Indian J. pure appl. Math., 11(8) : 1085–1094, August 1980

ATTENUATION OF NON-UNIFORM SHOCK WAVE

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(*Received* 20 *November* 1979)

An attempt has been made to study the attenuation of one dimensional shock wave produced by the flying plate impact in solids. This study is based on the assumption that the behaviour of the solids is hydrodynamic in the range of pressure considered and the attenuation of shock pressure occurs due to its interaction with the rarefaction wave. The expressions for the pressure profile of the rarefaction wave and attenuation of peak pressure of the shock wave have been established and the results obtained therefrom have been compared with those obtained else where.

NOTATIONS

A, n = constants of Murnaghan's equation of state,

a = hydrodynamic sound velocity $(\partial P/\partial \rho)_S^{1/2}$ at constant entropy and with respect to the medium ahead, in the region of rarefaction wave

a_1^2 = hydrodynamic sound velocity-square which is assumed to be equal to the slope of Hugoniot $(\partial P/\partial \rho)_{Hug.}$ at pressure P_1 and measured with respect to the compressed medium moving with velocity U_1 in laboratory coordinates

C_0, S_0 = constants of the linear relation between shock and particle velocity

S = distance of a point on the pressure profile of the rarefaction wave as measured from the head of this wave

R = distance travelled by rarefaction head in time t

R_1 = distance at which rarefaction head catches the shock front

U = mass velocity in rarefaction wave

U_1, ρ, P_1 = mass velocity, density and pressure behind the shock front, before the commencement of attenuation

λ = length of the rarefaction wave extending from pressure P_1 to pressure zero

N, m, m_1, m_2 = constants of attenuation equation

U_0 = flying plate velocity

x, t = position and time independent variables

Y = variable function of pressure, $[(P + A)/A]^{(n-1)/2n}$

$Y_1 = [(P_1 + A)/A]^{(n-1)/2n}$

$Z = x/t$.

Introduction

Interaction of shock wave with solids has yielded numerous engineering and fundamental applications, like explosive forming of metals and determination of the behaviour of solids at very high pressures. The shock wave employed is generally non-uniform either due to the particular method of its generation used or due to the effect of rarefaction waves generated at the boundaries of the specimen. The study of decay of non-uniform strong shock wave has, therefore, received attention of many investigators (Drummond 1957, Fowles 1960, Erkman and Christensen 1967). Drummond (1957) has made hydrodynamic calculations of attenuation of shock wave generated by detonating an explosive in contact with the metal.

This paper presents an analytical method of calculating attenuation of shock wave produced by the impact of a flying plate in a solid target. Taylor's approach (1950) has been extended to obtain the complete pressure profile of the shock wave at any time before the shock front is overtaken by the rarefaction front. Attempt has also been made to illustrate the method by giving a numerical example of attenuation calculations.

Formulation of the Problem

Let us consider a plane shock wave of rectangular pressure profile produced in a metal target by the impact of flying plate of the same metal of thickness d. It is assumed that at time $t = 0$, the back of this rectangular shock wave coincides with that of the flying plate and its front lies $2d$ cms away from the surface $x = 0$ as shown in Fig. 1. The front of this wave moves forward with shock velocity U_s while the head of the rarefaction wave caused by the expansion of rear face of flying plate moves into the shock compressed medium with sound velocity a_1. The velocity a_1, being greater than U_s, catches the shock front at a distance R_1, from the point of its initiation. The problem that confronts us at present is to find out the pressure profile of the wave at a particular distance and also to find out the attenuation of the peak pressure of the shock wave during its propagation.

It would be assumed in this analysis that yield strength of the material is negligible in comparison with the shock pressures considered and attenuation occurs only due to rarefaction wave which is initiated at the rear face of the flying plate and moves with hydrodynamic sound velocity into the shock compressed metal. The shock front has also been assumed to be plane to result only in one dimensional flow. The

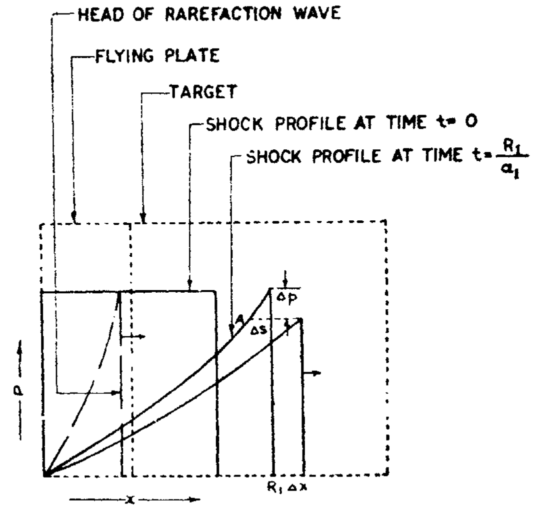

FIG. 1. Attenuation of rectangular shock wave in solids.

Murnaghan's equation of state and the Hugoniot of the metal would also be considered to be the same (Courant and Friedrichs 1948) so long as the change in entropy in shock transition is negligible.

ANALYSIS OF SHOCK WAVE PROFILE

The flow parameters in unidirectional rarefaction wave can be obtained by considering the conservation of mass and momentum in this region. If, U, ρ, and P denote mass-velocity, density and pressure, then the conservation laws can be expressed as

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho U)}{\partial x} = 0 \qquad \ldots(1)$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + \frac{a^2}{\rho}\frac{\partial \rho}{\partial x} = 0 \qquad \ldots(2)$$

where $a = (\partial P/\partial \rho)_S^{1/2} = (\partial P/\partial \rho)_{Hug.}^{1/2}$ has been used as hydrodynamic sound velocity (Fowles 1960) of the metal. Following Taylor's approach (1950), these equations can be changed to the forms

$$(U - Z + a)\left[\left(\frac{a}{\rho}\right)\frac{\partial \rho}{\partial Z} + \frac{\partial U}{\partial Z}\right] = 0 \qquad \ldots(3)$$

$$(U - Z - a\left[\left(\frac{a}{\rho}\right)\frac{\partial \rho}{\partial Z} - \frac{\partial U}{\partial Z}\right] = 0 \qquad \ldots(4)$$

by introducing a transformation

$$\left.\begin{aligned} Z &= x/t \\ \text{and} \qquad R &= a_1 t \end{aligned}\right\} \qquad \ldots(5)$$

where a_1 is the velocity of rarefaction front and R is the distance travelled by this front in a time t. Combining these equations with Murnaghan's equation of state for the metal, namely

$$P = A\,[(\rho/\rho_0)^n - 1] \qquad \ldots(6)$$

One readily obtains the solution of these equations as

$$(U - U_1) + a = x a_1/R \qquad \ldots(7)$$

$$(U - U_1) = \frac{2}{n-1}\left(a - a_1\right) \qquad \ldots(8)$$

This yields the pressure profile of the rarefaction wave as

$$P = A\left[\frac{a_1 - (n-1)\,a_1 S/R(n+1)}{(nA/\rho_0)^{1/2}}\right]^{2n/(n-1)} - A \qquad \ldots(9)$$

where $S = R - x$ is the distance of an arbitrary point, such as A in Fig. 1, measured from the head of rarefaction wave. The length of the rarefaction wave λ which is defined as the distance between its head at pressure P_1 and the tail at pressure zero can be easily obtained by setting $P = 0$ in equation (9). Thus one obtains

$$\lambda = \left(\frac{n+1}{n-1}\right) R\left[1 - \frac{1}{a_1}\left(\frac{nA}{\rho_0}\right)^{1/2}\right] \qquad \ldots(10)$$

This evidently shows that the length λ increases in a direct proportion with distance travelled by the rarefaction head, provided a_1 remains constant. To see the variation of the shape of this wave, the differential of eqn. (9) has been combined with the expression for the velocity of rarefaction head

$$a_1 = (nA/\rho_0)^{1/2}\,[(P_1 + A)/A]^{(n-1)/2n} \qquad \ldots(11)$$

This gives the slope of the wave at $S = 0$ as

$$\frac{dP}{ds} = -\left(\frac{2nA}{n+1}\right)\frac{1}{R}\left(\frac{P_1 + A}{A}\right) \qquad \ldots(12)$$

It can be seen from the above equation that, until pressure of the shock wave changes, the slope of the rarefaction wave at $S = 0$ decreases with distance R only. The rarefaction head, however, on catching the shock front, changes the shock pressure and as such this slope becomes dependent on both, the distance travelled by rarefaction head and the shock wave pressure. Also at this moment, the head of rarefaction wave coincides with the front of the shock wave and is forced to move along with it. To calculate the slope of the rarefaction wave now, therefore, one inserts a variable pressure P and distance $R = R_1 + X$ in eqn. (12) which gives,

$$\frac{dP}{ds} = -\left(\frac{2nA}{n+1}\right)\frac{1}{R_1+X}\left(\frac{P+A}{A}\right) \qquad \text{...(13)}$$

where R_1 is the maximum distance which has been travelled by the rarefaction head before catching the shock front and X is the distance measured from this point onward.

Remembering that initial distance between shock front and rarefaction front is $2d$ and it is being covered up with a relative velocity $(u_1 + a_1 - U_s)$ and a_1 is velocity of rarefaction front, the distance R_1 can be easily obtained (Fowles 1960) to be given by

$$R_1 = \frac{2a_1 d}{U_1 + a_1 - U_s} \qquad \text{...(14)}$$

where d is the thickness of the flying plate and u_1 and a_1 are particle and sound velocities behind the shock front and U_s is the velocity of the shock front. Making use of eqns. (9) and (14), one can obtain the profile of rarefaction wave at a time when head of this wave has travelled a distance R_1.

Analysis of Attenuation of Shock Pressure

The shock front moves a distance R_1 with constant velocity and pressure. The kinematic and thermodynamic parameters of the shock front, however, begin to decrease after its interaction with the head of the rarefaction wave. Hereafter the propagation of the shock wave, whose pressure decreases with distance, conforms to Fig. 1. As shown in this figure, a rarefaction wave which is initiated at a point A moves a distance $(\triangle S + \triangle X)$ with velocity $(u + a)$ with respect to unshocked medium in a time $\triangle t$ which is also equal to the time taken by the shock front in moving a distance $\triangle X$ with velocity U_s. Therefore

$$\frac{\triangle S + \triangle X}{U + a} = \frac{\triangle X}{U_s} = \triangle t$$

which in the limit $\triangle X \to \triangle t \to 0$ gives

$$\frac{dP}{dX} = \left(\frac{u+a}{U_s} - 1\right)\frac{dP}{dS} \qquad \text{...(15)}$$

in order to relate a single shock parameter with distance, this equation has been further transformed by using equation (6), and the equations

$$U_s = C_0 + S_0 u \qquad \text{...(16)}$$

$$U = U_1 - \int_P^{P_1} \frac{dP}{\rho a} \qquad \text{...(17)}$$

where U_1 and P_1 are the particle velocity and pressure behind the shock front before the commencement of attenuation and P is the arbitrary pressure of the rarefaction wave. C_0, and S_0 are constants of the medium. The linear relation (16) between shock velocity U_s and particle velocity U is well established for metals and has been assumed to be equivalent to Murnaghan's equation of state under the assumption of low shock pressures (Fowles 1960). Equation (17) shows that unattenuated particle velocity U_1 decreases by an amount $\int_P^{P_1} dP/\rho a$ when the rarefaction wave reduces the shock pressure from its unattenuated value P_1 to a new value P. Substituting eqns. (6), (16) and (17) in eqn. (15) and integrating one gets the solution

$$\log (R_1 + X) = -N\,[m_1 \log y + m_2 \log (m + y)] + \alpha \qquad \ldots(18)$$

where

$$y = \left(\frac{P + A}{A}\right)^{(n-1)/2n}$$

and

$$N = (n + 1)/(n - 1)$$

$$m = \left[\left(U_1 - \frac{2a_1}{n-1}\right)(1 - S_0) - C_0 \right] \Big/ \left[\left\{1 + \frac{2(1 - S_0)}{n-1}\right\}\left(\frac{nA}{\rho_0}\right)^{1/2}\right]$$

$$m_1 = \left\{C_0 + S_0 U_1 - \frac{2a_1 S_0}{n-1}\right\} \Big/ \left\{\left(U_1 - \frac{2a_1}{n-1}\right)(1 - S_0) - C_0\right\}$$

$$m_2 = \left(\frac{2S_0}{n-1}\right) \Big/ \left[1 + \frac{2(1 - S_0)}{(n - 1)}\right] - m_1. \qquad \ldots(19)$$

The constant of integration α can be evaluated from the initial conditions that at $X = 0$,

$$y = y_1 = \left(\frac{P_1 + A}{A}\right)^{(n-1)/2n}.$$

This yields a relation between Y — a function of shock pressure P, and the distance X travelled by shock wave during course of attenuation.

$$\log\left(\frac{R_1 + X}{R_1}\right) = N\left[m_1 \log\frac{y_1}{y} + m_2 \log \frac{m + y_1}{m + y}\right] \qquad \ldots(20)$$

The constants N, m, m_1, m_2 and R_1 can be obtained from initial conditions set for the experiment and the equation of state used for the metal.

Example — Attenuation of Shock Wave in Aluminium

Combining the expressions for conservations of mass and momentum across the shock front with the equation of state (14) one gets the relations

$$\left(\frac{U_0}{2}\right)^2 = \frac{A}{\rho_0}\left[\frac{(\rho_1/\rho_0) - 1}{(\rho_1/\rho_0)}\right][(\rho_1/\rho_0)^n - 1] \quad \text{...(21)}$$

$$U_s = \frac{U_0}{2}\left[\frac{1}{1 - (\rho_0/\rho_1)}\right] \quad \text{...(22)}$$

$$a_1 = \left(\frac{nA}{\rho_0}\right)^{1/2} (\rho_1/\rho_0)^{(n-1)/2} \quad \text{...(23)}$$

where U_0 is the velocity of the flying plate and equals the double of the particle velocity which it induces after impacting the target of the same material and ρ_0 is initial density of the target and flying plate. The constants of equation of state of aluminium has been abtained from the published literature (Fowles 1960, Mcqueen *et al.* 1970) as tabulated below:

TABLE I

Constants of equation of state of aluminium

A dynes/cm^2	n	ρ_0 gm/cm^3	C_0 cm/sec	S_0
188.96×10^9	4.266	2.785	5.328×10^5	1.338

Let us assume that the impacting velocity and thickness of the flying plate are given as $U_0 = 3.7 \times 10^5$ cm/sec and $d = 0.41$ cms. These values, when substituted in eqns. (21), (22), (23) and (14), yields $\rho_1/\rho_0 = 1.308$, $a_1 = 8.34070 \times 10^5$ cm/sec, $U_s = 7.85649 \times 10^5$ cm/sec, $R_1 = 2.93$ cm. Using these values, one easily gets the shock wave profile from the solution of eqn. (9). The results of these calculations have been graphically represented in Fig. 2. The values of constants of attenuation eqn. (20) have been obtained by using eqn. (19) and the constants of Table I. These values in Table II show that their variation in the range of pressure considered do not exceed $\pm$ 2% in m_1 and $\pm$ .05% and $\pm$ 0.7% in m and m_2 respectively from their average values.

TABLE II

Constants for attenuation equation for aluminium

Pressure of shock wave (Kbar)	m_1	m	m_2	N	Y_1
404.78	—0.229	—0.991	1.262	1.612	1.550
146.24	—0.220	—0.992	1.253	1.612	1.245

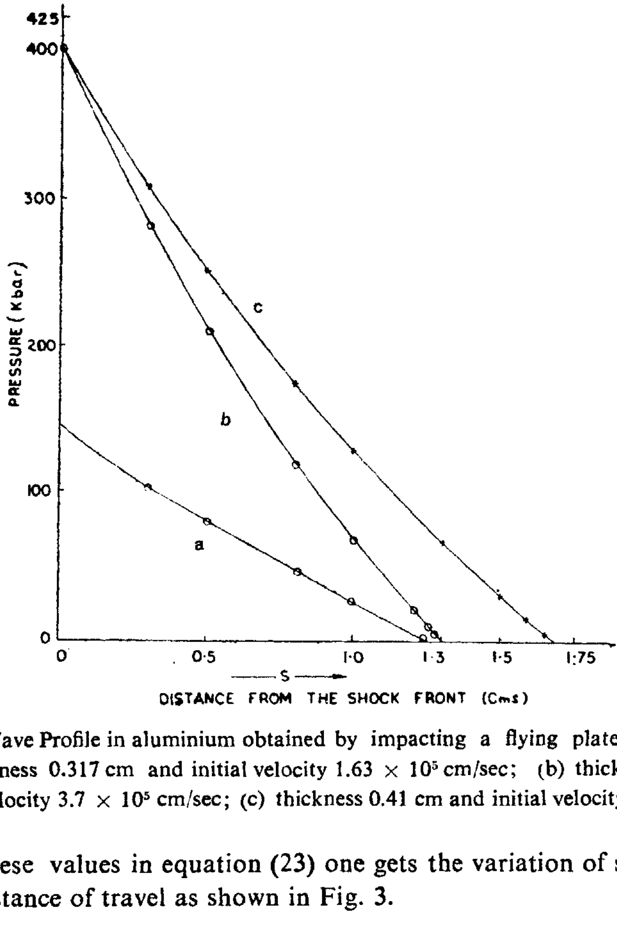

FIG. 2. Shock Wave Profile in aluminium obtained by impacting a flying plate of aluminium of (a) thickness 0.317 cm and initial velocity 1.63×10^5 cm/sec; (b) thickness 0.317 cm and initial velocity 3.7×10^5 cm/sec; (c) thickness 0.41 cm and initial velocity 3.7×10^5 cm/sec.

Substituting these values in equation (23) one gets the variation of shock wave pressure with its distance of travel as shown in Fig. 3.

DISCUSSION

Profile of the shock wave in aluminium has been calculated using eqn. (9) for these typical initial conditions. In all these cases, the distance R_1, where shock profile has been obtained is the one where head of the rarefaction wave catches the shock front. This distance can be easily obtained to be given by

$$R_1 = \frac{2a_1 d}{a_1 - U_1 (S_0 - 1) - C_0}$$

which, on differentiation, shows that dR/da_1 is negative provided $S_0 > 1$ which is generally the case for metals. In other words, this distance decreases with increasing initial pressures of the shock wave, and as such higher the initial pressure or higher the impacting velocity of the same thickness of flying plate, shorter will be the distance where attenuation of shock wave starts and greater will be the value of the maximum slope of the rarefaction which means faster attenuation as given by eqn. (12) and

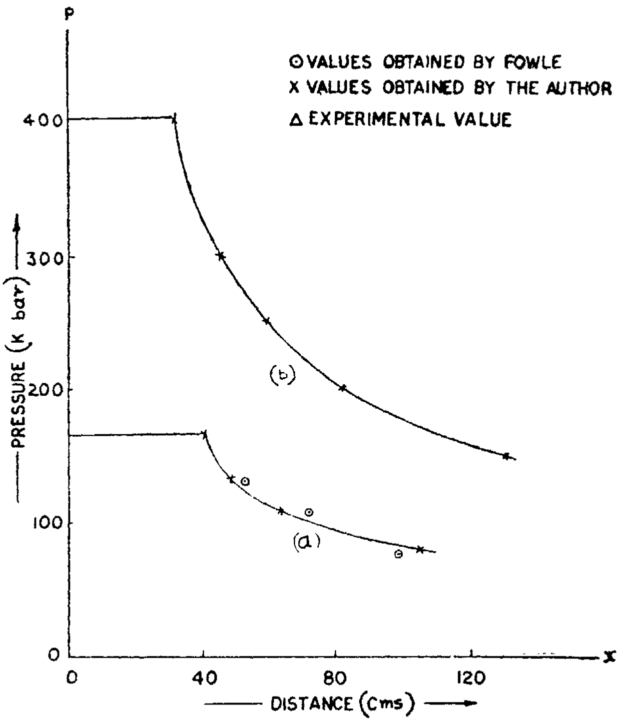

FIG. 3. Attenuation of shock wave in aluminium produced by impacting a flying plate of aluminium of (a) thickness 0.317 cm and initial velocity 0.815×10^5 cm/sec; (b) thickness 0.41 cm and initial velocity 1.85×10^5 cm/sec.

depicted graphically in Figs. 2 and 3. It can also be observed from this figure that the length of the wave λ at the distance considered does not significantly change in the range of present pressures. The attenuation of shock wave has been calculated for two typical initial conditions. These values, when compared with those obtained elsewhere (Fowles 1960) and with some of preliminary experimental results, are found to be in close agreement as shown in Fig. 3. The important feature of the attenuation curve is that attenuation is very fast in the beginning conforming to the steep slope of the rarefaction wave at high pressures and slows down very much in the later stages. The constants of eqn. (20) which have been used to calculate the attenuation curve and enlisted in Table No. 2 show that their values do not change significantly throughout the present range of pressures.

ACKNOWLEDGEMENT

The author feels indebted to the Director, TBRL, Chandigarh, for giving permission to publish this work. He is also grateful to Shri P. K. Misra and Shri D. S. Murty for their suggestions and encouragement throughout this work. The author is also thankful to Shri C. P. S. Tomar and Dr S. S. Sachdeva for whole hearted cooperation in the completion of this work.

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