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TANGENTIALLY STRESSED CHARGED CYLINDERS IN GENERAL RELATIVITY

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An interior solution of tangentially stressed charged cylinders has been presented here. It is regular everywhere inside the cylinder and joins smoothly to the exterior solution. The mass per unit length of the cylinder has been shown to be related to a parameter C and the charge of the cyliner.

1. INTRODUCTION

Krori and Paul (1977) have obtained an interior solution of a tangentially stressed cylinder. In continuation of this paper, we have here derived an interior solution of charged tangentially stressed cylinders. The solution is regular everywhere and joins smoothly to exterior (Som 1964) solution. $|\sigma/\rho|$ is not a constant quantity here. The mass per unit length of the cylinder is related to a parameter C and to the charge of the cylinder.

2. SOLUTION OF FIELD EQUATIONS

The general static cylindrically symmetric line-element is given by

$$ds^2 = g_{11}dr^2 + g_{22}d\phi^2 + g_{33}dz^2 + g_{44}dt^2 \qquad \ldots(1)$$

The field equations are

$$R^{\nu}_{\mu} - \tfrac{1}{2} Rg^{\nu}_{\mu} = -8\pi T^{\nu}_{\mu} \qquad \ldots(2)$$

$$T^{\nu}_{\mu} = \rho V_{\mu} V^{\nu} + \delta^{\nu}_{\mu} p^{\nu}_{\mu} + E^{\nu}_{\mu} \qquad \ldots(3)$$

$$E^{\nu}_{\mu} = \frac{1}{4\pi}[-F^{\nu\alpha}F_{\mu\alpha} + \tfrac{1}{4}\delta^{\nu}_{\mu} F^{\alpha\beta}F_{\alpha\beta}] \qquad \ldots(4)$$

$$F^{\mu\nu}_{;\nu} = 4\pi\sigma V^{\mu} \qquad \ldots(5)$$

$$F_{[\mu\nu,\alpha]} = 0 \qquad \ldots(6)$$

Since we are considering the case of tangentially stressed charged cylinders, $T^1_1 + T^3_3 = 0$. Thus the line-element (1) must be of Weyl's canonical form (Synge 1960).

Therefore the cylinderically symmetric line-element is written as

$$ds^2 = -e^{2\beta-2\alpha}(dr^2 + dz^2) - r^2e^{-2\alpha}d\phi^2 + e^{2\alpha}dt^2 \qquad ...(7)$$

where $\alpha = \alpha(r)$ and $\beta = \beta(r)$. Here r, z, ϕ and t are numbered 1, 2, 3 and 4 respectively.

The field eqns. (2) with the help of (7) are written as

$$e^{2(\alpha-\beta)}\left(\frac{\beta_1}{r} - \alpha_1^2\right) = -E^2 \qquad ...(8)$$

$$e^{2(\alpha-\beta)}(\beta_{11} + \alpha_1^2) = E^2 + 8\pi p_\phi \qquad ...(9)$$

$$e^{2(\alpha-\beta)}\left(2\alpha_{11} - \beta_{11} - \alpha_1^2 + \frac{2\alpha_1}{r}\right) = 8\pi\rho + E^2 \qquad ...(10)$$

where $$E^2 = -F^{14}F_{14}. \qquad ...(11)$$

The suffix 1 means differentiation with respect to r. Here ρ_σ and p_ϕ are the mass-density and tangential stress respectively in eqn. (5) is electric charge density.

Now eqns. (5) and (11) give

$$E^2 = \frac{4}{r}\frac{\mathcal{E}^2(r)}{e^{2\beta-4\alpha}} \qquad ...(12)$$

where $$\mathcal{E}(r) = \int_0^a 2\pi r e^{2\beta-3\alpha}\sigma\, dr. \qquad ...(13)$$

Now since there are four equations and six variables, let us assume

$$\alpha_1 = \frac{c}{r} \qquad ...(14)$$

and $$\psi = Br^2 \qquad ...(15)$$

where $c = c(r)$, B is a constant and ψ is electric potential related to F_{14} as follows,

$$F_{14} = -\psi_1 \qquad ...(16)$$

Equation (14) is quite general since $c(r)$ is an arbitrary function of r. The form of c will have to be determined on Physical ground. This has been done later. On the other hand eqn. (15) ensures that the electric field is zero on the axis ($r = 0$).

Now inorder that the solution is continuous with the exterior (Som 1964) solution, we have from eqns. (8) - (10)

$$\alpha = \int_a^r \frac{c}{r}\, dr + \log\frac{a^C}{c_1 + c_2a^{2C}} \qquad ...(17)$$

$$\beta = \int_a^r \frac{c^2}{r}\,dr + \int_a^r \frac{4}{r}\, e^{2\alpha}\mathcal{E}^2(r)\,dr + C^2 \log a + \log A \qquad \text{...(18)}$$

where c, c_1 and c_2 are the constants related to mass per unit length and charge of the cylinder, a is the radius of the cylinder and A is another constant.

From (9) and (10) we have

$$8\pi p_\phi = e^{2(\alpha-\beta)}\left[\frac{2cc_1}{r} + e^{-2\alpha}(8B^2cr^2 - 16B^2r^2)\right] \qquad \text{...(19)}$$

$$8\pi\rho = e^{2(\alpha-\beta)}\left[(1-c)\left(\frac{2c_1}{r} + 8B^2r^2e^{-2\alpha}\right)\right] \qquad \text{...(20)}$$

Now in order that the mass-density and tangential stress be free from singularity at $r = 0$ let us assume

$$c = Kr^2 \qquad \text{...(21)}$$

where K is a constant.

Hence eqns. (19) and (20) give

$$8\pi p_\phi = e^{2(\alpha-\beta)}\,[4K^2r^2 + 2e^{-2\alpha}(4B^2Kr^4 - 8B^2r^2)] \qquad \text{...(22)}$$

$$8\pi\rho = e^{2(\alpha-\beta)}\,[1 - Kr^2)\,(4K + 8B^2r^2e^{-2\alpha})] \qquad \text{...(23)}$$

Thus ρ is positive for $Ka^2 < 1$. p_ϕ is zero at $r = 0$.

From eqn. (5) we have

$$8\pi\sigma = 8B(1 - Kr^2)\, e^{\alpha-2\beta} \qquad \text{...(24)}$$

Thus eqns. (20) and (24) give

$$\left|\frac{\sigma}{\rho}\right| = \frac{8B}{(4K + 8B^2r^2e^{-2\alpha})\, e^{\alpha}} \qquad \text{...(25)}$$

Here $|\sigma/\rho|$ is not a constant quantity. It is a function of r.

The mass per unit length of the cylinder

$$M = \int_0^a \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \rho\, dV \qquad \text{...(26)}$$

where dV = proper elementary volume

$$= re^{2\beta-3\alpha}dr\, dz\, d\phi \qquad \text{...(27)}$$

Thus from (26) using (23) and (27) we have

$$M = \frac{1}{2} C - \frac{1}{4} C^2 + \int_0^a 2B^2 r^3 (1 - Kr^2)\, e^{-2\alpha} dr \qquad \text{...(28)}$$

to zero order in β.

Thus the mass per unit length of the cylinder is related to C and to the charge of the cylinder.

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