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Book Review

The Continuation of Ancient Mathematics: Wang Xiatong's Jigu suanjing, Algebra and Geometry in 7th Century China by Tina Su Lyn Lim and Donald B Wagner, NIAS Press, Nordic Institute of Asian Studies, Copenhagen, Denmark, Pages, xii+220

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In ancient China (from about 7th century CE), the sons of the civil and military officials (plus some others) were given mathematics courses in its national university called Guozi Xue (School for the Sons of the State). The textbooks followed were collectively called *Suanjing shishu* or the *Ten mathematical* (or *computational*) *Classics* (or *Canons*) (*jing*). The following is a list of the textbooks.

List of Ancient Chinese Text Books

1. *Zhoubi suanjing* (Zhoubi dynasty Canon of Gnomonic Computations) compiled during the Han period (206 BCE to 220 CE) or later.
2. *Jiuzhang suanshu* (Arithmetic in Nine Chapters) (1st cent. CE). It is the most popular ancient mathematical Chinese classic (often called Chinese mathematical bible!).
3. *Haidao suanjing* (Sea Island Computational Canon) by Liu Hui (3rd cent CE)
4. *Sunzi suanjing* (Sunzi's Computational Canon) (5th cent roughly)
5. *Wucao suanjing* (Computational Canon of the Five Administrative Sections) (5th cent. very roughly)
6. *Zhang Qiujian suanjing* (Zhang Qiujian's Computational Canon) (compilation period about 466-485)
7. *Xiahou Yang suanjing* (Xiahou Yang's Computational Canon) (1st half of the sixth century)
8. *Wujing suanjing* (Computational Prescriptions of the Five classics) (about 566 CE ?)
9. *Zhui shu* (Interpolation methods?) by Zu Chongzhi (429-500) and his son Zu Xuan who expanded and edited his father's work. (Not extant now)
10. *Jigu suanjing* (Continuation of Ancient Mathematics) by Wang Xiatong (7th cent.)

In addition to the above ten textbooks, the following two were also studied in certain courses (see below).

11. *Shushu jiyi* (Traditions of Arithmo – Numerological Processes) by Xu Yue (3rd cent. CE)
12. *Sandeng shu* (Numbers of Three Degrees) by Dong Quan (sixth-seventh cent.)

The present joint book (by Lim and Wagner) which is being reviewed here, is concerned fully with the canonical textbook no. 10 in the above List. Its original Chinese title was

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Jigu suanshu ('computational methods which continue the ancient'). It was presented by its author (Wang Xiatong) to the Royal Throne of China in CE 628 (or so) when the former's hair was 'turning white'. It is interesting to point out that in CE 628, another mathematically significant event took place in Asia – the completion of the *Brāhmasphuṭa Siddhānta* by the great Indian mathematician Brahmagupta. Wang's work was included among the official Canons (*jing*) in 656 when mathematics specialization was re-established in the national university after a short gap.

Wang had taken great pains to complete his *Jigu*. He was so confident of its correctness that he even declared a reward of a thousand *jin* (equivalent to about 40 kg of silver) to any one who could find a fault in it! Wang was also reputed for his knowledge of astronomy and was Assistant to the Grand Astrologer. In China, with the establishment of the Tang Dynasty (618-907), a new Calendar called the Wuyin Calendar was compiled by Fu Renjun. But it's predicted eclipse for 620 CE did not occur. Wang was ordered to investigate the matter. He corrected more than 30 errors in Fu's calendrical system (pp.3 and 121).

The core of the present book (under review) comes from Lim's M.Sc. thesis in mathematics (University of Copenhagen, 2006) (Prof. Wagner was one of her advisers). The book is presented in three parts:

Part I: Background (pp. 3-32)

Part II: The mathematics of *Jigu* (pp.35-115)

Part III: The Chinese text and translation (pp.119-204)

In the beginning of the book, the detailed Contents (pp. vii-x) is followed by a short preface and at the end, there are three useful Apprendices. Then follows a good Bibliography (pp. 211-216) of about 130 items and finally there is a handy 'Índex and Glossary'.

Part I provides background material which is necessary and relevant to understand Wang's book including its history and purpose. The part also contains a description of the general Chinese mathematics of his time. The following are the Sections of Part I:

1.1. Wang Xiatong and his book
1.2. Public works planning in ancient China.
1.3. Mathematics education in Tang China.
1.4. Mathematical background.
1.5. Extraction of the roots of cubic equations ('Horner's method')
1.6. The history of the text of *Jigu suanjing*.

Exact dates of Wang's birth and death are not known. Since he was quite aged (hair turning grey) in about 628, his birth may be placed near the start of the Sui Dynasty (581-618). The present book tentatively assigns him the dates (579? - 638?).

Under the Tang Dynasty mathematics was officially taught in the School for the Sons of the State. From about 656 there were two courses for mathematics specialization – one elementary and the other advance. In the first course, textbooks no. 1 to 8 (in the above List) were studied and in the second course no. 9 and 10 were studied along with no. 11 and 12. The second course involved both theoretical and practical Chinese mathematics and mathematical techniques of those days in China. For example, the textbook no. 11, *Shushu jiyi* has three degrees of large number-words (or characters) which denote very big numbers. These can be expressed respectively by $10^{(n+4)}$ (lower), 10^{8n} (middle) and 10^{k} (higher) where $k = 2^{n+2}$. Here n takes the values 1,2,3,…10 and 10^4 (*wan*) is the common starting number in all the three series. Thus the largest number will be 10^{4096}. While in textbook no. 4, *Sunzi suanjing* (for first course) we have the series from 10^4 to 10^{80}. Also note the title of the textbook no. 12 (for second course).

Most of the text books (in the above list) were commented either by Zhen Luan (6th cent.) or by Li Chunfeng (602—670) or by both. They were gathered together and collated by Li Chunfeng and his team. He was the director of the astronomical observation service. He was also the primary author of the 'Çalendrical treatise' in the *Sui Shu* (official History of the Sui Dynasty). It is interesting to note that in this treatise he mentions that Zu Chongzhi's *Zhui shu* (which is not extent now) contained Zu's calculation of the very accurate limits of π as follows :

$$3.1415926 < \pi < 3.1415927$$

Another interesting thing is that Zu in his memorial introducing his new *Daming* calendrical system states that he had consulted both Chinese and foreign (books). Here the reference to 'foreign' (books) has been taken to mean "translations of the Indian mathematical and/or astronomical texts" (p.17). In this connection, a footnote mentions reviewer's paper which was first published in 1989, but a better reference will be his paper on "Indian mathematical sciences in ancient and medieval China" (*Gaṇitabhāratī* 27, 2005, 26-63) which has been recently reprinted in his selected work *Gaṇitānanda* (2015).

It is said that the *Suanjing shishu* (Ten Mathematical classics) edited by Li Chunfeng (7th cent) was first printed in 1084. Bao Hanzhi (or Huanzhi) printed it again in 1213 after gathering the works with great difficulty. Out of the several editions of Wang's *Jigu* made during the Qing Dynasty (1644-1911), that of Li Huang (1832) is considered the best (p.31). Modern text of the work is included in the edition of the *Suanjing shishu* by Qian Baocong (Peking, 1963) as well as by Guo Shuchun and Liu Dun (Shenyang, 1998). Lin Yanquan translated the text in modern Chinese in his *Study* of the work (Taiwan, 2001). So the present book, with original Chinese text and a translation in the modern European language, by Lim and Wagner is most welcome.

In Part II, the authors provide us a detailed analysis of the mathematical techniques used by Wang in his *Jigu* which has 20 problems (called *shu* or 'method'). The first problem is astronomical. Due to its out-of-tune nature, it may be regarded as a later interpolation. Problems 2 to 14 are on practical solid geometry (being concerned with earthworks etc.) and the remaining six are on abstract geometry related to right angled triangles. These problems may be considered as a continuation and an advancement of mathematics dealt in the amout *Jiuzhang suanshu* (no. 2 in the list). For example Wang used π = 3 (used in *JZSS*) only once and the better value $\pi = \frac{22}{7}$ invariably on all other occasions.

Formation of a cubic equation is involved as an intermediary state in nearly all of the Wang's problems. Appendix 1 (p.205) lists all the cubic equations along with their real roots. They are of the type

$$x^3 + px^2 + qx = r \qquad \ldots(1)$$

where p, q, r are real and positive (but p and q can be 0 also). Such equations have been shown to have exactly one positive root (p.29).

How Wang solved such equations is not known (p.27). But his algorithm is said to be similar or equivalent to what is now called Horner's or rather Ruffini-Horner method for approximating the real roots of polynomical equations and named after Paolo Ruffini (1803) and William George Horner (1819). It may be that Wang did not include details of the method because (it is presumed that) it was already expounded in the sister textbook *Zhui shu* (no. 9 in the list) and because it was a well-known and well-practiced procedure in his time.

Happily, Lim and Wagner's presentation and exposition of Wang's problems in Part II is very simple, clear and explanatory. A large number of diagrams, figures and also pictures help the reader to enjoy and understand the mathematical steps easily. The illustrative diagrams are neatly drawn and is a distinct feature of the present book.

As a sample of Wang's formation of his problems and obtaining their solutions, we take the first part of his problem 7. This is related to a granary in the form of a *fangting* i.e. a truncated square pyramid. We first give a convenient modern exposition using modern symbology. Let a and b (see **Fig. 1**) be the sides of the bottom and top squares and h be the height of the pyramid.

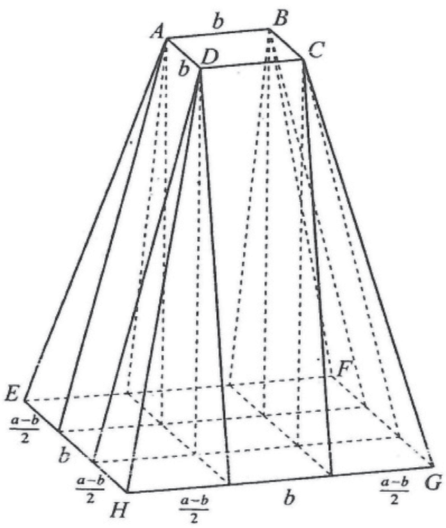

Fig. 1

In problem 7, the following are given (p.168): Difference of sides,

$a - b = 6$ *chi* …(2)

Excess of h over b, $h - b = 9$ *chi* …(3)

Grain content of granary = 187.2 *hu* …(4)

To find values of a, b, and h is the problem. By using the norm for *hu* (= 2.5 cubic *chi* per *hu*), the volume of the pyramid will be,

V = 187.2 × 2.5 = 486 cubic *chi*).

Thus we see that from modern point of view, Wang's problem amounts to finding of a, b and h when

(i) $a - b = x$, say …(5)

(ii) $h - b = y$, say …(6)

and *(iii)* $h(a^2 + ab + b^2)/3 = V$ …(7)

are given or known. To find b, we put the value of a from (5) and that of h from (6) into (7) to get $(b + y)\,[(b + x)^2 + (b + x)b + b^2] = 3V$ …(8)

which on simplification becomes the cubic

$$b^3 + (x+y)/b^2 + (xy + x^2/3)b = V - \frac{yx^2}{3} \qquad \text{…(9)}$$

Using the numerical values $x = 6$, $y = 9$, $v = 468$, we get

$b^3 + 15b^2 + 66b = 360$ …(10)

which is exactly what Wang got (p.169). He solves the equation (10) to get $b = 3$, and thence $a = 9$ and $h = 12$ easily. In fact he had also arrived at the more general form which may be considered equivalent to (9) (see the cubic on p. 169 with $k_1 = x^2/3$). Herein lies the greatness and ingenuity of Wang.

As is usual with ancient texts, the details of Wang's reasoning is not found in *Jigu*. But Lim and Wagner have correctly concluded that the cubic (8) was derived using volume dissection method. Fig. 1 clearly shows the nine subsolids standing on the nine subparts of the base.

On the other hand our Fig. 2 shows a square pyramid (of height h) formed by reassembling the four corner pyramids of Fig. 1 (whose points A, B, C and D merge to form the

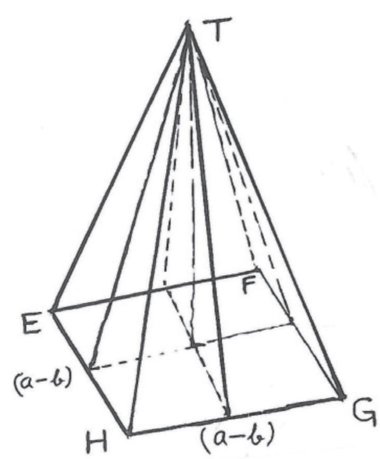

Fig. 2

common vertex T). Now we brief by describe the exposition of Lim and Wagner in our symbols (p.35):

'Area for the Corner *Yangma*' is

$$K_1 = (a - b)^2/3 = x^2/3 \qquad \ldots(11)$$

They say that K_1 "does not corresponds to any 'area' in the geometric situation." But the reviewer my point out that K_1 actually represents what may be called (followed ancient practice) the 'Éffective area of the combined corner *Yangma* (i.e. pyramid of Fig.2)' so that its volume

$$v_1 = \left(\frac{x^2}{3}\right)h = \left(\frac{x^2}{3}\right)(y+b) \qquad \ldots(12)$$

Similarly, the combined volume of the four lateral right prisms (Fig.1)

$$v_2 = 4b\ h(a-b)/4 = xb(y+b) \qquad \ldots(13)$$

Finally the volume of the remaining central solid

$$v_3 = b^2\ h = b^2(y+b) \qquad \ldots(14)$$

By adding (12), (13) and (14) and noting that $(v_1 + v_2 + v_3)$ is equal to V, we get the required equation (9) by rearranging the terms !

In the same and similar manner the present book has successfully and succinctly been able to expose and explain Wang's problems. Authors have given us this excellent book which fully demonstrates the 7th century Chinese skill in handling practical problems leading to cubic equations.

In the last section of Part II, the book discusses 'The traditional Chinese algebra of Polynomials'. In particular the question is raised as to whether Wang's 'reasoning about calculations' should be called algebra. G.H.F Nesselmann in his *Die algebra der Griechen* (1842) (reprinted, Frankfurt, 1969) had defined three stages in the development of algebra viz. *rhetorical*, *syncopated* and *symbolic* algebra. Wang is seemingly fit for the first category "but in China this was not a 'stage' in the development of the other two kinds of algebra" (p.10).

Actually, symbolic algebra is a powerful tool. We used it to derive equation (10) for b. By similar process the equation for a comes out to be (it can also be derived from that for b):

$$a^3 - 3a^2 - 6a = 432 \qquad \ldots(15)$$

whose one root is $a = 9$. However (14) involves negative coefficients and is not of the type (1) covered in the book.

In part III, the Chinese text and English translation appear conveniently side by side. But the text (included here) is reported to be very fragmentary at the end due to which original problems 18, 19 and 20 are incomplete. What has been given is only from reconstructions made by Zhang Dunren (1754-1834) and the Korean mathematician Nam Pyong-Gil (1820-1869) along with suitable comments (p.104). Significantly, the present book also includes the comments in smaller characters in the text along with their translation although their authorship is an open question. In all, we have a very fine book here and the reviewer strongly recommends it for all libraries and individuals concerned with History of Mathematics.