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HEAT TRANSFER PROBLEMS OF FORCED CONVECTION IN NON-CIRCULAR DUCTS

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In this paper, heat transfer problems of forced convection in non-circular ducts are studied retaining the dissipation terms in the energy equation using complex variable techniques. A uniform procedure has been presented for the determination of velocity and temperature fields which is different from that of Tao (1961). The results can be extended to doubly connected regions. Expressions for the average velocity, average temperature, mixed mean temperature, friction-factor, pressure drop, and normal stress have been derived. The results for cardioid and circular ducts have been deduced as particular cases.

INTRODUCTION

Tao (1961) has investigated the forced convection problems of fully developed laminar flow in a duct of the form of Pascal's limacon section with linearly varying axial wall temperature and with constant heat generation using the method of conformal mapping. The results were compared with those calculated from a circular pipe with an appropriate radius based upon equal area, equal circumferential length and equivalent hydraulic diameter. Dissipation term in the energy equation has been neglected.

1. MATHEMATICAL FORMULATION

Consider a steady fully developed laminar flow with arbitrary thermal energy sources in a duct of cross-section D bounded by a closed curve Γ. Let the axis of the duct be in the Z-direction. Suppose the fluid is incompressible, and the fluid properties ρ, c_p, k are constants and there is no mass diffusion, chemical reaction and electromagnetic effects. The governing differential equations for the determination of the velocity and temperature profiles are given by

$$\nabla^2 u = \frac{g_c}{\mu} \frac{\partial p}{\partial Z} = c_1 \qquad \text{...(1.1)}$$

$$\nabla^2 T = \frac{\rho c_p}{k} \frac{\partial T}{\partial Z} u - \frac{S}{k} - \frac{\mu'}{k g_c} \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 \right]. \qquad \text{...(1.2)}$$

The above equations are to be solved subject to the boundary conditions

$$u = 0,\ T = T_w. \qquad \text{...(1.3)}$$

The last term in eqn. (1.2) on the right-hand side represents part of the work done by the fluid on adjacent layers due to the action of shear stress. Using complex variables $z = x + iy$, $\bar{z} = x - iy$ eqns. (1.1) and (1.2) can be written as

$$4\,\partial^2 u/\partial z \partial \bar{z} = c_1 \qquad \text{...(1.4)}$$

$$4\,\partial^2 T/\partial z \partial \bar{z} = c_2 u - c_3 - 4(\mu/k)\,(\partial u/\partial z)\,(\partial u/\partial \bar{z}). \qquad \text{...(1.5)}$$

Solution of the Problem

The solution of (1.1) subject to the first of the conditions (1.3) can be derived in the form

$$u = c_1 z\bar{z}/4 + \phi(z) + \overline{\phi(z)} \qquad \text{...(1.6)}$$

where $\phi(z)$ is holomorphic in the cross-section and $\overline{\phi(z)}$ is its conjugate.

Since many investigators studied the class of problems neglecting viscous dissipation terms in the energy equation we present very general solution of such problems and dissipation terms will be taken into consideration in the particular example. Using (1.6) in (1.5) and integrating we obtain

$$T = \frac{c_4 z^2 \bar{z}^2}{64} - \frac{c_3 z\bar{z}}{4} + \frac{c_2}{2}\,\mathrm{Re}\left(\bar{z}\int \phi(z)\,dz + \psi(z) + \overline{\psi(z)}\right) \qquad \text{...(1.7)}$$

where $\psi(z)$ is analytic in the cross-section and $\overline{\psi(z)}$ is its conjugate. In the case of doubly connected regions we must select the constants of integration such that the functions $\phi(z)$ and $\psi(z)$ are single valued in the cross-section.

Using complex Stokes' theorems we can easily deduce the following results:

$$A = \frac{1}{2i}\int \bar{z}\,dz \qquad \text{...(1.8)}$$

$$Au_m = \iint u\,dx\,dy = \frac{c_1}{16i}\int z\bar{z}^2\,dz + \frac{1}{i}\int z\,\phi(z)\,dz \qquad \text{...(1.9)}$$

$$AT_m = \iint T\,dx\,dy = \frac{c_4}{384i}\int z^2\bar{z}^3\,dz - \frac{c_3}{16i}\int z\bar{z}^2\,dz$$

$$+ \frac{1}{i}\int \bar{z}\,\psi(z)\,dz + \frac{c_2}{8i}\int \bar{z}^2\left(\int \phi(z)\,dz\right)dz \qquad \text{...(1.10)}$$

$$Au_mT_m = \iint uT\,dx\,dy = \frac{c_1 c_4}{2048i}\int z^3\bar{z}^4\,dz$$

$$- \frac{c_1 c_3}{96i}\int z^2\bar{z}^3\,dz + \frac{c_1}{8i}\int z\bar{z}^2\psi(z)\,dz +$$

(*equation continued on p.* 338)

$$+ \frac{c_4}{192i} \int z^2 \bar{z}^3 \phi \, dz - \frac{c_3}{8i} \int z \bar{z}^2 \phi(z) \, dz$$

$$- i \int \left[\bar{z} \psi(z) \, \phi(z) + \psi(z) \int \overline{\phi(z)} \, d\bar{z} \right] dz$$

$$+ \iint \left[\frac{c_4}{16} z\bar{z} \int \bar{z} \phi(z) \, dz + \text{conjugate} \right] dx \, dy \qquad ...(1.11)$$

and the integrals are to be evaluated in the t-plane using the transformation formula $z = w(t)$, which maps the section on to the unit circle in the t-plane. The constants in the analytic functions $\phi(z)$ and $\psi(z)$ can be determined uniquely using the conditions (1.3). The functions can be determined using Cauchy's integral formulae or by Taylor's expansion of the function and comparing like powers of σ on the boundary of the unit circle.

We get the heat transfer rate by integrating (1.5) over the section as

$$q = kA(c_2 u_m - c_3) - 4\mu \iint \left(\frac{\partial u}{\partial z}\right) \left(\frac{\partial u}{\partial \bar{z}}\right) dx \, dy. \qquad ...(1.12)$$

The heat transfer coefficient h and the Nusselt number Nu based on T_M may be evaluated using the following formulae:

$$\text{(a)} \quad h = - \frac{q}{S} T_M, \quad \text{(b)} \quad Nu = hD_e/k. \qquad ...(1.13)$$

Pressure Drop

In a long duct in which fluid enters at a uniform velocity, the effect of the entrance region is to increase the pressure drop compared to that of fully developed flow. This is denoted by $k^{(\infty)}$ and defined by

$$\Delta p \Big/ \left(\frac{\rho u_m^2}{2g_c} \right) = f_{fd} \frac{L}{r_h} + k^{(\infty)}. \qquad ...(1.14)$$

In the heat exchanger analysis the knowledge of entrance length and f_{fd} is enough to calculate the total pressure drop (London and Shah 1971). For ducts of arbitrary section Lundgren *et al.* (1964) derived the expression $k^{(\infty)}$ in the form

$$k^{(\infty)} = (2/A) \iint \left[\left(\frac{u}{u_m}\right)^3 - \left(\frac{u}{u_m}\right)^2 \right] dx \, dy. \qquad ...(1.15)$$

Normal stress

We can easily derive the expression for the normal stress in the form

$$N = \left| \frac{\partial u}{\partial n} \right|^2 = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2. \qquad ...(1.16)$$

In terms of the complex variables it can be written as

$$N = 4\left(\frac{\partial u}{\partial z}\right)\left(\frac{\partial u}{\partial \bar{z}}\right). \qquad ...(1.17)$$

Calculation of f_{Re}-Friction Factor

We can show that

$$f_{Re} = -8c_1A^2/(S^2u_m). \qquad ...(1.18)$$

2. Pascal's Limacon Section

Let us take the case of an indented pipe in the form of Pascal's limacon with constant thermal energy source within the fluid. In the physical plane the parametric equations of the boundary are given by

$$x = a\,(\cos\theta + m\cos 2\theta),\; y = a\,(\sin\theta + m\sin 2\theta) \qquad ...(2.1)$$

$$0 \leqslant m \leqslant 0.5,\; -\pi < \theta < \pi,\; a > 0$$

which can be mapped conformally on to the unit circle in the t-plane by the formula

$$z = w(t) = a(t + mt^2). \qquad ...(2.2)$$

Using (2.2) in the general formulae (1.6), (1.8) and (1.9) we obtain

$$\phi(z) = -\tfrac{1}{8}\,c_1a^2\,[(1 + 2m^2) + 2mt] \qquad ...(2.3)$$

$$A = \pi a^2(1 + 2m^2) \qquad ...(2.4)$$

$$u = \tfrac{1}{4}\,c_1a^2\,[t\bar{t}(1 + mt)(1 + m\bar{t}) - (1 + m^2) - m(t + \bar{t})] \qquad ...(2.5)$$

$$u_m = -\,c_1a^2(1 + 4m^2 + 2m^4)/8(1 + 2m^2). \qquad ...(2.6)$$

Using these results in the formulae (1.15), (1.17) and (1.18) we easily obtain

$$k^{(\infty)} = \frac{4(5 + 60m^2 + 270m^4 + 612m^6 + 718m^8 + 368m^{10} + 40m^{12})}{15(1 + 4m^2 + 2m^4)^3} \qquad ...(2.7)$$

$$N = \frac{c_1a^2(1 + 6m^2 + 4m^4) + 4m(1 + 2m^2)\cos\theta + 2m^2\cos 2\theta}{4(1 + 4m\cos\theta + 4m^2)} \qquad ...(2.8)$$

$$f_{Re} = \frac{64\pi^2a^2(1 + 2m^2)^3}{S^2(1 + 4m^2 + 2m^4)}. \qquad ...(2.9)$$

The element of the arc-length of the duct is given by

$$\left(\frac{ds}{d\theta}\right)^2 = \left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2. \qquad ...(2.10)$$

Using (2.1) and integrating we find

$$S = 4a(1 + 2m) \int_0^{\pi/2} \left[1 + \frac{8m}{(1 + 2m)^2} \cos^2 \frac{\theta}{2}\right] d\theta \qquad \text{...(2.11)}$$

which is to be used in the formula (2.9).

3. Determination of the Temperature Field

Using (1.6) in (1.5) and integrating we find

$$T = \psi(z) + \overline{\psi(z)} + \frac{c_4'}{64} z^2\bar{z}^2 - \frac{c_3}{4} z\bar{z} - \frac{\mu}{k} \phi(z)\, \overline{\phi(z)}$$

$$+ \frac{c_2}{4}\left(\bar{z}\int \phi\, dz + z\int \bar{\phi}\, d\bar{z}\right) - \frac{\mu c_1}{4k}\left(\bar{z}\int z\, d\phi + z\int \bar{z}\, d\bar{\phi}\right). \qquad \text{...(3.1)}$$

Now we establish the following results:

$$\frac{c_4'}{64} z^2\bar{z}^2 = \frac{c_4' a^4}{64}\Big[(1 + 4m^2 + m^4) + 2m(1 + m^2)$$

$$\times \left(\sigma + \frac{1}{\sigma}\right) + m^2\left(\sigma^2 + \frac{1}{\sigma^2}\right)\Big] \qquad \text{...(3.2)}$$

$$\frac{c_3}{4} z\bar{z} = \frac{c_3}{4} a^2\left[(1 + m^2) + m\left(\sigma + \frac{1}{\sigma}\right)\right] \qquad \text{...(3.3)}$$

$$\frac{c_2}{4}\left[\bar{z}\int \phi\, dz + z\int \bar{\phi}\, d\bar{z}\right] = -\frac{c_2 a^4}{96}\,[6(1 + 3m^2 + m^4) + 4m^2(\sigma^2 + \sigma^{-2})$$

$$+ (9m + 10m^3)(\sigma + \sigma^{-1})] \qquad \text{...(3.4)}$$

$$\frac{\mu\phi\phi'}{k} = \frac{\mu c_1^2 a^4}{64k}\,[(1 + 6m^2 + m^4) + 2m(1 + m^2)(\sigma + \sigma^{-1})] \qquad \text{...(3.5)}$$

$$\frac{\mu c_1}{4k}\left(\bar{z}\int z\, d\phi + z\int \bar{z}\, d\bar{\phi}\right) = \frac{\mu c_1^2 a^4 m}{96k}\,[(6m + (3 + 2m^2)(\sigma + \sigma^{-1})$$

$$+ 2m(\sigma^2 + \sigma^{-2})]. \qquad \text{...(3.6)}$$

Using the second of the boundary conditions (1.3) and (3.2) to (3.6) in (3.1) we find

$$\psi + \bar{\psi} = T_w + \frac{c_4 a^4}{64}(3 + 8m^2 + 3m^4) + \frac{c_3 a^2(1 + m^2)}{4}$$

$$+ \frac{\mu c_1^2 a^4(1 + 3m^2 + m^4)}{32k} + (\sigma + \sigma^{-1})\Big[\frac{c_3 a^2 m}{4}$$

$$+ \frac{c_4 a^4 m(6 + 7m^2)}{96} + \frac{\mu c_1^2 a^4 m(3 + 5m^2)}{96k}\Big]$$

$$+ (\sigma^2 + \sigma^{-2})\left[\frac{a^4 m^2}{192}\left(5c_4 - \frac{\mu c_1^2}{k}\right)\right]. \qquad \text{...(3.7)}$$

Assuming

$$\psi(z) = B_0 + B_1 t + B_2 t^2 \qquad \text{...(3.8)}$$

and using in (3.7) we obtain

$$2B_0 = \frac{c_4 a^4(3 + 8m^2 + 3m^4)}{64} + \frac{c_3 a^2(1 + m^2)}{4} + \frac{\mu c_1^2 a^4(1 + 3m^2 + m^4)}{32k} + T_w \qquad \text{...(3.9)}$$

$$B_1 = \frac{c_3 a^2 m}{4} + \frac{c_4 a^2 m(6 + 7m^2)}{96} + \frac{\mu c_1^2 a^4 m(3 + 5m^2)}{96k} \qquad \text{...(3.10)}$$

$$B_2 = \frac{a^4 m^2}{192}\left(5c_4 - \frac{\mu c_1^2}{k}\right). \qquad \text{...(3.11)}$$

Thus the temperature profiles have been exactly determined. Solving (2.2) for t we get

$$t = [-1 + (1 + 4mz/a)^{1/2}]/2m. \qquad \text{...(3.12)}$$

We can also express the function $\phi(z)$ and $\psi(z)$ in terms of z using (3.12)

$$\phi(z) = -\tfrac{1}{8} c_1 a^2 [m^2 + (1 + 4mz/a)^{1/2}]$$

$$\psi(z) = B_0 - \left[\frac{B_1}{2m} - \frac{B_2}{2m^2}\right]\left[1 - \left(\frac{a + 4mz}{a}\right)^{1/2}\right] + \frac{B_2 z}{am}. \qquad \text{...(3.13)}$$

The mean temperature T_m and the mixed mean temperature T_M are given by

$$AT_m = \iint T\,dx\,dy \qquad \text{...(3.14)}$$

$$Au_m T_M = \iint uT\,dx\,dy. \qquad \text{...(3.15)}$$

After simplification we find

$$T_m - T_w = \frac{c_4 a^4}{48(1 + 2m^2)}\left[(1 + 6m^2 + 8m^4 + 2m^6) + 6\beta(1 + 4m^2 + 2m^4) + \frac{c_1 \mu}{2c_2 k}(1 + 6m^2 + 10m^4 + 2m^6)\right] \qquad \text{...(3.16)}$$

$$T_M = \frac{c_4 a^4}{384(1 + 4m^2 + 2m^4)}[11 + 88m^2 + (596m^4/3) + (2072m^6/15) + 22m^8 + 64\beta(1 + 6m^2 + 8m^4 + 2m^6)] + \frac{c_1^2 a^4 [5 + 40m^2 + 98.66m^4 + 72.66m^6 + 10m^8]}{384k(1 + 4m^2 + 2m^4)}. \qquad \text{...(3.17)}$$

Now integrating the energy equation over the section we find

$$q = \frac{-\pi c_4 k a^4}{8}\left[(1 + 4m^2 + 2m^4) + 8\beta(1 + 2m^2) + \frac{\mu c_1(1 + 4m^2 + 2m^4)}{c_2 k}\right]. \quad \ldots(3.18)$$

The heat transfer coefficient h and the Nusselt number Nu can be calculated using (2.11), (3.17) and (3.18) in the formulae (1.13). Neglecting the terms containing μ in eqns. (3.16), (3.17) and (3.18) we find that our results agree with the results obtained by Tao (1961).

4. Special Cases

Cross-section a Circle

Writing $m = 0$ in the above formulae we find

$$u_m = -\tfrac{1}{8} c_1 a^2, \ \phi(z) = -0.125 c_1 a^2,$$

$$\psi(z) = 0.5T_w + 0.02344 c_4 a^4 + 0.125 c_3 a^2 + 0.01562(\mu c_1^2 a^4/k),$$

$$(T_m - T_w) = \frac{c_4 a^4}{48}\left[1 + 6\beta + \frac{\mu c_1}{2kc_2}\right],$$

$$T_M = \frac{c_4 a^4}{384}(11 + 64\beta) + 0.01302(\mu c_1^2 a^4/k),$$

$$k^{(\infty)} = 4/3, \ N = c_1^2 a^2/4, \ f_{Re} = 16,$$

$$q = -0.125\pi c_4 k a^4(1 + 8\beta + (\mu c_1/kc_2)),$$

$$Nu = \frac{48(1 + 8\beta + (\mu c_1/kc_2))}{11 + 64\beta + 5(\mu c_1/kc_2))}. \quad \ldots(4.1)$$

Cross-section a Cardioid

In this problem simply writing $m = \frac{1}{2}$ in (2.3) to (3.18) we get

$$u_m = -0.17708 c_1 a^2, \ \phi(z) = -c_1 a^2(5 + 4t)/32,$$

$$\psi(z) = B_0 + B_1 t + B_2 t^2 \quad \ldots(4.2)$$

where

$$B_0 = 0.5T_w + 0.04053 c_4 a^4 + 0.15625 c_3 a^2 + 0.02832(\mu c_1^2 a^4/k),$$

$$B_1 = 0.125 c_3 a^2 + 0.04036 c_4 a^4 + 0.022135 \mu c_1^2 a^4/k,$$

$$B_2 = a^4(0.00651 c_4 - 0.0013 \mu c_1^2/k),$$

$$T_m - T_w = c_4 a^4 [0.0421 + 0.1771\beta + 0.021918(\mu c_1/kc_2)],$$

(equation continued on p. 343)

$$T_M = c_4a^4(0.05841 + 0.2377\beta) + 0.02742(\mu c_1^2\, a^4/k),$$

$$k^{(\infty)} = 1.37871,\ N = \frac{c_1a^2(11 + 12\cos\theta + 2\cos 2\theta)}{32(1 + \cos\theta)},$$

$$f_{Re} = 15.6878,$$

$$q = -\pi c_4ka^4\,[0.26562 + 1.5\beta + 0.26562(\mu c_1/kc_2)],$$

$$Nu = \frac{\pi^2\,[20.3203 + 114.75\beta + 20.3203(\mu c_1/kc_2)]}{47.6609 + 194\beta + 22.3724(\mu c_1/kc_2)}\,. \qquad \text{...(4.3)}$$

The various approximate results for non-circular ducts can be derived from eqns. (4.1) using equal area, equal circumferential length and equal area circumference ratio.

TABLE I

Increment in pressure drop for various values of m (Fig. 1)

m	$k^{(\infty)}$
0	1.33333
0.1	1.33335
0.2	1.33397
0.3	1.33848
0.4	1.35225
0.5	1.37871

TABLE II

Friction factor (Fig. 1)

m	f_{Re}
0	16.0000
0.1	15.9435
0.2	15.9016
0.3	15.8235
0.4	15.6756
0.5	15.6878

TABLE III

Variation in normal stress : $N/(4c_1^{-2}\,a^{-2})$ (Fig. 2)

θ / m	0.1	0.2	0.3	0.4	0.5
0°	1.0336	1.7665	1.23765	1.38716	1.56250
30°	1.0269	1.3042	1.2041	1.33904	1.50000
45°	1.0197	1.0763	1.16533	1.28305	1.42678
60°	1.0116	1.0503	1.11755	1.21246	1.33333
90°	1.0004	1.0055	1.02382	1.06244	1.12500
120°	0.8465	1.0189	1.01895	1.00762	1.00000
150°	1.0191	1.1519	1.35955	1.54642	1.50000
180°	1.0506	1.2844	2.1025	6.76000	∞

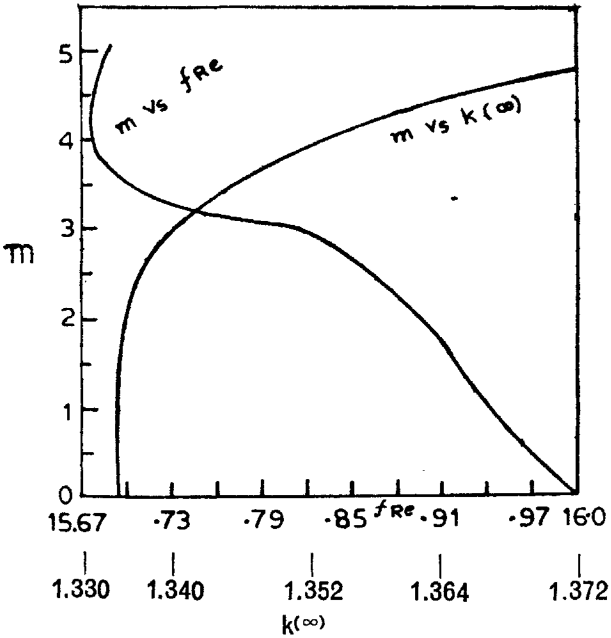

FIG. 1. Variations of f_{Re} and $k^{(\infty)}$ with m.

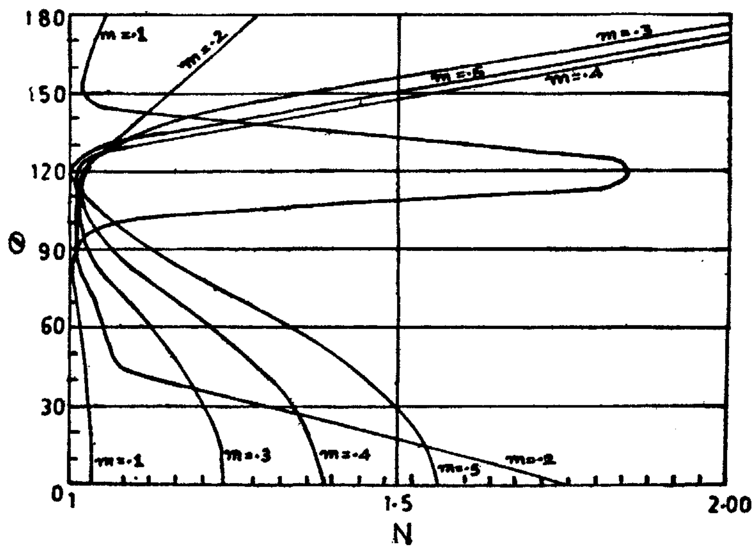

FIG. 2. Variation in normal stress.

5. Equivalent Circular Pipes

Engineering problems of non-circular ducts are usually approached using solutions derived for a circular tube with an appropriate radius. Here we evaluate the average velocity, the average temperature, the mixed mean temperature and the heat transfer rate of Pascal's limacon section based on the solution of a circular pipe of equal area, equal circumferential length and equal area circumference ratios respectively.

(i) For equal areas we have

$$\pi R^2 = \pi a^2(1 + 2m^2).$$

Substituting $R^2 = a^2(1 + 2m^2)$ in eqns. (4.1) we obtain the following results:

$$u_m = -0.125c_1a^2(1 + 2m^2) \quad \text{...(5.1)}$$

$$T_m = 0.0208c_4a^4(1 + 2m^2)^2 [1 + 6\beta_1 + (\mu c_1/2kc_2)] \quad \text{...(5.2)}$$

where

$$\beta_1 = c_3/[c_4a^2(1 + 2m^2)] \quad \text{...(5.3)}$$

$$T_M = 0.002604c_4a^4(1 + 2m^2)^2 [11 + 64\beta_1 + (5\mu c_1/kc_2)] \quad \text{...(5.4)}$$

$$\frac{q}{k} = -0.125\pi c_4a^4(1 + 2m^2)^2 [1 + 8\beta_1 + (\mu c_1/kc_2)] \quad \text{...(5.5)}$$

$$Nu = \frac{48 [1 + 8\beta_1 + (\mu c_1/kc_2)]}{11 + 64\beta_1 + (5\mu c_1/kc_2)}. \quad \text{...(5.6)}$$

(ii) For equal circumferential length we have

$$2\pi R = 4a(1 + 2m)E \quad \text{...(5.7)}$$

where

$$E = \int_0^{\pi/2} \frac{(1 - 8m \sin^2 \theta)^{1/2} \, d\theta}{(1 + 2m)^2}. \quad \text{...(5.8)}$$

Using (5.7) we obtain the following results:

$$R = \frac{2a}{\pi} (1 + 2m) E \quad \text{...(5.9)}$$

$$u_m = -\frac{c_1a^2(1 + 2m)^2 E^2}{2\pi^2} \quad \text{...(5.10)}$$

$$T_m = \frac{c_4a^4(1 + 2m)^4 E^4}{3\pi^4} \left(1 + 6\beta_2 + \frac{\mu c_1}{2kc_2}\right) \quad \text{...(5.11)}$$

where

$$\beta_2 = \pi^2c_3/[c_4 4a^2(1 + 2m)^2 E^2] \quad \text{...(5.12)}$$

$$T_M = c_4a^4(1 + 2m)^4 E^4 [11 + 64\beta_2 + 5(\mu c_1/kc_2)]/(24\pi^4) \quad ...(5.13)$$

$$\frac{q}{k} = - 2c_4a^4(1 + 2m)^4 E^4 [1 + 8\beta_2 + (\mu c_1/kc_2)] \pi^{-3}. \quad ...(5.14)$$

(iii) In the case of equal area circumference ratio we get

$$\frac{\text{Area}}{\text{Circumference}} = \frac{\pi R^2}{2\pi R} = \frac{R}{2} = \frac{\pi a^2(1 + 2m^2)}{4a(1 + 2m) E}.$$

Therefore

$$R = \frac{\pi a(1 + 2m^2)}{2(1 + 2m) E} \quad ...(5.15)$$

$$u_m = - \frac{c_1\pi^2a^2(1 + 2m^2)^2}{32(1 + 2m)^2 E^2} \quad ...(5.16)$$

$$T_M = \frac{c_4\pi^4a^4(1 + 2m^2)^4}{768(1 + 2m)^4 E^4} [1 + 6\beta_3 + (\mu c_1/2kc_2)] \quad ...(5.17)$$

where

$$\beta_3 = \frac{4c_3(1 + 2m)^2 E^2}{c_4\pi^2a^2(1 + 2m^2)^2} \quad ...(5.18)$$

$$T_M = \frac{c_4\pi^4a^4(1 + 2m^2)^4}{6144(1 + 2m)^4 E^4} [11 + 64\beta_3 + 5(\mu c_1/kc_2)] \quad ...(5.19)$$

$$\frac{q}{k} = - \frac{c_4\pi^5a^4(1 + 2m^2)^4}{128(1 + 2m)^4 E^4} [1 + 8\beta_3 + (\mu c_1/kc_2)] \quad ...(5.20)$$

$$Nu = \frac{48 [1 + 8\beta_3 + (\mu c_1/kc_2)]}{11 + 64\beta_3 + 5(\mu c_1/kc_2)}. \quad ...(5.21)$$

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