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FUNDAMENTAL FREQUENCY OF VIBRATION OF A RECTANGULAR PLATE ON A NONLINEAR ELASTIC FOUNDATION

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Fundamental frequency of vibration of a simply-supported rectangular plate resting on a hardening nonlinear elastic foundation is deduced by taking resort to a perturbation procedure introduced by Lindsted in the analysis of weakly nonlinear autonomous dynamical system subjected to certain initial conditions. Numerical result is presented graphically.

INTRODUCTION

Following Lindsted's perturbation technique (Meirovitch 1970) in the analysis of quasi-harmonic system with the hardening spring, the author has obtained the fundamental frequency of vibration of a simply-supported rectangular plate resting on a nonlinear elastic foundation. The fundamental frequency of vibration of elastic plates on nonlinear foundation is important from the standpoint of designs of pavement and airport runway.

MATHEMATICAL ANALYSIS

For an elastic plate resting on a hardening nonlinear elastic foundation, the equation of motion, neglecting damping, is governed by the following nonlinear partial differential equation (Reissner 1970)

$$D\left(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}\right) + \rho h \frac{\partial^2 W}{\partial t^2} + k_1 W + k_3 W^3 = 0 \qquad \ldots(1)$$

where $D = Eh^3/12(1 - \nu^2)$ is the flexural rigidity of the plate; E, ν are respectively the elastic modulus, Poisson's ratio; h, a and b denote thickness, length and breadth of the plate, ρ being plate-density; k_1, k_3 are foundation-parameters and $w(x, y, t)$ is the lateral deflection of the plate at any instant t. Equation (1) may be reduced to a set of nonlinear ordinary differential equations with the help of modified Galerkin procedure. Take in the first approximation

$$W(x, y, t) = c\Phi(x, y)\,\theta(t) \qquad \ldots(2)$$

where c is the arbitrary amplitude, $\Phi(x, y)$ is a suitably chosen function satisfying the boundary conditions of the plate and $\theta(t)$ is a unspecified function of time t. For a plate with all its edges simply-supported, take $\Phi(x, y) = \sin(\pi x/a) \sin(\pi y/b)$ and employ eqn. (2) in eqn. (1) to obtain the following nonlinear equation

$$\frac{d^2\theta}{dt^2} + \bar{\omega}_0^2 \, \theta + \lambda\epsilon\theta^3 = 0 \qquad \text{...(3)}$$

in which

$$\left.\begin{aligned} \bar{\omega}_0^2 &= (\rho h)^{-1} \left\{ D \left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 \right]^2 + k_1 \right\} \\ \lambda &= 0.5625 \, k_1/\rho h \\ \epsilon &= k_3/k_1 \; (\ll 1). \end{aligned}\right\} \qquad \text{...(4)}$$

Following Lindsted's procedure (Meirovitch 1970), a periodic solution of eqn. (3) is sought in the form

$$\theta(t) = \theta_0 + \epsilon\,\theta_1 + \epsilon^2\theta_2 + \ldots \qquad \text{...(5)}$$

where each $\theta_i (i = 0, 1, 2, \ldots)$ is taken to be periodic. Again, by virtue of dependence of the period of oscillation on ϵ, the unknown frequency can be represented as

$$\Omega = \bar{\omega}_0 + \epsilon\,\bar{\omega}_1 + \epsilon^2\bar{\omega}_2 + \ldots \,. \qquad \text{...(6)}$$

Further setting $\tau = \Omega t$, eqn. (3) is transformed in the form

$$\Omega^2\ddot{\theta} + \bar{\omega}_0^2\,\theta + \epsilon\,\lambda\theta^3 = 0 \qquad \text{...(7)}$$

where dots denote differentiation with respect to τ. Use of (5) and (6) in eqn. (7) yields the following system of ordinary differential equations:

$$\left.\begin{aligned} &\ddot{\theta}_0 + \theta_0 = 0 \\ &\ddot{\theta}_1 + \theta_1 = -\,\alpha\theta_0^3 - 2(\bar{\omega}_1/\bar{\omega}_0)\,\ddot{\theta}_0 \\ &\ddot{\theta}_2 + \theta_2 = -\,3\alpha\theta_0^2\,\theta_1 - \{2(\bar{\omega}_2/\bar{\omega}_0) + (\bar{\omega}_1/\bar{\omega}_0)^2\}\,\ddot{\theta}_0 - 2(\bar{\omega}_1/\bar{\omega}_0)\,\ddot{\theta}_1 \\ &\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ &\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \end{aligned}\right\} \qquad \text{...(8)}$$

where

$$\alpha = \lambda/\bar{\omega}_0^2 = 0.5625\,k_1/\{D\,[(\pi/a)^2 + (\pi/b)^2]^2 + k_1\}. \qquad \text{...(9)}$$

In consistence with autonomous system, take the initial conditions as

$$\tau = 0, \;\; \dot{\theta}_n = 0 \quad (n = 0, 1, 2, \ldots) \qquad \text{...(10)}$$

The solution of the first equation of system (8) subject to condition (10) is

$$\theta_0 = C\cos\tau, \;\; C \text{ being integration constant} \qquad \text{...(11)}$$

and which reduces the second equation of (8) as follows

$$\ddot{\theta}_1 + \theta_1 = C(2(\bar{\omega}_1/\bar{\omega}_0) - 0.75\alpha C^2)\cos\tau - 0.25C^3\cos 3\tau \qquad \text{...(12)}$$

It can be easily observed that the first term on the right hand side of (12) may lead to resonance and consequently may produce mathematically unbounded solution. To get rid of this term, equate the coefficient of cos τ in (12) to zero. This establishes

$$\bar{\omega}_1/\bar{\omega}_0 = 0.375\alpha C^2 \quad \text{...(13)}$$

and

$$\theta_1 = 0.3125\alpha C^3 \cos 3\tau. \quad \text{...(14)}$$

Make use of eqns. (11), (13) and (14) in the third eqn. of (8). This will yield

$$\bar{\omega}_2/\bar{\omega}_0 = 0.0585937\alpha^2 C^4 \quad \text{...(15)}$$

and

$$\theta_2 = 0.0205078\alpha^2 C^5 \cos 3\tau + 0.0009765\,\alpha^2 C^5 \cos 5\tau. \quad \text{...(16)}$$

Finally, substitute the above results in (2) and (6) to get the following expressions of deflection function $w(x, y, t)$ and Ω, the frequency of vibration

$$W(x, y, t) = [A \cos(\Omega t + \psi) + 0.375\epsilon\alpha A^2(1 - 0.65625\epsilon\alpha A^2) \times \cos 3(\Omega t + \psi) + 0.0009765\epsilon^2\alpha^2 A^4 \cos 5(\Omega t + \psi)] \times \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} + O(\epsilon^3) \quad \text{...(17)}$$

$$\Omega = \bar{\omega}_0 [1 + 0.375\epsilon\alpha A^2 - 0.0585937\epsilon^2\alpha^2 A^4] + O(\epsilon^3) \quad \text{...(18)}$$

where $\epsilon C^i = A^i (i = 1, 2, \ldots)$ is the amplitude and ψ the phase angle.

An integration of eqn. (3) yields

$$\left(\frac{d\theta}{dt}\right)^2 + \bar{\omega}_0^2\,\theta^2 + \epsilon\lambda\,\frac{\theta^4}{2} = E, \text{ a constant} \quad \text{...(19)}$$

where E represents the total energy per unit mass of the system.

It is apparent that the only equilibrium point of the system is at the origin of the phase space $\theta = \dot{\theta} = 0$ which is the centre. For a given value of E, a value that depends on the initial conditions, the motion takes place along the level curve $E =$ constant, where the level curve represents a closed trajectory enclosing the centre.

NUMERICAL RESULT

A graph of relative frequency $\Omega/\bar{\omega}_0$ versus amplitude $|A|$ has been plotted (Fig. 1) for a simply-supported concrete slab of dimensions $10' \times 10' \times 1'$ ($E = 2 \times 10^6$ psi) resting over a firm soil ($k = 614.4$ lbs/in^3).

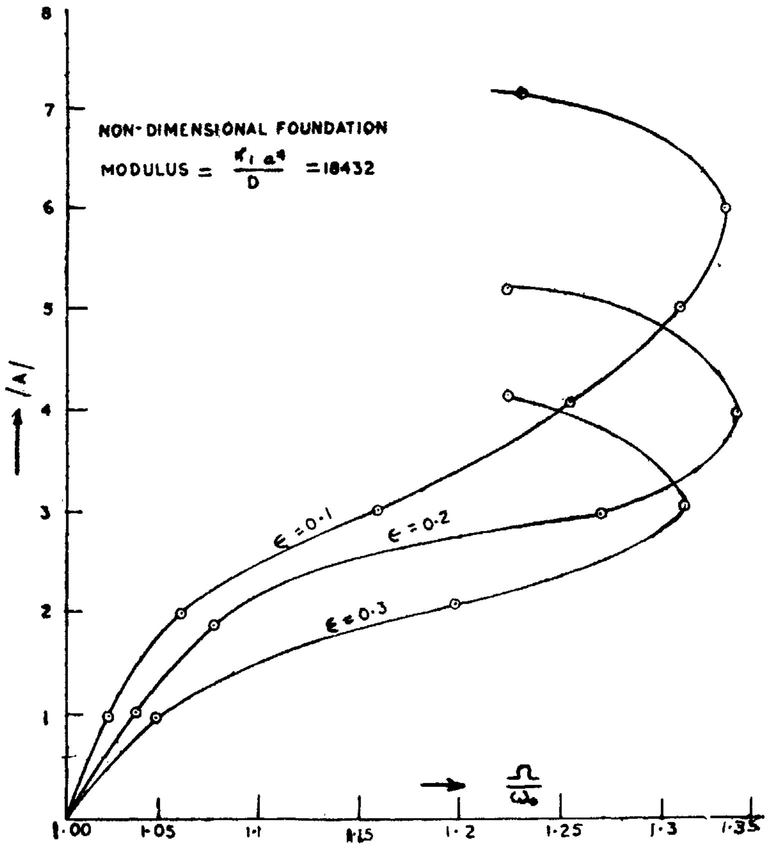

FIG. 1. $\Omega/\bar{\omega}_0$ as function of $|A|$.

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