b) [Gold -measures]:

(i)[2]

5 *guñjā*	=	1 *māṣa*
16 *māṣa*	=	1 *karṣa*
1 *karṣa* of gold	=	1 *suvarṇa*
4 *suvarṇa*	=	1 *pala*

(ii)[3]

3 *guñja*	=	1 *valla*
16 *valla*	=	1 *gadyāṇaka*

3(c-d)-4: [Grain-measures (Dhānya-māna)[4]]:

1 *kuḍapa* (or *kuḍava*)	= 3×3× 1½ cu. aṅgulas
	$= \left(\frac{1}{4}\right)^5$ cu. cubits
4 *kuḍupa*	= 1 *prastha*
4 *prastha*	= 1 *āḍhaka*
4 *āḍhaka*	= 1 *droṇa*
16 *droṇa*	= 1 *khārī* (of Magadha)
	= 1 cu. cubit.

5. [Money-measures][5]:

20 *varāṭaka* (cowries)	= 1 *kākiṇī*
4 *kākiṇī*	= 1 *paṇa*
16 *paṇa*	= 1 *dramma*
16 *dramma*	= 1 *niṣka*

6-7 [Land-measures][6]:

3 vertical *yava*[7]	= 1 *aṅgula* (digit)
24 *aṅgula*	= 1 *kara* (cubit)
4 *kara*	= 1 *daṇḍa*
2000 *kara*	= 2 *krośa*
4 *krośa*	= 1 *yojana*
10 or 8 *kara*	= 1 *vaṃśa* (bamboo)

Area of a square field on the side of 20 *vaṃśa*

(200 or 160 cubits)[8] = 1 *nivartana*.

The above definitions agree with those given in the *Līlāvatī* of Bhāskara II. They agree only partially with those given in the *Pāmīgaṇita* of Śridharācārya. Śridharācārya mentions *purāṇa* in place of *dramma*, but does not mention *valla*, *gadyāṇaka, niṣka*, *vaṃśa* and *nivartana*. So the author's assertion in the opening stanza that he is setting out in the present work the essence of Śridharācārya's *Pāṭīgaṇita* is not true. This also throws doubt as to its author being Śridharācārya. The rules that follow are also mostly based on the *Līlāvatī* of Bhāskara II, sometimes even the words used are the same. They are certainly not based on the *Pāṭīgaṇita* of ŚŚridharācārya.

Name of Places

8-9. *Eka*, *daśa*, *śata*, *sahasra*, *ayuta*, *lakṣa*, *prayuta*, *koṭi*, *arbuda*, *padma*, *kharva*, *nikharva*, *mahapadma*, *śanku*, *jaladhi*, *antya*, *madhya* and *parārdha* - these are the denominations of places whose values increase in the ratio[9] of 10.

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1 Work on *Pāṭīgaṇita* deals with arithmetic and mensuration.
2 Cf. *L* (= *Līlāvatī*), Ānandāśarma Sanskrit Series edition, vs. 4; PG (*Pāṭīgaṇita* of Śridharācārya)
3 Cf.*L*, vs. 3. This table does not occur in *PG.*
4 Cf. *L* vss. 7-8; *PG*, 11
5 Cf. *L,* vs. 2; *GT*(= *Gaṇitatilaka* of Śrīpati , H R Kapadia's edition, p.2 vs.4 *PG* has *purāṇa* in place of *dramna* but *nicka* is not there.
6 Cf. *L*, vss. 5-6; *PG*, 12 does not mention *vaAœa* and *nivartana*.
7 frFkZX;oksnjk.;'Vk v/okZ ok czhg;L=;%A ¼Le'fr%½
8 200 cubits according to *Līlāvatī* and 160 cubits according to *Gaṇitatilaka*.
9 Cf. *L*. vss. 10-11; *PG*, 7-8

This list of names of places is the same as given in the *Līlāvatī* with *abja* in place of *padma*. The *Pāmīgaṇita* has *abja* in place of *padma*, *mahāsaroja* in place of *mahāpadma*, and *saritpati* in place of *jaladhi*.

Section One: Operations (*Pakikarma*)

Addition and Subtraction

10 (a, b). Addition and subtraction are performed (starting from the unit's place on the right towards the left; or from the last place on the left, towards the right).[10] When zero is added to or subtracted from a number, the number remains unchanged.[11]

Example. What is obtained when 60, 9 and 12 are added together, and what results when the sum obtained is subtracted from 100? (Answer: sum 81; remainder 19).

Multiplication

10 (c, d). Multiply the digit of the multiplicand standing in the last and other places by the multiplier.[12]

Example 1 (c, d). What is obtained when 127 is multiplied by 24, and what when 16132 is multiplied by 123? (Answer: 3048 and 1984236.)

Division

11(a, b). In the case of division, the quotient is the (greatest possible) number by which the divisor being multiplied the resulting product is subtractable from the dividend.[13]

Example. What will be the result when th dividend is 3048 and divisor 24; and when dividend 1984236 and divisor 123? Results are 127 and 16132.

Square and Cube

11 (c, d). The product of two equal numbers is the square[14], and the (continued) product of three equal numbers is the cube.[15]

Example 2 (a, b). Friend,What are the squares and cubes of 9 and 123. (Answer: squares 81 and 15129; cubes 729 and 1860867)

Square-root

12-13 (a-b). Having subtracted from the last odd place (the greatest possible) square number (and then having set down twice its square-root in a line), divide the next even place by twice the square-root (which has been set down in a line); then subtract the square of the resulting quotient from the next odd place and set down twice that quotient in the line. Now starting with "divide the next even place" repeat the process (until all the digits of the given number are exhausted). Half of the number (standing in the line) is the (required) square-root.[16]

Example. What is the square root of 81 and 15129?; The square root is 9 and 123.

Cube-root

13 (c-d)-15 (a-b). To begin with, divide the digits starting from the unit's place into periods of one cube (*ghana*) and two non-cube (*aghana*) places. Then from the last cube place subtract (the greatest possible) cube and set down the cube-root of that separately (in line). Then by thrice the square of that divide the next (non-cube) place; and put down the quotient in the line (of the cube-root).

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[10] Cf. *L*.vs. 12. There is no such rule in *PG*.
[11] Cf. *L*.vs. 45 (a,b); PG, vs. 21 (a,b).
[12] Cf. *L*.vs. 14 (a,b). There is no such rule in PG.
[13] Cf. *L*. vs. 18 (a,b). There is no such rule in PG.
[14] Cf. *L*.vs. 19 (a).
[15] Cf. *L*.vs. 24 (a).
[16] 1. Cf. L, vs. 22. for details see PG, English translation, p.10.

Then subtract the square of that quotient, multiplied by thrice the last (i.e. the cube-root), from the next (non-cube) place. Then subtract the cube of that quotient from the next (cube) place. (The number in the line of the cube-root gives the required cube-root.) (If there are more places), repeat this process (until all the places are exhausted) to get the cube-root of the given number.[17]

Example. What is the cube root of 729 and 1860867?; the result is 9 and 123.

Fractions

Addition and subtraction of simple fractions

15 (c-d)-16 (a-b). To reduce the (two) given fractions to a common denominator, multiply the numerator and denominator of each fraction by the denominator of the other. Then add or subtract (as the case may be) the numerators of the fractions with equal denominators. It shall be noted that the denominator of a whole number is 1.[18]

Example. What is obtained when the unit fractions 1/2, 1/3, 1/4 are added together and what they are subtracted from 3? [Answer: sum is 13/12 (or 11/12) and difference is 23/12 (or $1\frac{11}{12}$)].

Reduction of "Fraction of a Fraction" and Multiplication of Two Fractions

16 (c-d). In the case of simplification of a fraction of a fraction or multiplication (of two fractions), divide the product of the numerators (of the fractions) by the product of the denominators (of the fractions).[19]

That is, a/b of c/d = a/b × c/d = ac/bd.

Example 3 (a-b). What is 1/3 of 1/2 of 1/4 and what 1/2 multiplied by 1/3? (Answer: 1/24 and 1/6).

Division of one fraction by another

17 (a-b) Invert the numerator and denominator of the divisor and then multiply (the two fractions).[20]

Example. What is obtained by dividing than previous result (1/6) by 1/3?; dividend 1/6 divisor 1/3, result 1/2.

Square and square-root of a fraction

17 (c-d) To find the square (of a fraction), square the numerator and the denominator; to find the cube, cube the numerator and the denominator; to find the (square or cube) root, extract the (square or cube) root of the numerator and the denominator.[21]

Example 3 (c-d). What are the square and cube of $3\frac{1}{2}$ and what are the square - root and cube-root of those (square and cube)?[22]

(Answer: square 49/4 and cube 343/9; square-root 7/2 and cube-root 7/2.)

Simplification of $\left(a+\frac{b}{c}\right)$ (*rūpabhāgānubandha*) or $\left(a-\frac{b}{c}\right)$ (*rūpabhāgāpavaha*)

18 (a-b) The numerator of the (given) fraction should be added to or subtracted from (as the case may be) the (given) whole number multiplied by the denominator of the (given) fraction. This is the case when one fraction is a whole number.

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[17] Cf. L, vss. 28-29, for details see PG, English Translation, pp. 13-14
[18] Cf. L. Vss. 30 and 37. Also see PG. rule 32.
[19] 2. Cf. PG. rule 33a.
[20] Cf. *L*. vs. 41.
[21] Cf. *L*. vs. 43
[22] Same example occurs in *L*, vs. 41.

Simplification of a/b + c/d of a/b written as $\boxed{\begin{array}{c}a\\b\\c\\d\end{array}}$ (*bhāgabhāgānubandha*) or a/b - c/d of a/b written as $\boxed{\begin{array}{c}a\\b\\c\\d\end{array}}$ (*bhāgabhāgā, unāha*)

Example. What is 3 plus 1/3 and 3 minus 1/3?; result is 10/3 and 8/3. What is 3/1+1/3 of 3/1 and what is 3/1-1/3 of 3/1?; result is 4 and 2.

18 (c-d) Multiply the denominator (of the upper fraction) by the denominator of the lower fraction, and the numerator (of the upper fraction) by the same denominator (of the lower fraction) increased for decreased (as the case may be) by its own numerator; (and then rule out the lower fraction).[23]

That is, a/b + c/d of a/b = a (d+c) / bd

and a/b - c/d of a/b = a(d-c)/bd.

Example 4 (a-b). What is the value of 3±1/3 and of 3±1/3 of 3? (Answer: 10/3 and 8/3; 4 and 2).

Operations with Zero

19 (a-b). Multiplication by zero yields zero; the square and cube etc. of zero are also zero.

Rule of Inversion

19 (c-d)-20(a-b). Starting from the visible quantity (backward), reverse the operation of addition and subtraction, multiplication and division, square and square-root, cube and cube-root, and if there is increase or decrease by a fraction of itself, divide by one plus minus the fraction.[24]

Example 4 (c-d). What is that number which being diminished by 2, then multiplied by 3, then squared, then increased by 1/3 of itself, and then divided by 3 yields 16? (Answer: 4.)

Miscellaneous Types of Problems

(1) The visible type (*dṛśyajāti*)

20 (c-d). The visible quantity divided by 1 multiplied, divided, or increased or decreased by the fraction, in the manner stated in the (given) problem, (yields the desired result).[25]

Example 5 (a-b). What is that number which multiplied by 3, then divided by 5, then increased by one-fifth of the same number, and then divided by 4, yields 2?[26] (Answer 10)

$$\text{The required number} = \frac{2}{\frac{\frac{3}{5}+\frac{1}{5}}{4}} = 10$$

(2) The remainder type (*śeṣajāti*)

For rule vide Sripati's *Gaṇitalilaka* ed. M.R. Kapadu, p 44.

Example 5 (c-d). One-sixth (of the parrots), one half of the remainder, and one-third of those left thereafter having gome away, there remain 10 parrots. (Find the number of the parrots.) (Answer: 36 parrots.)

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[23] Cf.this rule and the previous one with *L*, vs. 31 and *PG*, rules 39, 40

[24] Cf. *L*, vs. 48.

[25] Cf. *GK* (=*Gaṇita-Kaumdi*) Part I, p. 17, rule 37 (c-d)-38. L, vs. 51 gives a more general rule. But PG does not contain this rule.

[26] After this example P (*Pāṭīgaṇita*) adds:

Example on the "visible" type. One-sixth of a number of parrots went to the sky, one-third to the water, and the remaining 6 are visible. What was the number of those parrots?

Rule for the "remainder" type. Divide the "visible" number by the product of the denominators, each diminished by the corresponding numerator, as divided by the product of the denominators.

[Applying this rule to the problem given above under the remainder type, we get $\text{no. of parrots} = \frac{10}{\frac{(6-1)\,(2-1)\,(3-1)}{6\times2\times3}} = 36$.]

In view of 20 b above, this rule is irrelevant:

(3) The difference type (*viśleṣajāti*)

Example 6. What is the strength of a herd of cow, if the difference of and there of having been lost 6 cows are to be seen? (Answer: 9 cows.)

(4) The visible, square-root and fraction type (*dṛśyamūlāmsajāti*)[27]

21. Rule: Multiply the result obtained by adding or subtracting (the multiple of) the square-root by 4, to that add the square of (the multiple of) the square-root, then extract the square-root of that, then diminish or increase that by (the multiple of) the square-root, then reduce that to half, and then square it: what is thus obtained is the (required) result.[28]

This rule gives the solution of the quadratic equation

$$x \pm m\sqrt{x} = n,$$

where m is the multiple of the square-root. The solution is correctly stated as

$$x = \left[\frac{\sqrt{m^2 \mp 4n} \mp m}{2}\right]^2$$

Example 7. What is the number which diminished by its square-root, then diminished by one-sixth of the remainder, and then added to the square-root of the remainder, yeilds 30?[29] (Answer: 36).

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[27] P calls this type *mulajāti* instead of *dṛsyamūlāmṣajāti*.

[28] Cf. *L*, vs. 65. Also see *PG*, rule 75.

[29] After this P adds the following:

Rule for *dṛṣṭamūlāṃśa-jāti*:

When some quantity is stated to be increased or decreased by a fraction, then it should be obtained by dividing the "visible" number by 1 increased or decreased by that fraction. If that is increased or decreased by its square-root also, one should proceed as stated before.

Example. O proficient mathematician, quickly say the number which when increased by 1/27, 1/3 and 1/9 of itself and also by its square-root, becomes 129.

[Let the number be x. Then

$$x\,(1+1/27+1/3+1/9) + \sqrt{x} = 129$$

or $x + \dfrac{\sqrt{x}}{1+1/27+1/3+1/9} = \dfrac{129}{1+1/27+1/3+1/9}$ or $x + \dfrac{27}{40}\sqrt{x} = \dfrac{27\times 129}{40}$ [vide rule of vs. 13]

Rule for *māḍajāti*:

On the floors of the multiple-storeyed structure write down a series of equal numbers (one above the other). (Starting from the lowest number) divide by the *iṣṭa* plus 1, successively (increasing in the ratio of itself). (The equal numbers are to be chosen in such a way that after this division there is 1 in the last but one storey.) Then the lowest number gives the number of people in the lowest storey; this number multiplied by *iṣṭa* and added to the number above it gives the number of people in the second storey; and so on. Listen to this unique rule propounded by Narmada. Example. O pupil, listen. On every floor of the king's seven-storeyed beautiful gate-pyramid there were people.

For fear of collapse of the building, people in each lower storey cried aloud and said to those in the next upper storey said: "Come as many people as we are here to this storey." The number of people in each storey then became equal. Tell me. O friend, the equal number of the people (and also the number of people who were in the seven storeys in the beginning).

The equal number of people = 2^{7-1} = 64. The number of people in the seven storeys in obtained as follow

	Series of equal number	After successive division	People in the seven storeys
Seventh Storey	64	64	127
Sixth Storey	64	1	63
Fifth Storey	64	2	62
Fourth Storey	64	4	60
Third Storey	64	8	56
Second Storey	64	16	48
First Storey	64	32	32

Let x be the required number. Then

$$x - \sqrt{x} - \frac{1}{6}\left(x - \sqrt{x}\right) + \sqrt{x - \sqrt{x} - \frac{1}{6}\left(x - \sqrt{x}\right)} = 30$$

which can be written as

$$x - \sqrt{x} = y \qquad (1)$$

$$y - \frac{1}{6}y = z \qquad (2)$$

$$z + \sqrt{z} = 30 \qquad (3)$$

Solving (3) we get z = 25; then solving (2) we get y = 30; and finally solving (1) we get x = 36.

Rule of Three etc.

22-23. In the case of (problems on proportion involving) three, five, seven or more quantities, the argument and the requisition are of the same denomination. (Set down the quantities associated with the argument on one side and those associated with the requisition on the other and then) carry the fruit and the denominators (if any) from one side to the other; when the proportion is invesrse, the fruit is not to be transposed. Then the product of the quantities on the side having larger number of quantities divided by the product of the quantities on the side having lesser number of quantities, gives the (desired) fruit.[30]

Example 8 (a-b). If 100 mangoes are obtained for $6\frac{1}{2}$ paṇas, how many of them will be obtained for 1 *dramma* (or 16 paṇas)?

Indian method of working: argument side requisition side

argument side	requisition side
13	16
2	
——	——
100	x

Transposing fruit and denominator to the other side:

13	16
	2
——	——
x	100

Hence the required number of mangoes =

$$\frac{16 \times 2 \times 100}{13} = 246\frac{2}{13}$$

Example 8 (c-d). If the interest on 100 for $1\frac{1}{2}$ months is 5, what will be the interest on 60 for 1 year?

Indian method of working:

argument side	requisition side
100	60
——	——
3	12
2	
——	——
5	x

Transposing fruit and denominator:

100	60
——	——
3	12
	2
——	——
x	5

Therefore, the required interest =

$$\frac{60 \times 12 \times 2 \times 5}{100 \times 3} = 24$$

Therefore example of *Pañcarāśika, saptanāśika* in the original but not translated into English.

Inverse rule of three

Example 10 (a-b). When 1/2 *māṣa* of 8 *varṇas* is given, how much of 10 *varṇas* will be obtained in exchange? (Answer: 2/5 *māṣa*)

[30] Cf. L, vss. 73a, 82; PG, 45,

Barter of commodities

Example 10 (c-d). If 30 camels are obtained for 100 horses, how many camels will be obtained for 30 horses? (Answer : 9 camels.)

Section Two: Determinations (Vyavahara)

1. Separation of mixed Quantities (Miśraka)

1. Simple Interest

24. By the time corresponding to the argument multiply the argument, and by the time past (severally) multiply the interest etc.; divide (each product) by their sum and multiply by the mixed amount. Then are obtained the capital and interest etc. separately.[31]

Example 11. Find the capital and interest when their sum amounts to 8 in a year, the rate of interest being 5 percent (per months).

Here argument = 100, corresponding time =1 month, other time =12 months, fruit =5, amount =8.

Hence

$$\text{capital} = \frac{100\times1\times8}{100\times1+5\times12} = 5$$

$$\text{interest} = \frac{5\times12\times8}{100\times1+5\times12} = 3.$$

Example 12. On 100, the monthly interest is 10, the commission of the broker 1/2, and the fee of the scribe 1/3. If a certain sum amounts to 495 in half a year, give out the capital and interest etc. (separately).

$$\text{Capital} = \frac{100\times1\times495}{100\times1+6(10+1/2+1/3)} = 300$$

$$\text{Interest} = \frac{10\times6\times495}{100\times1+6(10+1/2+1/3)} = 180$$

$$\text{Brokerage} = \frac{1/2\times6\times495}{100\times1+6(10+1/2+1/3)} = 9$$

$$\text{Scribes fee} = \frac{1/3\times6\times495}{100\times1+6(10+1/2+1/3)} = 6$$

2. The Jewel problem

25 (a-b). Diminish the denominators by their own numerators and reduce them to their lowest common denominator. Then the resulting numerators are the values of the fractional parts of the men's own capitals.[32]

Example. One (man) said $\frac{1}{2}$ (of his capital), the other $\frac{1}{3}$ (of his capital), and the other $\frac{1}{4}$ (of his capital). Each of these (severally) increased by the capitals of the other two brothers is the price of a jewel. How is this equality possible?

The fractions associated with the three men are

1/2, 1/3, 1/4

Diminishing the denominators by their own numerators, we set

1/1, 1/2, 1/3

Reducing them to their lowest common denominator, we get

6/6, 3/6, 2/6

Therefore,

$$\frac{\text{capital of first man}}{2} = 6$$

$$\frac{\text{capital of second man}}{3} = 3$$

$$\frac{\text{capital of third man}}{4} = 2$$

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[31] Cf. *L*, vs. 90; *PG*, 47.

[32] A similar rule occurs in BM(= Bakshali Manuscript), ed. Svami Satya Prakash Sarasvati and Dr.Usha Jyotismati, p.10, sutra (iii). But there is no such rule in PG

Hence the capitals of the three men are 12, 9 and 8 respectively, and the price of the jewel is 23.

3. The fruit problem

25 (c-d). The number of men plus 1 gives the number of fruits with the last man and that increased (successively) by 2 with the men (successively) ahead of him.

Example 14. Four men going one behind the other are carrying fruits. Each man gives one fruit, out of his own fruits, to each man behind him (if any). When the fruits have been taken in this way, all men possess equal fruits. Tell the number of fruits with them. (Answer : 5,7,9, 11 fruits.)

4. Partnership

26 (a-b). The investments (of the partners) (severally) divided by their sum and multiplied by the mixed profit give the individual profits (of the partners).[33]

Example 15 (a-b). If the total produce be 63 (*prasthas*), what are the (individual) shares (of the partners) who sowed 2, 3 and 4 *prasthas* (of grain)? (Answer:14, 21 and 28 *prasthas*.)

5. Alligation

26 (c-d) The sum of the products of weight and *varṇa* of the several pieces of gold, being divided by the sum of the weights (of those pieces of gold), gives the *varṇa* (of the alloy).[34]

Example 15 (c-d). What is the *varṇa* of the alloy formed by melting together two pieces of gold whose weights are 2 and 8 *maṣas* and *varṇas* 8 and 12 (respectively)? (Answer: $11\frac{1}{5}$)

27 (a-b). The sum of the products of *varṇa* and weight of the several pieces of gold) when divided by the *varṇa* of the refined gold gives the weight of the refined gold, and when divided by the weight of the refined gold gives the *varṇa* of the refined gold.[35]

Example 16 (a-b). The two pieces of gold of the previous example are melted together and refined. If the weight of the refined gold is 8 (*māṣas*) what is its *varṇa*; if the varṇa is 16 (lit. devoid of *kṣaya*), what is its weight? (Answer: *varṇa* 14, weight 7 *māṣas*)

27 b-28 (a-b). Find the product of the *varṇa* of the refined alloy (of all the pieces of gold) and the sum of the weights (of all those pieces of gold whose weighs are known) also find the sum of the products of *varṇa* and weight (of those pieces of gold whose *varṇa* and weight both are known). Their difference divided by the weight of the piece of gold of unknown *varṇa*, gives the unknown *varṇa*.[36]

That is, if n pieces of gold of weights w_1, w_2, ...w_n and *varnas* v_1, v_2,..........., v_n are mixed up with another piece of gold of weight w but of unknown *varṇa* and refined and the *varṇa* of the refined alloy is found to by e, then the unknown *varṇa* v is given by the formula

$$v = \frac{(w_1 + w_2 + \ldots w_n + w)v' - (w_1v_1 + w_2v_2 + \ldots + w_nv_n)}{w}$$

28 (c-d) (The same difference) divided by the difference of the varṇa of the refined alloy and the varṇa of the unknown weight, gives the unknown weight.[37]

That is, if n pieces of gold of weights w_1, w_2,..., w_n and *varṇa* v_1, v_2,..........., v_n are mixed up

[33] Cf. *L*, vs. 94.
[34] Cf. *L*, vs. 103 (a-b); *PG*, 52 (c-d).
[35] Cf. *L*. vs. 103 (x-d); *PG*, 53;
[36] Cf. *L*, vs. 106; *PG*, 54.
[37] Cf. *L*, vs. 108; *PG*, 55.

with another piece of gold for *varṇa* v but of unknown weight and refined and the *varṇa* of the refined alloy is found to be the unknown weight w is given by the formula

$$w = \frac{(w_1 + w_2 + ...w_n + w)v' - (w_1v_1 + w_2v_2 + ... + w_nv_n)}{v - v'}$$

Let the weight and varṇa of two pieces of gold and those of the refined alloy, be

weight	2	8	
varṇa	x	12	$11\frac{1}{5}$

Then

$$x = \frac{(2+8)\times 56/5 - 8\times 12}{2} = 8$$

And if the weight and varṇa of the two pieces of gold and those of the refined alloy be

weight	y	8	
varṇa	8	12	$11\frac{1}{5}$

$$y = \frac{8\times 56/5 - 8\times 12}{8 - 56/5} = 2$$

29. Divide the weight of the gold-stick by the difference of the *varṇas* of the two pieces of gold and multiply by the difference of the higher *varṇa* and the desired *varṇa*: the result is the measure of weight of the lower *varṇa*. The weight of the stick diminished by that gives the measure of weight of the higher *varṇa*.[38]

Two pieces of gold are mixed up and a gold stick of given weight is formed out of the alloy. The *varṇas* of the two pieces and the *varṇa* of the gold stick being known, the rule tells how to find weighs of the lower and higher *varṇas* in the gold stick.

Example 16. The *varṇas* of two pieces of gold are 15 and 6. (The weight of the gold stick is 6.) If the *varṇa* is reduced by 1/4m find the weights of gold of the lower and higher *varṇas* (in the stick).

Weight of lower *varṇa* = $\frac{6\times 1/4}{15-6} = 1/6$

Weight of higher *varṇa* = $6 - \frac{1}{6} = 5\frac{5}{6}$

2. Series (*Śreḍhī*)

30. Diminish the number of terms by 1, then multiply by half the common difference, then add the first term, and then multiply by the number of terms: the result is the sum of the series in arithmetic progression.[39] That is if *a* be the first term, *d* the common difference and *n* the number of terms, then the sum of the sets is given by

$$s = n\left[\frac{d}{2}(n-1) + a\right]$$

Example 17 (a-b). The initial sum is 8, and the daily increment is 5. What will be the amount at the end of a month? (Answer: 2415.)

3. Plane Figures (*Kṣetra*)

1. Triangle and quadrilateral

31 (a-b). In triangle etc. (i.e. in triangle and quadrilateral), the product of half the sum of sides and countersides gives the area.[39]

That is, if *a*, *b*, *c*, *d* be the sides of a quadrilateral, taken in order, then

$$\text{Area} = \frac{a+c}{2}\times\frac{b+d}{2}$$

Similarly, if *a* be the base of a triangle and *b*, *c* the other two sides, then

[38] Cf. *L*, vs. 110.

[39] This rule is the same as stated in *BrSpSi* (= *Brāhma-sphuṭa - siddhānta*), xii. 21 (a-b), but criticized by Śridharācārya in PG, rule 112-114.

$$\text{area} = \frac{a}{2} \times \frac{b+c}{2}$$

These results are gross.

Example 17 (c-d)-18 (a-b). What is the area of the triangle whose sides are 13, 14 and 15; of the square whose sides are each 5; and of the quadrilateral whose sides are 5, 7, 9 and 8? (Answer: 98, 25 and 105/2.)

31 (c-d). Set down half the sum of the (four) sides (of the quadrilateral) in four places, then diminish them (respectively) by the (four) sides (of the quadrilateral) and then take the square-root of their product: the result is the accurate area of the quadrilateral (if it is cyclic).[40]

Example 18 (c-d)-19 (a-b). What is the accurate area of the quadrilateral whose base is 60, face 25 and the lateral sides 3×13 and 4×13. (Answer: 1764.)

2. Circle

32 (a-b). The diameter and the square of the semi-diameter, when divided by 7 and multiplied by 22, give the circumference and the area (respectively).[41]

It is noteworthy that Śridharācārya in his *Pāṭīgaṇitasāra* or *Triśatikā* takes $\pi = \sqrt{10}$, whereas in above rule use has been made of π = 22/7.

Example 19 (c-d). What is the circumference of the circle having 10 for its diameter, and what is its accurate area? (Answer : circumference $31\frac{3}{7}$ and area $78\frac{4}{7}$).

3. Segment of a Circle

32 (c-d). In the case of the segment of a circle, the area is equal to the arrow as multiplied by half the sum of the *jyā* and arrow increased by one-twentieth of itself.

That is, area of the segment of a circle

= (1+1/20)× arrow (*jyā* arrow)/2.

This formula seems to have been derived from Śridharācārya's formula[42]

Area of the segment of a circle

$$= \frac{\sqrt{10}}{3} \text{ arrow } (jy\bar{a} \text{ arrow})/2$$

by replacing $\sqrt{10}$ by 22/7 and ultimately taking the round figuer 1/20 in place of 1/21.

Example 20 (a-b). What is the accurate area of the segment of a circle in which *jyā* = 13 and arrow = 3?[43] (Answer : $25\frac{1}{5}$.)

[40] Cf. *PG*, rule 117 (a-b); L, vs. 169

[41] Based on L, vs .199 (c-d).

[42] See Triśatikā (ed. S. Dvivedi), p. 35, rule 47

[43] After this P adds the following rules and examples which are taken from Bhāskara II's *Līlāvatī* (vss. 204, 205, 135-137): Rule for finding the arrow: Take the square-root of the product of the sum and difference of diameter and chord, diminish the diameter by that: one –half of it is the arrow. Diminish the diameter by the arrow and then multiply by the arrow: the square root of that multiplied by 2 is the chord. The square of half the chord divided and increased by the arrow is said to be the measure of the diameter in the case of circle.
Example. A circle of diameter 10 has a chord equal to 6. Tell the corresponding arrow. Lso derive the arrow from the chord and the diameter from the chord and the arrow.
Rule of right - angled triangle:
The side which is adjacent to the base (*bhūja*) but is another (orthogonal) direction is the other side. In aright angled triangle or rectangle, it is called the upright (*koṭi*) by the learned. The square root of sum of their squares is the hypotenuse; the square root of the difference of the base and the hypotenuse is the upright; and the square-root of the difference of the squares of the upright and the hypotenuse is the base.
Example. What is the hypotenuse when the upright is 4 and the base 3? Also tell the upright from the base and the hypotenuse and base from the upright and hypotenuse.

4. Excavations (*Khāta*)

33 (a-b). In the case of excavation, the area of the transverse section multiplied by the depth gives the volume in terms of cubic cubits.

Example 20 (c-d). What are the volume in cubic cubits of excavations of depth 3 (cubits) whose transverse sections are the previously stated triangle and rectangle ? (Answer: 294 and 75 cu. cubits.)

5-6. Piles of Bricks and Sawing of Wood (*Citi* and *Krakaca*)

33 (c-d). In the case of piles of bricks, multiply the (base) area by the height;[44] and in the case of sewing, multiply the (sectional) area by the number of sections.[45]

Example 21 (a-b). What are the volumes of the piles of bricks of height 2.when the bases are the previously stated triangle and rectangle? (Answer: 168 and 50.)

Example 21 (c-d). If the length (of the section) is 10 and breadth 3 and the number of sections 6, what is the total area sawn? (Answer: 180.)

7. Heaps of Grain (*Rāśi*)

34 (a-b). In the case of heaps of grain, multiply one-ninth of the circumference by the square of one-sixth of the circumference.[46]

Example 22 (a-b). The circumference of the base of a heap of grain is 36 cubits. How many cubic cubits is its volume? (Answer: 144 cu. cubits or *māgadhakhārīs*.)

8. Shadow (*Chāyā*)

34 (c-d). Half the gnomon divided by gnomon plus its shadow is the day elapsed or to elapse.

Example 22 (c-d). How much of the day has or is to elapse when the shadow of the eight- digit (*aṣṭaṅgula*) gnomon is 24 digits (*aṅgulas*)?[47] (Answer : $\frac{1}{8}$.)

Appreciation of the Work and its Author

35. That you may not fall into the trap laid by the cunning authors of imitative compositions, you should read this chapter in arithmetic composed by the poet Śridhara and delightful to men of taste.

36. From the abode of the gods in the north to the Malaya mountain in the south and between the oceans in the east and west there exists no mathematician other than Śridhara.

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[44] Cf. *Triśatikā*, p.41, rule 58 (a-b); L, vs. 220 (a-1)

[45] Cf. Triśatikā, p.42, rule 59 (a-b).

[46] Cf. *L*, vs. 227. The above rule is for heaps of bearded grain in which the height of the heap is supposed to the one-sixth of the circumference of the base.

[47] Here P adds the following rules and examples which are taken from Bhāskara II's *Līlāvatī* (vss, 234-237):
Rule for knowing the shadow:
The gnomon multiplied by the distance between the foot of the lamp-post and the foot of the gnomon and divided by the height of the lamp minus the height of the gnomon gives the shadow.
Example. If the space intervening between thegnomon and the lamp-post is 3 cubits and the height of the lamp 3 cubits, say quickly how much is the shadow of that gnomon which is 12 digits (*aEgulas*) in length.
Rule for knowing the height of the lamp:
The gnomon divided by the shadow and multiplied by the distance between the foot of the gnomon and the lamp-post, when increased by the gnomon, gives the height of the lamp.
Example. If the space between the lamp-post and the gnomon is 3 cubits and the shadow (of the gnomon) equal to 16 digits (*aṅgulas*), tell me what is the height of the lamp and therefrom the distance between the lamp-post and the gnomon.