Indian J. pure appl. Math., **11**(8) : 1055–1059, August 1980

ELECTRO-SCALARLY CHARGED FLUID SPHERES IN GENERAL RELATIVITY

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(*Received* 14 *May* 1979; *after revision* 17 *March* 1980)

An interior solution of an electro-scalarly charged fluid sphere is presented here. The solution is physical in all respects. The geometric mass of the sphere has been calculated.

1. INTRODUCTION

Teixeira *et al.* (1976) have obtained an interior solution of electro-scalarly charged static 'dust' spheres and have shown that the geometric mass of an electro-scalarly charged sphere is $m = (q^2 - \gamma b^2)^{1/2}$ where q is the electric charge, b the scalar charge and $\gamma = \pm 1$. Florides (1977) has shown that the geometric mass of a charged sphere is $m = \mu(a) + \mathcal{E}(a)$, where $\mu(a)$ is the contribution from the mass-density and $\mathcal{E}(a)$ is that from electric charge. Som *et al.* (1977) have found Reissner-Nordstrom solution from the Schwarzschild solution by a co-ordinate transformation and have shown that the geometric mass of a charged sphere is $m = (m_s^2 + q^2)^{1/2}$ where m_s is the Schwarzschild mass and q is the charge of the sphere. In the present paper, we have investigated the interior solution of an electro-scalarly charged "fluid" sphere and have shown that the geometric mass of the sphere has contribution from its mass-density and scalar and electric charges. The solution is singularity-free, physical in all respects and is more general than the one due to Teixeira *et al.* (1976).

2. SOLUTION OF FIELD EQUATIONS

The following spherically symmetric line-element is taken

$$ds^2 = e^{2\eta} dt^2 - e^{2\alpha} dr^2 - r^2 e^{\alpha-\eta}(d\theta^2 + \sin^2\theta\, d\phi^2) \qquad \text{...(1)}$$

where $\alpha = \alpha(r)$ and $\eta = \eta(r)$. Here r, z, ϕ and t are numbered 1, 2, 3 and 4 respectively.

Einstein-Maxwell scalar field equations are

$$R_\mu^\nu = -8\pi(T_\mu^\nu - \tfrac{1}{2}\delta_\mu^\nu T) \qquad \text{...(2)}$$

$$T_\mu^\nu = (\rho + p)\, V_\mu V^\nu - \delta_\mu^\nu p + \frac{1}{4\pi}[- F^{\nu\alpha}F_{\mu\alpha} + \tfrac{1}{4}\delta_\mu^\nu F^{\alpha\beta}F_{\alpha\beta}] + k_\mu^\nu \qquad \text{...(3)}$$

$$4\pi\gamma k^{\nu}_{\mu} = s^{;\nu}s_{;\mu} - \tfrac{1}{2}\delta^{\nu}_{\mu}\, s^{;\alpha}s_{;\alpha} \qquad ...(4)$$

$$F^{\mu\nu}_{;\nu} = 4\pi\sigma V^{\mu} \qquad ...(5)$$

$$F_{[\mu\nu,\alpha]} = 0 \qquad ...(6)$$

$$S^{;\mu}_{;\mu} = -4\pi\gamma\beta \qquad ...(7)$$

where ρ, p, σ and β are the mass-density, pressure, charge density and scalar charge density respectively. S is the scalar field. The matter is at rest in the co-ordinate system of (1) so that $V^{\mu} = \delta^{\mu}_{4}\,(g_{44})^{-1/2}$. The electric field is such that only $F_{41} = +\psi_1$ exists along radial direction where ψ is the electric potential. The suffix 1 indicates differentiation w.r.t. r.

Equation (2) with the help of (1) gives

$$\eta_{11} + \frac{2\eta_1}{r} = 4\pi(\rho + 3p)\, e^{2\alpha} + e^{-2\eta}\psi_1^2 \qquad ...(8)$$

$$\alpha_{11} - \frac{\alpha_1^2}{2} - \alpha_1\eta_1 + \tfrac{3}{2}\,\eta_1^2 - \frac{2\eta_1}{r} = -4\pi(\rho - p)\, e^{2\alpha} + e^{-2\eta}\psi_1^2 - 2\gamma s_1^2 \qquad ...(9)$$

$$\tfrac{1}{2}\,\eta_{11} - \tfrac{1}{2}\,\alpha_{11} + \frac{\eta_1}{r} - \frac{\alpha_1}{r} - \frac{1}{r^2} + \frac{1}{r^2}\, e^{\eta+\alpha} = 4\pi(\rho - p)\, e^{2\alpha} + e^{-2\eta}\psi_1^2 \qquad ...(10)$$

$$\frac{d}{dr}\,(r^2 e^{-2\eta}\psi_1) = -4\pi\sigma r^2 e^{2\alpha-\eta} \qquad ...(11)$$

$$\frac{d}{dr}\,(r^2 s_1) = 4\pi\gamma\beta r^2 e^{2\alpha}. \qquad ...(12)$$

Now since there are five equations and eight variables, let us assume

$$\left.\begin{aligned} 2\alpha &= Ar^2 \\ 2\eta &= Br^2 \\ \text{and} \qquad s &= Fr^2 \end{aligned}\right\} \qquad ...(13)$$

where A, B and F are constants. The first two assumptions of (13) ensure flatness at the centre and the third makes the scalar field zero at $r = 0$.

Now with the help of (13), eqns. (8) to (12) give

$$16\pi p = \left[\tfrac{3}{2}(A + B) - \frac{1}{r^2}\,(e^{(A+B)r^2/2} - 1)\right] e^{-Ar^2} \qquad ...(14)$$

$$8\pi\rho = \left[\tfrac{1}{2}(17B - 7A) - \tfrac{1}{2}(3B^2 - A^2 - 2AB + 16\gamma F^2)\, r^2 + \frac{1}{2r^2} \times (e^{(A+B)r^2/2} - 1)\right] e^{-Ar^2} \quad ...(15)$$

$$\psi_1^2\, e^{-2\eta} = \left[\tfrac{1}{2}(B - A) + \tfrac{1}{4}(3B^2 - A^2 - 2AB + 16\gamma F^2) r^2 + \frac{1}{2r^2} \times (e^{(A+B)r^2/2} - 1)\right] \quad ...(16)$$

$$4\pi\sigma = \left(\frac{d}{dr}\, F^{41} + \frac{2}{r}\, F^{41} + 2FrF^{41}\right) e^{(1/2)Br^2} \quad ...(17)$$

$$4\pi\beta = \sigma\gamma F e^{-Ar^2} \quad ...(18)$$

Now at $r = 0$ we have

$$8\pi\rho_0 = \tfrac{1}{2}(9B - 3A) \quad ...(19)$$

$$16\pi p_0 = A + B \quad ...(20)$$

$$4\pi\sigma_0 = (3B^2 - A^2 - 2AB + 16\gamma F^2)^{1/2} \quad ...(21)$$

$$4\pi\beta_0 = \sigma\gamma F \quad ...(22)$$

We assume that the constants A and B are positive. Hence ρ_0 [eqn. (19)] and p_0 [eqn. (20)] are both positive. If $B \geqslant A$, $\rho_0 \geqslant 3p_0$. For

$$3B^2 - A^2 - 2AB + 16\gamma F^2 > 0,$$

σ_0 in eqn. (21) is real. ψ_1^2 in eqn. (16) and ρ in eqn. (15) are positive althroughout the sphere under the condition,

$$a^2 < \frac{9B - 3A}{3B^2 - A^2 - 2AB + 16\gamma F^2} \quad ...(23)$$

The boundary $r = a$, at which $p = 0$ will satisfy the equation

$$\tfrac{3}{2}(A + B)\, a^2 - e^{(A+B)a^2/2} + 1 = 0 \quad ...(24)$$

The geometric mass of the sphere is

$$M = \int_0^a \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \rho\, dV \quad ...(25)$$

where dV (the proper elementary volume)

$$= r^2 e^{2\alpha-\eta} \sin\theta\, d\theta\, d\phi\, dr. \quad ...(26)$$

Thus the geometric mass of the sphere using (15) and (26) in (25)

$$M = \int_0^a [\tfrac{1}{4}(9B - 3A) + (\tfrac{3}{2}A^2 - \tfrac{9}{2}B^2 + 3AB)\, r^2]\, e^{-\eta} r^2 dr$$

$$+ \tfrac{1}{4} \int_0^a (2e^{-2\eta}\psi_1^2 + \sigma\gamma s_1^2)\, e^{-\eta}\, r^2 dr$$

$$= \mu(a) + \mathcal{E}(a) + S(a) \qquad \text{...(27)}$$

Thus the mass-density, electric charge and scalar charge densities contribute to the geometrical mass of the charged fluid sphere as is evident from (27).

3. Continuity with the Exterior Metric

The exterior metric is given by (Teixeira *et al.* 1976)

$$\left.\begin{aligned} g_{44} &= \left[\cosh c/r + \left(1 + \frac{q^2}{c^2}\right)^{1/2} \sinh c/r\right]^{-2} \\ g_{rr} &= \frac{1}{g_{44}}[-(f/r)^4 \sinh^{-4}(f/r)] \\ g_{\theta\theta} &= -r^2\left(\frac{g_{44}}{g_{rr}}\right)^{1/2} \\ \psi_1 &= -qr^{-2}g_{44} \\ S_1^2 &= \gamma r^{-4}(f^2 - c^2) \end{aligned}\right\} \qquad \text{...(28)}$$

where q, c and f are constants associated with the gravitational, electric and scalar charges respectively.

Now for the interior metric (13) to be continuous with the exterior metric (28), i.e. matching $g_{\mu\nu}$ and their derivatives at the boundary, we have

$$(e^{Br^2})_{r=a} = \left[\cosh c/r + \left(1 + \frac{q^2}{c^2}\right)^{1/2} \sinh c/r\right]^{-2}_{r=a} \qquad \text{...(29)}$$

$$(e^{Ar^2})_{r=a} = \left[\left(\frac{f}{r}\right)^4 \sinh^{-4} f/r \left\{\cosh c/r + \left(1 + \frac{q^2}{c^2}\right)^{1/2} \sinh c/r\right\}^2\right]_{r=a} \qquad \text{...(30)}$$

$$(Br)_{r=a} = \left[\frac{c}{r^2}\, \frac{\sinh c/r + \left(1 + \frac{q^2}{c^2}\right)^{1/2} \cosh c/r}{\cosh c/r + \left(1 + \frac{q^2}{c^2}\right)^{1/2} \sinh c/r}\right]_{r=a} \qquad \text{...(31)}$$

$$(Ar)_{r=a} = \left[\frac{2f}{r^2} \coth\frac{f}{r} - \frac{2}{r^2} - \frac{c}{r^2}\, \frac{\sinh c/r + \left(1 + \frac{q^2}{c^2}\right)^{1/2} \cosh c/r}{\cosh c/r + \left(1 + \frac{q^2}{c^2}\right)^{1/2} \sinh c/r}\right]_{r=a} \qquad \text{...(32)}$$

Since these four conditions involve six arbitrary quantities viz., A, B, a, c, f and q, it is possible for these quantities to satisfy them (i.e. the four conditions (29) to (31)).

ACKNOWLEDGEMENT

The author is thankful to Professor K. D. Krori, Cotton College, Gauhati, Assam, for guidance and to U.G.C. for financial help for carrying out this work.

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