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COMBINED FREE AND FORCED CONVECTIVE HYDROMAGNETIC FLOW THROUGH A POROUS CHANNEL WITH HALL AND WALL CONDUCTANCE EFFECTS

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An investigation of the hydromagnetic free and forced convection in a parallel plate channel formed by two finitely conducting parallel porous walls taking Hall effects into account, the liquid being permeated by a transverse magnetic field, is made. There is uniform axial temperature variation along the walls. An exact solution of the governing equations is obtained. The flow phenomenon has been characterized by the non-dimensional numbers like M^2 (Hartmann number), G (Grashof number), v_0 (suction parameter), ϕ_1, ϕ_2 (conductance parameters) and m (Hall parameter). The induced magnetic field and the heat transfer characteristics in the flow are also determined. Expressions for the shearing stress components have also been sought. The effect of Hall parameter on the velocity, the induced magnetic field and shearing stress is interpreted with the aid of graphs and a table.

INTRODUCTION

Many authors have paid their attention in problems associated with MHD channel flows for their wide application in technology. The heat transfer aspect of such flows has been studied in case of fully developed flows by Siegel (1958), Gershuni and Zhukhovitskii (1958), Sherman and Sutton (1961) and Alpher (1961). In all these investigations, the effects of the buoyancy forces have not been taken into account. This was done with the assumption that the effects of the gravitation field are almost negligible in horizontal flows. But this is not the real situation. It was obtained by Gill and Casal (1962) that viscosity variations and temperature differences on the horizontal forced convection flow between two parallel infinite plates can induce such forces as might significantly increase or decrease the tendency towards instability. Many authors have shown that the fluids with low Prandtl number are to a good deal more sensitive to gravitational field effects than the fluids with high Prandtl numbers. This is due to the fact that the low Prandtl number fluids are characterized by thicker thermal boundary layers. All fluids with low Prandtl number are electrically conducting and hence the flow pattern is affected by an externally imposed magnetic field. This property has been utilised profitably in magnetohydrodynamic channel flows. Liquid metals are electrically conducting and have low Prandtl number (say, $Pr \sim 1/40$ for murcury at ordinary temperature and $\sim 1/135$ for liquid sodium at 200°C). It is therefore of some interest to investigate the effect of buoyancy forces on a forced convection flow of an electrically conducting liquid. But attempts to analyse the MHD flows with buoyancy forces does not seem to have attracted much attention. The effect of the buoyancy forces on a forced convection flow of an electrically conducting liquid in a horizontal channel with a linear axial temperature variation along the walls under the action of a transverse magnetic field has been investigated by Gupta (1969). Mishra and Muduli (1976) have studied the above work in case of porous plate channel. In these investigations, the effects of Hall currents were ignored. But these effects have a pronounced effect when the strength of the magnetic field is very large. Mazumder *et al.* (1976) and Kant (1977) studied Gupta's and Mishra and Muduli's problems respectively by taking Hall effects into account.

In all the above investigations, the plates of the channel have been assumed to be perfect insulators. However, it is interesting to find the effect of wall conductances on the convective flow. Dutta and Jana (1977) extended the work of Mazumder *et al.* (1976) by including wall conductance effect. But they ignored the thickness of the plates. In this paper an attempt has been made to extend the work of Dutta and Jana (1977) in case of porous channel. This analysis also includes the thickness of the plates. An exact solution of the governing equations has been obtained. The fluid is assumed to be non-magnetic so that the phenomena like magnetostriction etc. may be ignored. The injection of fluid at one wall is taken to be equal to the fluid suction at the other wall. There is uniform axial temperature variation along the walls. This study will be significant for the problems of cooling of nuclear reactors, where liquid metals (which are electrically conducting) are used as coolant.

Mathematical Formulation of the Problem and Its Solution

Consider the flow of an incompressible electrically conducting fluid filling the gap between two porous horizontal perfectly conducting parallel plates under the action of a uniform transverse magnetic field H_0. The planes $y' = -L$ and $y' = L$ are identified with the planes of the plates, so that $2L$ is the width between the plates. Choose a Cartesian co-ordinate system with x'-axis along the plates, y'-axis perpendicular to it. Let (u', v', w') and $(H_{x'}, H_{y'}, H_{z'})$ be the components of the velocity $\bar{q}'$ and the magnetic field $\bar{H}'$. The physical model of the problem is shown in Fig. 1.

At a large distance from the entry section the flow will be fully developed and in the steady state all the physical variables (except pressure) depend on y' only. Let us assume that σ_1, σ_2 and d_1, d_2 are respectively electrical conductivity and thickness of the upper and lower plates. It is also assumed that the rate of suction at the lower plate is equal to that of injection at the upper plate. The equations

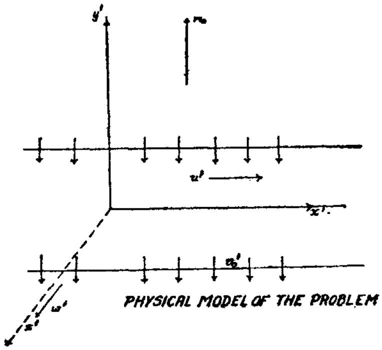

FIG. 1.

of continuity, momentum and the magnetic induction equations in rationalized MKS units can be written as Kant (1977)

$$v' = \text{constant} = -v_0' \qquad \ldots(1)$$

$$-v_0' \rho_0 \frac{du'}{dy'} = -\frac{\partial p'}{\partial x'} + \mu \frac{d^2u'}{dy'^2} + \mu_e H_0 \frac{dH_{x'}}{dy'} \qquad \ldots(2)$$

$$0 = -\frac{\partial p'}{\partial y'} - \rho g - \mu_e \left(H_{x'} \frac{dH_{x'}}{dy'} + H_{z'} \frac{dH_{z'}}{dy'} \right) \qquad \ldots(3)$$

$$-v_0' \rho_0 \frac{dw'}{dy'} = \mu \frac{d^2w'}{dy'^2} + \mu_e H_0 \frac{dH_{z'}}{dy'} \qquad \ldots(4)$$

$$-\frac{d^2H_{x'}}{dy'^2} + m \frac{d^2H_{z'}}{dy'^2} = \sigma\mu_e \left(H_0 \frac{du'}{dy'} + v_0' \frac{dH_{x'}}{dy'} \right) \qquad \ldots(5)$$

$$\frac{d^2H_{z'}}{dy'^2} + m \frac{d^2H_{x'}}{dy'^2} = -\sigma\mu_e \left(H_0 \frac{dw'}{dy'} + v_0' \frac{dH_{z'}}{dy'} \right) \qquad \ldots(6)$$

where μ is the coefficient of viscosity, μ_e the magnetic permeability, ρ the fluid density, σ the fluid conductivity and $m = \omega_e\tau_e$; ω_e the cyclotron frequency, τ_e the electron collision time. If we assume uniform axial temperature variation along the channel walls, we may take the temperature in the flow as

$$T - T_0 = Nx' + \phi(y') \qquad \ldots(7)$$

where N is the uniform temperature gradient of the walls, T is the fluid temperature and T_0 is the temperature in the reference state. Using (7) and the equation of state

$$\rho = \rho_0 [1 - \beta(T - T_0)] \qquad \ldots(8)$$

in (3) and integrating, we get

$$p' = -\rho_0 g y' + \rho_0 g \beta N x' y' + \rho_0 g \beta \int \phi \, dy' - \frac{\mu_e}{2} (H_{x'}^2 + H_{z'}^2) + F(x') \qquad \ldots(9)$$

where β is the coefficient of thermal expansion and ρ_0 the density in the reference state.

We introduce the following non-dimensional quantities:

$$\left.\begin{aligned} &y = \frac{y'}{L},\ u = \frac{u'L}{\nu P_x},\ w = \frac{w'L}{\nu P_x},\ v_0 = \frac{v_0'L}{\nu}, \\ &P_x = -\frac{L^3}{\rho_0\nu^2}\left(\frac{dF}{dx'}\right),\ H_x = \frac{H_{x'}}{\sigma\nu\mu_e H_0 P_x}, \\ &H_z = \frac{H_{z'}}{\sigma\nu\mu_e H_0 P_x},\ M^2 = \frac{\sigma\mu_e^2 H_0^2 L^2}{\rho_0\nu},\ G = \frac{gNL^4\beta}{\nu^2 P_x}. \end{aligned}\right\} \qquad \ldots(10)$$

Eliminating p' from (2) and (9) and introducing the non-dimensional quantities, we have

$$\frac{d^2u}{dy^2} + v_0\frac{du}{dy} + M^2\frac{dH_x}{dy} - Gy = -1. \qquad \ldots(11)$$

Equation (7) shows that positive or negative values of N correspond to heating or cooling along the channel walls. Considering $P_x > 0$, it follows from the definition of G given by (10) that $G \gtrless 0$ according as the channel walls are heated or cooled in axial direction. Equation (4) reduces to

$$\frac{d^2w}{dy^2} + v_0\frac{dw}{dy} + M^2\frac{dH_z}{dy} = 0. \qquad \ldots(12)$$

Equation (12) multiplied by i when added to (11) gives

$$\frac{d^2U}{dy^2} + v_0\frac{dU}{dy} + M^2\frac{dh}{dy} - Gy = -1 \qquad \ldots(13)$$

where

$$U = u + iw,\ h = H_x + iH_z. \qquad \ldots(14)$$

Similarly combining (5) and (6) and using (10), we have

$$(1 + im)\frac{d^2h}{dy^2} + v_0P_m\frac{dh}{dy} + \frac{dU}{dy} = 0 \qquad \ldots(15)$$

where

$P_m = \sigma\nu\mu_e$, being magnetic Prandtl number.

From (13) and (15) we have,

$$\frac{d^2U}{dy^2} + A_1v_0\frac{dU}{dy} - M_1^2U = G_1y^2 + G_2y + C_1 \qquad \ldots(16)$$

where

$$
\left.\begin{aligned}
A_1 &= 1 + \frac{P_m}{(1 + im)}, \quad M_1^2 = \frac{M^2 - P_m v_0^2}{(1 + im)} \\
G_1 &= \frac{P_m v_0 G}{2(1 + im)}, \quad G_2 = G - \frac{P_m v_0}{(1 + im)}
\end{aligned}\right\} \qquad \ldots(17)
$$

and C_1 is a constant of integration.

The no-slip conditions at the plates $y = \pm 1$ are

$$U(\pm 1) = 0. \qquad \ldots(18)$$

Since the plates are assumed to be perfectly conducting, the magnetic boundary conditions are

$$
\left.\begin{aligned}
y = 1: &\quad \frac{dh}{dy} + \frac{h}{\phi_1} = 0 \\
y = -1: &\quad \frac{dh}{dy} - \frac{h}{\phi_2} = 0
\end{aligned}\right\} \qquad \ldots(19)
$$

where

$$\phi_1 = \frac{\sigma_1 d_1}{\sigma L}, \quad \phi_2 = \frac{\sigma_2 d_2}{\sigma L}. \qquad \ldots(20)$$

Solutions of (13) and (16) satisfying (18) and (19) are

$$U(y) = \exp\left(-\frac{A_1 v_0 y}{2}\right)(C_2 \cosh B_1 y + C_3 \sinh B_1 y) - \frac{G_1 y^2}{M_1^2} - B_2 y - B_3 \qquad \ldots(21)$$

and

$$
\begin{aligned}
M^2 h = {} & K_1(y^2 - 1 - 2\phi_1) + K_2(y - 1 - \phi_1) - \exp\left(-\frac{A_1 v_0}{2}\right) \\
& \times \left[K_3 \left\{\cosh B_1 + \phi_1 \left(B_1 \sinh B_1 - \frac{A_1 v_0}{2} \cosh B_1\right)\right\}\right. \\
& \left. + K_4 \left\{\sinh B_1 + \phi_1 \left(B_1 \cosh B_1 - \frac{A_1 v_0}{2} \sinh B_1\right)\right\}\right] \\
& + \exp\left(-\frac{A_1 v_0 y}{2}\right)(K_3 \cosh B_1 y + K_4 \sinh B_1 y) \qquad \ldots(22)
\end{aligned}
$$

where

$$K_1 = \frac{G}{2} + \frac{G_1 v_0}{M_1^2}; \; K_2 = B_2 v_0 - 1 + \frac{2G_1}{M_1^2}$$

$$K_3 = K_5 C_2 - B_1 C_3; \; K_4 = K_5 C_3 - B_1 C_2$$

$$K_5 = \nu_0\left(\frac{A_1}{2} - 1\right);\ C_2 = \frac{(B_4 + B_3 \cosh \frac{1}{2} A_1\nu_0)}{\cosh B_1}$$

$$C_3 = \frac{(B_5 + B_3 \sinh \frac{1}{2} A_1\nu_0)}{\sinh B_1};\ B_1 = \sqrt{\frac{A_1^2\nu_0^2 + 4M_1^2}{4}}$$

$$B_2 = \frac{1}{M_1^2}\left(G_2 + \frac{2G_1A_1\nu_0}{M_1^2}\right)$$

$$B_3\left[\frac{\nu_0 B_1(A_1 - 1)}{\sinh 2B_1}\{(\phi_1 - \phi_2)\cosh 2B_1 + (\phi_2 e^{A_1\nu_0} - \phi_1 e^{-A_1\nu_0})\}\right.$$

$$+ \frac{4B_1 \sinh(\frac{1}{2}A_1\nu_0 + B_1)\sinh(\frac{1}{2}A_1\nu_0 - B_1)}{\sinh 2B_1}$$

$$\left. - (\phi_1 + \phi_2)(B_1^2 + \tfrac{1}{2}A_1\nu_0K_5)\right]$$

$$= 2K_1(\phi_2 - \phi_1) - K_2(\phi_1 + \phi_2 + 2) - \frac{B_1}{\sinh 2B_1}$$

$$\times\left[(\tfrac{1}{2}A_1\nu_0 + K_5)\cosh 2B_1\left\{\phi_1\left(\frac{G_1}{M_1^2} + B_2\right) - \phi_2\left(\frac{G_1}{M_1^2} - B_2\right)\right\}\right.$$

$$+ (\tfrac{1}{2}A_1\nu_0 + K_5)\left\{\left(\frac{G_1}{M_1^2} + B_2\right)\phi_2 e^{A_1\nu_0} - \phi_1 e^{-A_1\nu_0}\left(\frac{G_1}{M_1^2} - B_2\right)\right\}$$

$$\left. + \frac{4G_1\sinh(\frac{1}{2}A_1\nu_0 + B_1)\sinh(\frac{1}{2}A_1\nu_0 - B_1)}{M_1^2} + 2B_2\sinh A_1\nu_0\right]$$

$$+ (B_1^2 + \tfrac{1}{2}A_1\nu_0K_5)\left\{\phi_1\left(B_2 + \frac{G_1}{M_1^2}\right) + \phi_2\left(\frac{G_1}{M_1^2} - B_2\right)\right\} - 2K_5B_2$$

$$B_4 = \frac{G_1}{M_1^2}\cosh(\tfrac{1}{2}A_1\nu_0) + B_2\sinh(A_1\nu_0/2)$$

$$B_5 = \frac{G_1\sinh(\frac{1}{2}A_1\nu_0)}{M_1^2} + B_2\cosh(\tfrac{1}{2}A_1\nu_0).$$

Separating (21) and (22) into real and imaginary parts, we can readily find the x and z components of velocity and induced magnetic field.

The dimensionless shear stress for the primary and secondary flows at the upper and lower plates can be obtained by using the values in $\left.\frac{du}{dy}\right]_{y=\pm 1}$ and $\left.\frac{dw}{dy}\right]_{y=\pm 1}$.

When buoyancy forces are absent ($G = 0$), the shear stress components due to the primary and secondary flow do not vanish on either of the plates whether the porous plates are perfectly electrically conducting or non-conducting. Thus we arrive at the interesting conclusion that in the absence of the buoyancy forces, the primary and cross-flows do not show incipient flow reversal in case of porous channel formed by either electrically conducting or non-conducting plates. On the other hand, the cross-flow due to Hall currents shows incipient flow reversal although the primary flow does not when $G = 0$ and $v_0 = 0$. The incipient reversed flow for the primary and cross-flows at the upper and lower plates takes place corresponding to that value of G at which the dimensionless shear stress vanishes at the plates.

Heat Transfer

For fully developed flow, the equation of energy including viscous and ohmic dissipation is

$$u' \frac{\partial T}{\partial x'} - v_0' \frac{\partial T}{\partial y'} = K \frac{\partial^2 T}{\partial y'^2} + \frac{\mu}{\rho_0 C_p} \left[\left(\frac{du'}{dy'} \right)^2 + \left(\frac{dw'}{dy'} \right)^2 \right] + \frac{1}{\rho_0 C_p \sigma} \left[\left(\frac{dH_{x'}}{dy'} \right)^2 + \left(\frac{dH_{z'}}{dy'} \right)^2 \right] \qquad \text{...(23)}$$

where K is the thermal diffusivity, C_p is the specific heat at constant pressure. The last term in (23) arises out of the Joule heating.

Using (7) and (10), the above can be put in dimensionless form as

$$\frac{d^2\theta}{dy^2} + Pr\, v_0 \frac{d\theta}{dy} = Pr\, u - K_1^* \left[\frac{dU}{dy} \cdot \frac{d\bar{U}}{dy} + M^2 \frac{dh}{dy} \cdot \frac{d\bar{h}}{dy} \right] \qquad \text{...(24)}$$

where Pr is the Prandtl number ν/K and

$$K_1^* = \frac{\nu^3 P_x}{C_p K N L^3}, \quad \theta = \frac{\phi}{N L P_x} \qquad \text{...(25)}$$

and the over bar denotes a complex conjugate. As for the temperature boundary conditions we take the reference temperature T_0 in (7) in such a manner that the temperature of the lower plate $y = -1$ is $T_0 + Nx'$ and this implies that $\phi(-1) = 0$. Hence using (25), the boundary conditions for $\theta(y)$ are

$$\theta(-1) = 0; \theta(1) = \frac{\phi(1)}{N L P_x} = N_1 \qquad \text{...(26)}$$

where N_1 is taken as the wall temperature parameter.

The temperature distribution $\theta(y)$ can be obtained by solving the ordinary linear differential eqn. (24) of second order with constant coefficients after substituting the expressions for $U(y)$ and $h(y)$ from (21) and (22) and making use of the boundary conditions (26). We omit the details of calculation as they are quite cumbersome.

RESULTS AND DISCUSSION

In all the calculations, the quantities G, ν_0, M, ϕ_1 and ϕ_2 have been given fixed values, viz., $G = 1.0$, $\nu_0 = 1.0$, $M = 10$, $\phi_1 = \phi_2 = 0.5$.

In Figs. 2 and 3, the profiles of the primary and secondary flows respectively have been displayed for different values of Hall parameter m. In both the figures the velocity profiles are not symmetrical about the axis of the channel and a maximum is attained in the upper half of the channel. It is evident from Fig. 2 that near the lower plate the primary velocity decreases with increase in m while at the upper plate it increases with increase in m. It is seen from Fig. 3 that the secondary velocity increases numerically at any point in the channel with increase in m.

The profiles for the induced magnetic fields H_x and H_z versus y have been plotted in Figs. 4 and 5 respectively for different values of Hall parameter. In both the figures, the behaviour of induced magnetic fields is not symmetrical about the axis of the channel. Figure 4 shows that an increase in m increases the magnitude of H_x. Figure 5 shows that with increase in m, H_z increases near the lower plate of the channel while it decreases near the upper plate.

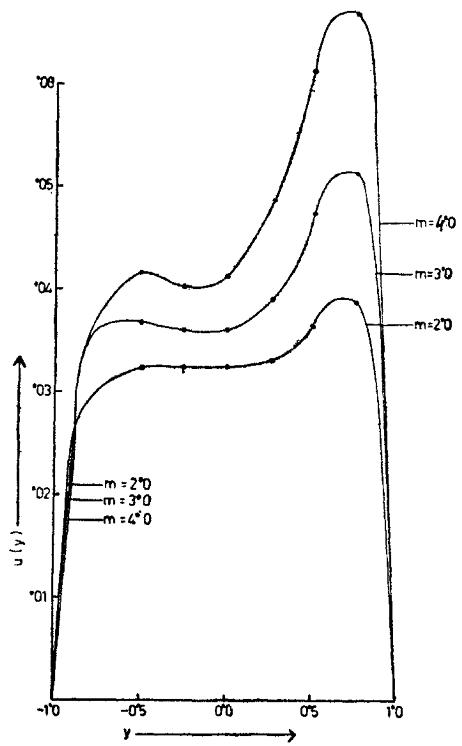

FIG. 2. Profiles of non-dimensional primary velocity $u(y)$.

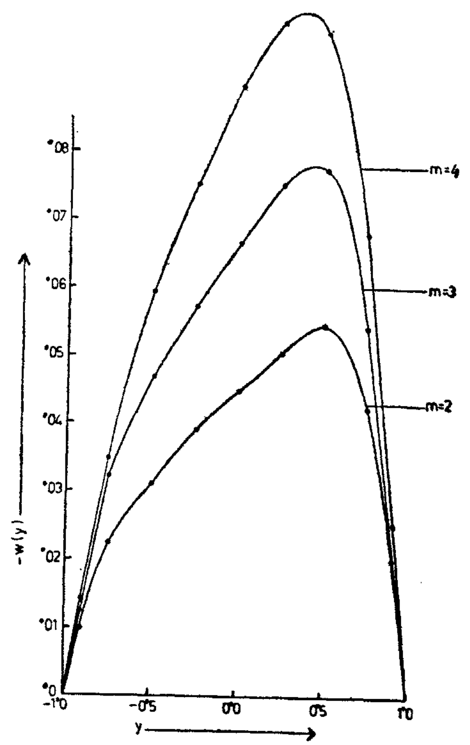

FIG. 3. Profiles of non-dimensional secondary velocity $w(y)$.

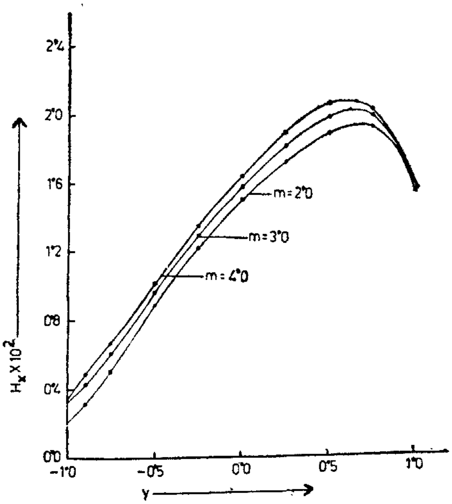

FIG. 4. Profiles of non-dimensional induced magnetic field H_x.

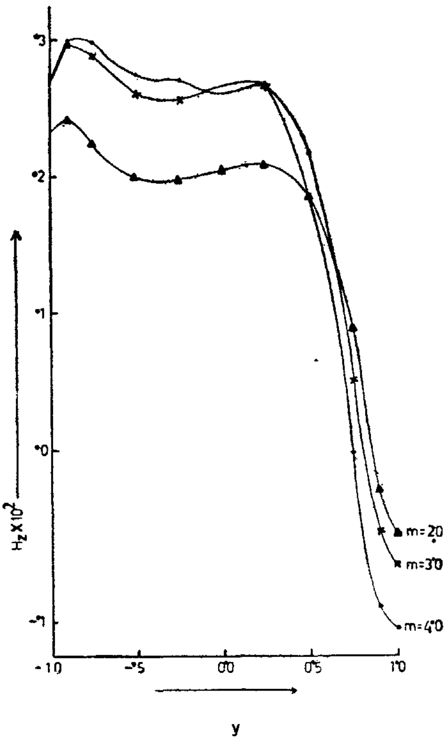

FIG. 5. Profiles of non-dimensional induced magnetic field H_z.

The values of shear stress at the upper and lower plates for the primary and cross-flows are given in Table I in order to show the effect of m. The table shows that the shear stress for the primary flow at the upper plate is negative and that at the lower plate is positive while a reverse result is observed for the cross-flow. The value of shear stress at both the plates for the primary and cross-flows increases numerically as m increases. Hence there is no possibility of the flow separation at both the plates when $G = 1.0$, $v_0 = 1.0$, $M = 10$, $\phi_1 = \phi_2 = 0.5$.

TABLE I

Values of shear stress

m	$\left.\frac{du}{dy}\right]_{y=1}$	$\left.\frac{du}{dy}\right]_{y=-1}$	$\left.\frac{dw}{dy}\right]_{y=1}$	$\left.\frac{dw}{dy}\right]_{y=-1}$
2.0	− 0.3971798	0.2155248	0.1981803	− 0.0927541
3.0	− 0.4576861	0.2733877	0.2467319	− 0.1066797
4.0	− 0.5145749	0.2858514	0.2703247	− 0.1288670

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