*Indian J. pure appl. Math.*, 11(4) : 487–491, April 1980

# A NOTE ON STEADY SLOW MOTION OF THERMOVISCOUS FLUID THROUGH A CIRCULAR TUBE

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(*Received* 24 *April* 1979; *after revision* 26 *June* 1979)

The steady slow motion of a second order incompressible thermoviscous fluid through a circular tube, under the influence of a constant pressure gradient is studied. It is observed that the thermoviscous effects are more predominant in a thin region around the boundary and for small values of thermoviscous parameter the flow is parabolic with an altered pressure gradient.

Koh and Eringen (1963) considered thermoviscous fluids exibiting the interaction between thermal and mechanical responses when the fluids are set in motion. For such a class of fluids, the stress tensor $t$ and heat flux bivector $h$ are postulated to be polynomials in rate of deformation tensor $d$ and the thermal gradient bivector $b$:

$$t = \alpha_1 I + \alpha_3 d + \alpha_5 d^2 + \alpha_6 b^2 + \alpha_8(db - bd) \qquad ...(1)$$

$$h = \beta_1 b + \beta_3(bd + db) \qquad ...(2)$$

where $2d_{ij} = u_{i,j} + u_{j,i}$; $b_{ij} = \epsilon_{ijk}\theta_{,k}$; $u_i$ being the $i$th component of velocity and $\theta$ the temperature field.

The constitutive coefficients $\alpha_i$ and $\beta_i$ are functions of scalar polynomials in the invariants of $d$ and $b$ and assumed to be independent of temperature.

In this paper, the steady slow motion of a second order incompressible thermoviscous fluid in a straight circular tube under the action of a constant pressure gradient is studied. The boundary $\Gamma$ of the tube is assumed to be rigid and fixed and is maintained at a constant temperature $\theta_1$. The $z$-axis is taken along the axis of the tube. The temperature gradient is assumed to be a constant. The velocity and temperature fields are $(0, 0, w(x, y))$ and $\theta(x, y)$.

In the absence of any external forces and heat sources within, the momentum equation reduces to

$$0 = -\frac{\partial p}{\partial z} + \frac{\alpha_3}{2}\nabla^2 w - \frac{\partial \theta}{\partial z}\alpha_6 \nabla^2 \theta \qquad ...(3)$$

and equation of energy to

$$\rho c \frac{\partial \theta}{\partial z} w = \frac{\alpha_3}{2}\left[\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] - \frac{\partial \theta}{\partial z}\alpha_6\left(\frac{\partial w}{\partial x}\frac{\partial \theta}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial \theta}{\partial y}\right) - \beta_1 \nabla^2\theta + \frac{\beta_3}{2}\frac{\partial \theta}{\partial z}\nabla^2 w \qquad ...(4)$$

together with boundary conditions

$$\left.\begin{aligned} w &= 0 \\ \theta &= \theta_1 \end{aligned}\right\} \text{ on } \Gamma: . \qquad ...(5)$$

Assuming slow motion characterized by neglecting the second and higher order terms in the velocity and temperature gradients, the equations of momentum and energy for such a motion now reduce to the non-dimensional form

$$\nabla^2 W - b_1 \nabla^2 T = c_1 \qquad ...(6)$$

$$\nabla^2 W - b_2 \nabla^2 T = b_3 W \qquad ...(7)$$

where

$$\left.\begin{aligned} & w = W\alpha_3/2\rho a, \; r = aR, \; \theta/\theta_1 = T \\ & \frac{\partial p}{\partial z} = \alpha_3^2 c_1/4\rho a^3, \; \frac{\partial \theta}{\partial z} = \frac{\theta_1}{a} c_2 \end{aligned}\right\} \qquad ...(8)$$

$r$ is the distance from $z$-axis and $c_1$ and $c_2$ are non-dimensional pressure and temperature gradients and

$$b_1 = 4\alpha_6 c_2 \theta_1^2 \rho/\alpha_3^2, \; b_2 = 4\rho a^2 \beta_1/\alpha_3\beta_3 c_2, \; b_3 = 2\rho c a^2/\beta_3.$$

From (6) and (7) we have

$$\nabla^2 W - m_1 W = -m_2 \qquad ...(9)$$

$$\nabla^2(\nabla^2 - m_1) T = B \qquad ...(10)$$

with $\quad m_1 = b_1 b_3/(b_1 - b_2), \; m_2 = b_2 c_1/(b_1 - b_2), \; B = b_3 c_1/(b_1 - b_2)$

together with boundary conditions

$$W = 0, \; T = 1 \text{ on } \Gamma: R = 1. \qquad ...(11)$$

These equations yield the velocity distribution

$$W = \frac{m_2}{m_1}\left[1 - \frac{I_0(\sqrt{m_1} R)}{I_0(\sqrt{m_1})}\right] \qquad ...(12)$$

and the temperature distribution

$$T = \left(1 + \frac{B}{4m_1}\right) \frac{I_0(\sqrt{m_1}R)}{I_0(\sqrt{m_1})} - \frac{B}{4m_1} R^2. \qquad ...(13)$$

The profiles of the velocity and temperature fields are illustrated in Figs. 1 and 2 for different values of $m_1$, the thermoviscous parameter (for $B = 1$). The velocity profiles become more flat as the thermoviscous parameter ($m_1$) increases whereas an opposite effect is observed for the temperature field. This effect may be attributed to the greater conversion of the fluid kinetic energy to thermal energy. It may be mentioned that the Newtonian viscous and Fourier-heat conducting type flow can be realized from eqns. (12) and (13) as the thermoviscous parameter $m_1$ tends to zero.

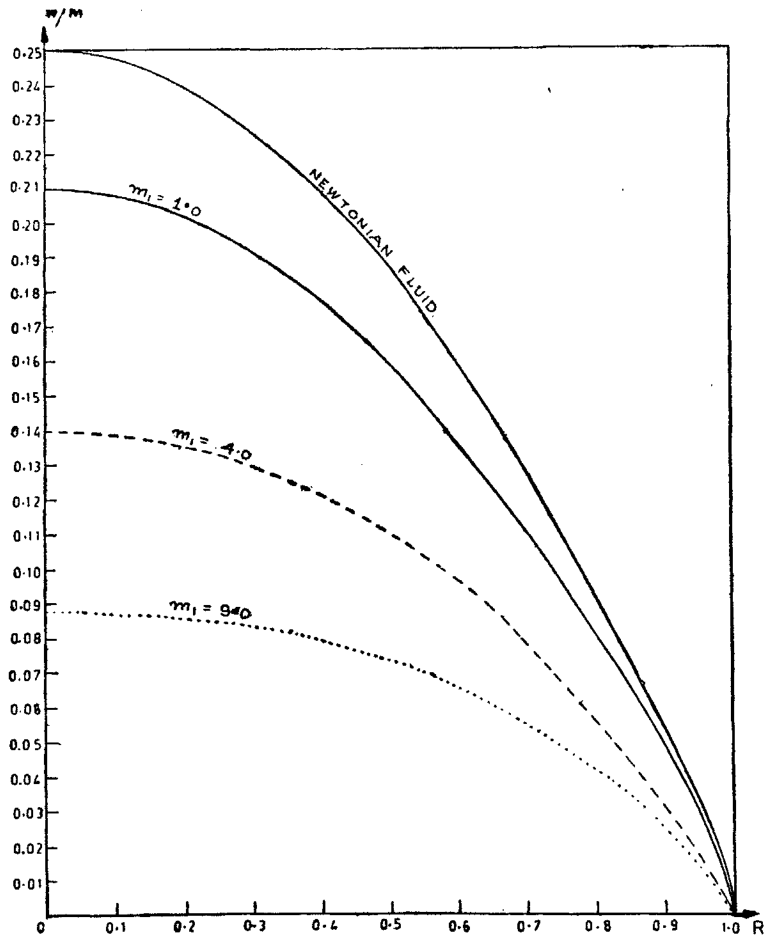


FIG. 1. Velocity distribution.

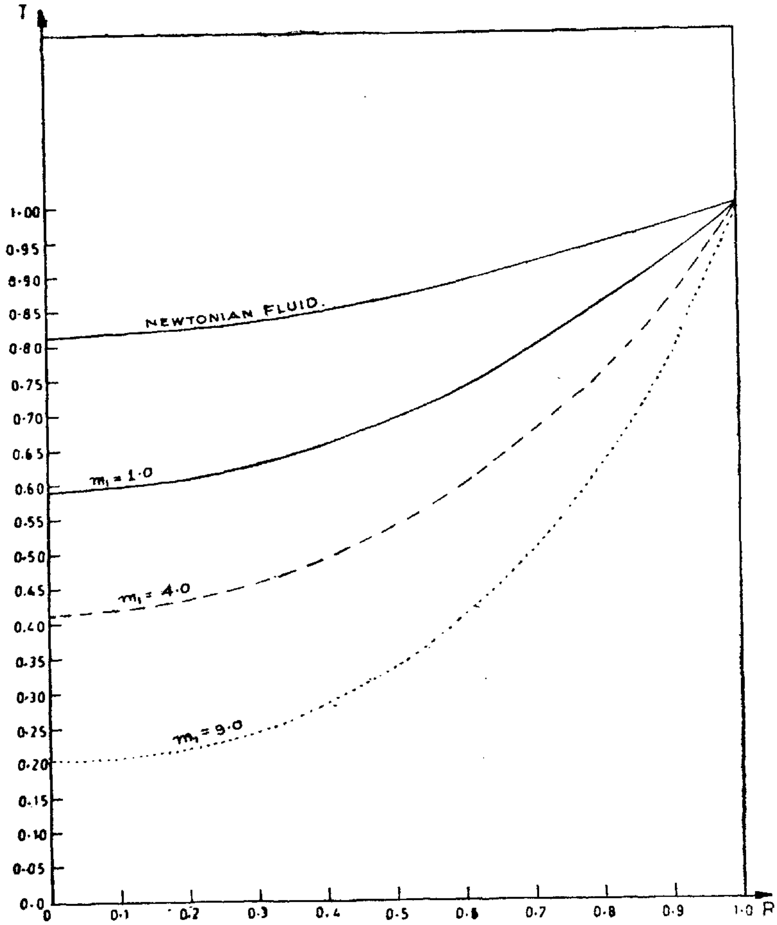


FIG. 2. Temperature distribution.

The heat transfer coefficient, Nussult number on the tube wall is given by

$$Nu = \left(1 + \frac{B}{4m_1}\right)\sqrt{m_1}\,\frac{I_1(\sqrt{m_1})}{I_0(\sqrt{m_1})} - \frac{B}{2m_1} \qquad \text{...(14)}$$

which increases with $m_1$.

*Flow for Large Values of* $m_1$

Following 9.7.1 of Abramowitz and Stegun (1964, p. 377),

$$I_0(\sqrt{m_1}R) \simeq \frac{\exp(\sqrt{m_1}R)}{\sqrt{2\pi\sqrt{m_1}R}}. \qquad \text{...(15)}$$

The velocity field (12) and temperature field (13) now reduce for large values of thermoviscous parameter $m_1$, to

$$W = \frac{m_2}{m_1}\left[1 - \frac{1}{\sqrt{R}} \exp\left(-\sqrt{m_1}(1-R)\right)\right] \qquad \ldots(16)$$

and

$$T = \left(1 + \frac{B}{4m_1}\right) \exp\left(-\sqrt{m_1}(1-R)\right) \frac{1}{\sqrt{R}} - \frac{B}{4m_1} R^2 \qquad \ldots(17)$$

i.e., the thermoviscous effects are more predominant in a thin region around the tube boundary, the width of which is given by $\delta = 1/\sqrt{m_1}$.

*Flow for Small Values of $m_1$*

In this case

$$I_0(\sqrt{m_1}R) = 1 + \frac{m_1}{4} R^2 + \ldots$$

and the velocity and temperature fields are

$$W = \frac{m_2}{4 + m_1} (1 - R^2) \qquad \ldots(18)$$

$$T = \left(1 + \frac{B}{4m_1}\right) \frac{4 + m_1 R^2}{4 + m_1} - \frac{B}{4m_1} R^2 \qquad \ldots(19)$$

The velocity distribution is parabolic with an altered pressure gradient given by $m_2/(4 + m_1)$.

ACKNOWLEDGEMENT

The financial support from CSIR is acknowledged.

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