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# PRICING AND ORDERING POLICY WITH GENERAL WEIBULL RATE OF DETERIORATING INVENTORY*

VIJAY P. GOEL†

*Department of Statistics, Panjab University, Chandigarh* 160014

AND

S. P. AGGARWAL

*Department of Operational Research, University of Delhi, Delhi* 110007

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In this paper an algorithm for determining the optimal pricing and ordering policy for 3-parameter Weibull rate of deterioration is developed.

## 1. INTRODUCTION

The inventory theory deals with the determination of optimal stocks of commodities to meet demands in the foreseeable future. Existing inventory models differ in many ways—in the nature of the demand, the nature of the information available with the decision making authorities at any given time, in relation to the costs associated with the systems. Some items are perishable whereas other items can be stored for an indefinite period without deteriorating.

The decrease in utility or loss for an inventory of goods subject to deterioration is usually a function of the total amount of inventory on hand. Optimal ordering policies for some particular classes of deteriorating inventory problems without prices being involved are derived by many authors (Covert and Philips 1973;. Ghare and Scharder 1963, Philip 1974). The study of price as an inventory decisions variable for a non-deteriorating product has also been undertaken by researchers (Kunreuther and others 1973, 1975; Thomas 1970, 1974). The analysis of pricing and ordering policies for a constant proportional rate of deteriorating products have also been studied exhaustively (Cohen 1977, Smith 1975).

The present paper develops an algorithm for determining the optimal pricing and ordering policy for 3-parameter Weibull rate of deterioration. The 3-parameter Weibull distribution can be applied for the items with any initial value of the rate of deteriorating and for items which start deteriorating only after a certain period of time. The exponential distribution is a particular case of Weibull distribution ($\beta = 1, \gamma = 0$). Both cases without shortages and with shortages have been discussed in this paper.

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†Present address : Department of Statistics, Khalsa College (Bombay University), Bombay.

## 2. Preliminaries

*Assumptions*

1. Demand is known and has a constant rate.
2. Replenishment is instantaneous.
3. Lead time is zero.
4. Shortages are allowed and fully backlogged.
5. The time to deterioration of the items has a 3-parameter Weibull distribution.
6. A deteriorating unit is lost.

*Notations*

$T$ = length of one cycle

$Q$ = order quantity in one cycle

$A$ = cost of placing an order

$C$ = cost price of one unit

$s$ = selling price of one unit

$h$ = inventory holding cost per unit per unit time

$\pi$ = shortage cost per unit per unit time

$\lambda(s)$ = demand rate (function of selling price), $s\lambda(s)$ is a concave function of $s$.

The objective of the present paper is to maximize profit rate function $P(T, s)$ where $T$ and $s$ are decision variables.

## 3. Mathematical Formulation and Solution

The 3-parameter Weibull density function is given by

$$f(t) = \alpha\beta(t - \gamma)^{\beta-1} \exp[-\alpha(t - \gamma)^{\beta}] \quad \text{...(3.1)}$$

where

$\alpha$ = the scale parameter, $\alpha > 0$

$\beta$ = the shape parameter, $\beta > 0$

$\gamma$ = the location parameter, $-\infty < \gamma < \infty$

$t$ = time to deterioration, $t \geqslant \gamma$; $t > 0$.

The instantaneous rate of deterioration of the on hand inventory is given by $\alpha\beta(t - \gamma)^{\beta-1}$.

*Model without shortages*

Let $I(t)$ be the inventory level of the system at any time $t$ $(0 \leqslant t \leqslant T)$. The instantaneous state of $I(t)$ over the cycle of length $T$ satisfies

$$\frac{dI(t)}{dt} + \alpha\beta(t-\gamma)^{\beta-1} I(t) = -\lambda(s)\ (0 \leqslant t \leqslant T). \quad \ldots(3.2)$$

The solution of the above differential equation is

$$I(t) = \frac{I(0)\exp[\alpha(-\gamma)^{\beta}] - \lambda(s)\int_0^t \exp[\alpha(t-\gamma)^{\beta}]\,dt}{\exp[\alpha(t-\gamma)^{\beta}]}. \quad \ldots(3.3)$$

The stock loss due to deterioration in the interval (0, $t$) is given by

$$L(t) = I(t)\Big[\exp[\alpha\{(t-\gamma)^{\beta} - (-\gamma)^{\beta}\}] - 1\Big] - \lambda(s)\,t$$
$$+ \lambda(s)\int_0^t \exp[\alpha\{(t-\gamma)^{\beta} + \gamma^{\beta}\}]\,dt. \quad \ldots(3.4)$$

Also

$$I(0) = Q = L(T) + \lambda(s)\,T$$
$$= \lambda(s)\int_0^T \exp[\alpha\{(t-\gamma)^{\beta} - (-\gamma)^{\beta}\}]\,dt \qquad (\text{since } I(T) = 0). \quad \ldots(3.5)$$

Hence

$$I(t) = \lambda(s)\exp\{-\alpha(t-\gamma)^{\beta}\}\,[\int_0^T \exp\{\alpha(t-\gamma)^{\beta}\}\,dt$$
$$- \int_0^t \exp\{\alpha(t-\gamma)^{\beta}\}\,dt]. \quad \ldots(3.6)$$

The total cost per unit time, viz. $K(T, s)$, is the sum of the set-up cost, cost of the units and the inventory holding cost, i.e.

$$K(T, s) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\int_0^T I(t)\,dt$$
$$= \frac{A}{T} + \frac{C\lambda(s)}{T}\int_0^T \exp[\alpha\{(t-\gamma)^{\beta} - (-\gamma)^{\beta}\}]\,dt$$
$$+ \frac{h\lambda(s)}{T}\int_0^T \exp\{-\alpha(t-\gamma)^{\beta}\}\Big[\int_0^T \exp\{\alpha(t-\gamma)^{\beta}\}\,dt$$
$$- \int_0^t \exp\{\alpha(t-\gamma)^{\beta}\}\,dt\Big]\,dt. \quad \ldots(3.7)$$

By using the Taylor series expansion for the exponential function

$$\exp[\alpha(t-\gamma)^{\beta}] = 1 + \alpha(t-\gamma)^{\beta} + \frac{[\alpha(t-\gamma)^{\beta}]^2}{2!} + \ldots$$

which is a valid approximation for small values of $\alpha(t-\gamma)^{\beta}$.

Equation (3.7) (ignoring the terms of $O(\alpha^2)$) can be written as

$$K(T, s) = \frac{A}{T} + \frac{C\lambda(s)}{T}\int_0^T [1 + \alpha\{(t-\gamma)^{\beta} - (-\gamma)^{\beta}\}]\,dt$$

$$+ \frac{h\lambda(s)}{T}\int_0^T [1 - \alpha(t-\gamma)^{\beta}]\left[\int_0^T \{1 + \alpha(t-\gamma)^{\beta}\}\,dt\right.$$

$$\left. - \int_0^t \{1 + \alpha(t-\gamma)^{\beta}\}\,dt\right] dt \quad \ldots(3.8)$$

$$= \frac{A}{T} + C\lambda(s)(1 - \alpha(-\gamma)^{\beta}) + \frac{h\lambda(s)}{2}T$$

$$+ \frac{C\alpha\lambda(s)}{(\beta+1)}\frac{1}{T}[(T-\gamma)^{\beta+1} - (-\gamma)^{\beta+1}]$$

$$- \frac{2h\alpha\lambda(s)}{(\beta+1)(\beta+2)}\frac{1}{T}[(T-\gamma)^{\beta+2} - (-\gamma)^{\beta+2}]$$

$$+ \frac{h\alpha\lambda(s)}{(\beta+1)}[(T-\gamma)^{\beta+1} + (-\gamma)^{\beta+1}].$$

Using Binomial expansion and further simplifying, we get

$$K(T, s) = \frac{A}{T} + C\lambda(s) + \frac{h\alpha\beta\lambda(s)}{(\beta+1)(\beta+2)}T^{\beta+1} + \frac{\alpha\lambda(s)}{(\beta+1)}$$

$$\times [C - h(\beta-1)\gamma]\,T^{\beta} - \alpha\gamma\lambda(s)\left[C - \frac{h(\beta-2)\gamma}{2}\right]T^{\beta-1} + \ldots$$

$$+ \frac{\alpha\beta(-\gamma)^{\beta-2}\lambda(s)}{6}[C(\beta-1) - h\gamma]\,T^2$$

$$+ \frac{\lambda(s)}{2}[h + C\alpha\beta(-\gamma)^{\beta-1}]\,T. \quad \ldots(3.9)$$

The profit rate function $P(T, s)$ is

$$P(T, s) = s\lambda(s) - \frac{A}{T} - C\lambda(s) - \frac{h\alpha\beta\lambda(s)}{(\beta+1)(\beta+2)}T^{\beta+1} -$$

(*equation continued on p.* 622)

$$- \frac{\alpha\lambda(s)}{(\beta+1)} [C - h(\beta-1)\gamma] T^{\beta}$$

$$+ \alpha\lambda(s) \left[C - \frac{h(\beta-2)\gamma}{2}\right] T^{\beta-1} + \ldots$$

$$+ \frac{\alpha\beta(-\gamma)^{\beta-2}\lambda(s)}{6} [C(\beta-1) - h\gamma] T^2$$

$$- \frac{\lambda(s)}{2} [h + C\alpha\beta(-\gamma)^{\beta-1}] T. \qquad \ldots(3.10)$$

The profit rate function is a concave function, since $s\lambda(s)$ is a concave function and $K(T, s)$ is a convex function. To find the optimal value of $T$ and $s$ it can be differentiated partially with respect to $T$ and $s$ respectively and set equal to zero, i.e.

$$\frac{\partial P(T,s)}{\partial T} = 0$$

$$\Rightarrow \frac{h\alpha\beta\lambda(s)}{(\beta+2)} T^{\beta+2} + \alpha\beta\lambda(s) [C - h(\beta-1)\gamma] T^{\beta+1}$$

$$- \alpha(\beta-1)\gamma\lambda(s) \left[C - \frac{h(\beta-2)\gamma}{2}\right] T^{\beta}$$

$$+ \ldots + \frac{\alpha\beta(-\gamma)^{\beta-2}\lambda(s)}{3} [C(\beta-1) - h\gamma] T^3$$

$$+ \frac{\lambda(s)}{2} [h + C\alpha\beta(-\gamma)^{\beta-1}] T^2 - A = 0$$

or

$$T^{\beta+2} + \frac{(\beta+2)}{(\beta+1)h} [C - h(\beta-1)\gamma] T^{\beta+1}$$

$$- \frac{(\beta+2)(\beta-1)\gamma}{h\beta} \left[C - \frac{h(\beta-2)\gamma}{2}\right] T^{\beta} + \ldots$$

$$+ \frac{(\beta+2)(-\gamma)^{\beta-2}}{3h} [C(\beta-1) - h\gamma] T^3$$

$$+ \frac{(\beta+2)}{2h\alpha\beta} [h + C\alpha\beta(-\gamma)^{\beta-1}] T^2 - \frac{A(\beta+2)}{h\alpha\beta\lambda(s)} = 0 \qquad \ldots(3.11)$$

and

$$\frac{\partial P(T,s)}{\partial s} = 0$$

$$\Rightarrow s = -\frac{\lambda(s)}{\lambda'(s)} + C + \frac{h\alpha\beta}{(\beta+1)(\beta+2)} T^{\beta+1} -$$

*(equation continued on p. 623)*

$$+ \frac{\alpha}{(\beta+1)} [C - h(\beta-1)\gamma] T^{\beta} - \alpha\gamma \left[ C - \frac{h(\beta-2)\gamma}{2} \right] T^{\beta-1}$$

$$+ \ldots + \frac{\alpha\beta(-\gamma)^{\beta-2}}{6} [C(\beta-1) - h\gamma] T^{2} + \tfrac{1}{2} [h + C\alpha\beta(-\gamma)^{\beta-1}] T. \quad \ldots(3.12)$$

(3.11) is a polynomial equation in $T$ of degree $(\beta + 2)$ and will determine different values of $T$ in terms of $s$ and $\lambda(s)$. Let $T = \phi(s)$. We choose those values of $T$ and $s$ [using (3.11) and (3.12)] which maximize the profit rate function $P(T, s)$. Again (3.12) becomes

$$s = -\frac{\lambda(s)}{\lambda'(s)} + C + \frac{h\alpha\beta}{(\beta+1)(\beta+2)} [\phi(s)]^{\beta+1}$$

$$+ \frac{\alpha}{(\beta+1)} [C - h(\beta-1)\gamma] [\phi(s)]^{\beta} - \alpha\gamma \left[ C - \frac{h(\beta-2)\gamma}{2} \right]$$

$$\times [\phi(s)]^{\beta-1} + \ldots + \frac{\alpha\beta(-\gamma)^{\beta-2}}{6} [C(\beta-1) - h\gamma] [\phi(s)]^{2}$$

$$+ \tfrac{1}{2} [h + C\alpha\beta(-\gamma)^{\beta-1}] \phi(s) = 0. \quad \ldots(3.13)$$

(3.13) can be solved by successive approximation (using numerical techniques) for various choices of demand rate function. Let the optimal approximate value of $s$ be $s_0$ then the approximate value of $T$ be $T_0 = \phi(s_0)$. Hence $(T_0, s_0)$ maximizes the profit rate function $P(T, s)$ and the optimal value $Q_0$ can be determined from (3.5). For $\beta = 2$, (3.11) reduces to

$$T^4 + \frac{4}{3h} (C - h\gamma) T^3 + \left( \frac{1}{\alpha} - \frac{2C\gamma}{h} \right) T^2 - \frac{2A}{h\alpha\lambda(s)} = 0$$

which can be solved by using theory of equations.

*Model with shortages*

The model is extended further to permit shortages which are fully backlogged. Stock decreases by a combination of deterioration and sales in the interval $(0, T_1)$ and is backlogged as a result of excess demand in the interval $(T_1, T)$. The instantaneous states of $I(t)$ over the cycle $(0, T)$ are given by

$$\frac{dI(t)}{dt} + \alpha\beta(t-\gamma)^{\beta-1} I(t) = -\lambda(s) \quad 0 \leqslant t \leqslant T_1 \quad \ldots(3.14)$$

and

$$\frac{dI(t)}{dt} = -\lambda(s) \quad 0 \leqslant t \leqslant T - T_1 \quad \ldots(3.15)$$

The solutions of (3.14) and (3.15) are

$$I(t) = \frac{I(0) \exp [\alpha(-\gamma)^{\beta}] - \lambda(s) \int_0^t \exp [\alpha(t-\gamma)^{\beta}]\, dt}{\exp [\alpha(t-\gamma)^{\beta}]} \quad (0 \leqslant t \leqslant T_1)$$

$$I(t) = -\lambda(s)\, t \quad 0 \leqslant t \leqslant T - T_1 \qquad \ldots(3.16)$$

The loss of stock due to deterioration within the cycle $T$ is

$$L(T_1) = -\lambda(s)\, T_1 + \lambda(s) \int_0^{T_1} \exp [\alpha\{(t-\gamma)^{\beta} - (-\gamma)^{\beta}\}]\, dt \qquad \ldots(3.17)$$

and backlogged demand is

$$B(T_1) = \lambda(s)\, (T - T_1) \qquad \ldots(3.18)$$

$$\therefore \quad Q = L(T_1) + \lambda(s)\, T$$

$$= \lambda(s)\, (T - T_1) + \lambda(s) \int_0^{T_1} \exp [\alpha\{(t-\gamma)^{\beta} (-\gamma)^{\beta}\}]\, dt \qquad \ldots(3.19)$$

$$I(0) = \lambda(s) \int_0^{T_1} \exp [\alpha\{(t-\gamma)^{\beta} - (-\gamma)^{\beta}\}]\, dt.$$

Hence

$$I(t) = \lambda(s) \exp \{-\alpha(t-\gamma)^{\beta}\} \left[ \int_0^{T_1} \exp \{\alpha(t-\gamma)^{\beta}\}\, dt - \int_0^{t} \exp \{\alpha(t-\gamma)^{\beta}\}\, dt \right] dt. \qquad \ldots(3.20)$$

The total cost per unit time is

$$K(T, T_1, s) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T} \int_0^{T_1} I(t)\, dt + \frac{\pi\lambda(s)}{T} \int_0^{T-T_1} |t\, dt$$

$$= \frac{A}{T} + \frac{C\lambda(s)\,(T - T_1)}{T} + \frac{C\lambda(s)}{T} \int_0^{T_1} \exp [\alpha\{(t-\gamma)^{\beta}$$

$$- (-\gamma)^{\beta}\}]\, dt + \frac{h\lambda(s)}{T} \int_0^{T_1} \exp \{-\alpha(t-\gamma)^{\beta}\}$$

$$\times \left[ \int_0^{T_1} \exp \{\alpha(t-\gamma)^{\beta}\, dt \right] dt -$$

(*equation continued on p. 625*)

$$- \frac{h\lambda(s)}{T} \int_0^{T_1} \exp\{-\alpha(t-\gamma)^\beta\} \left[ \int_0^t \exp\{\alpha(t-\gamma)^\beta\}\, dt \right] dt$$

$$+ \frac{\pi\lambda(s)}{T} \int_0^{T-T_1} t\, dt$$

$$K(T, T_1, s) = \frac{A}{T} + C\lambda(s) \frac{(T-T_1)}{T} + C\lambda(s)\{1 - \alpha(-\gamma)^\beta\} \frac{T_1}{T}$$

$$+ \frac{h\lambda(s)}{2} \frac{T_1^2}{T} + \frac{C\alpha\lambda(s)}{(\beta+1)} \frac{1}{T} [(T_1-\gamma)^{\beta+1} - (-\gamma)^{\beta+1}]$$

$$- \frac{2h\alpha\lambda(s)}{(\beta+1)(\beta+2)} \frac{1}{T} [(T_1-\gamma)^{\beta+2} - (-\gamma)^{\beta+2}]$$

$$+ \frac{h\alpha\lambda(s)}{(\beta+1)} \frac{T_1}{T} [(T_1-\gamma)^{\beta+1} + (-\gamma)^{\beta+2}]. \qquad \ldots(3.21)$$

Let $\eta = T_1/T$

$$K(T, s, \eta) = \frac{A}{T} + C\lambda(s) + \frac{2h\alpha(-\gamma)^{\beta+1}(1-\eta)\lambda(s)}{(\beta+1)}$$

$$+ \frac{h\alpha\eta^{\beta+1}(\beta - 2\eta + 2)\lambda(s)}{(\beta+1)(\beta+2)} T^{\beta+1}$$

$$+ \frac{\alpha\eta^\beta\lambda(s)}{(\beta+1)} [C\eta - h\gamma(\beta - 2\eta + 1)] T^\beta - \alpha\gamma\eta^{\beta-1}\lambda(s)$$

$$\times \left[ C - \frac{h\gamma(\beta - 2\eta)}{2} \right] T^{\beta-1}$$

$$+ \ldots - \frac{\alpha\beta(-\gamma)^{\beta-2}\eta^2\lambda(s)}{6} [C(\beta-1)\eta - h\gamma(3-2\eta)] T^2$$

$$+ \frac{\lambda(s)}{2} [h\eta^2 + C\alpha\beta(-\gamma)^{\beta-1}\eta^2 + \pi(1-\eta)^2\, T. \qquad \ldots(3.22)$$

The profit rate $P(T, s, \eta)$ is

$$P(T, s, \eta) = s\lambda(s) - K(T, s, \eta)$$

$$= s\lambda(s) - \frac{A}{T} - C\lambda(s) - \frac{2h\alpha(-\gamma)^{\beta+1}(1-\eta)\lambda(s)}{(\beta+1)}$$

$$- \frac{h\alpha\eta^{\beta+1}(\beta - 2\eta + 2)\lambda(s)}{(\beta+1)(\beta+2)} T^{\beta+1} - \frac{\alpha\eta^\beta\lambda(s)}{(\beta+1)}$$

$$\times [C\eta - h\gamma(\beta - 2\eta + 1)] T^\beta +$$

*(equation continued on p. 626)*

$$+ \alpha\gamma\eta^{\beta-1}\lambda(s)\left[C\eta - \frac{h\gamma(\beta - 2\eta)}{2}\right]T^{\beta-1} + \ldots$$

$$+ \frac{\alpha\beta(-\gamma)^{\beta-2}\eta^2\lambda(s)}{6}[C(\beta - 1)\eta - h\gamma(3 - 2\eta)]T^2$$

$$- \frac{\lambda(s)}{2}[h\eta^2 + C\alpha\beta(-\gamma)^{\beta-1}\eta^2 + \pi(1 - \eta)^2]T. \qquad \ldots(3.23)$$

Differentiating (3.23) partially with respect to $T$, $s$ and $\eta$ respectively and equating them to zero i.e.

$$\frac{\partial P(T, s, \eta)}{\partial T} = 0$$

$$\Rightarrow \frac{h\alpha\eta^{\beta+1}(\beta - 2\eta + 2)\lambda(s)}{(\beta + 2)}T^{\beta+2} + \frac{\alpha\beta\eta^{\beta}\lambda(s)}{(\beta + 1)}[C\eta - h\gamma(\beta - 2\eta + 1)]T^{\beta+1}$$

$$- \alpha(\beta - 1)\gamma\eta^{\beta-1}\lambda(s)\left[C\eta - \frac{h\gamma(\beta - 2\eta)}{2}\right]T^{\beta} + \ldots$$

$$- \frac{\alpha\beta(-\gamma)^{\beta-2}\eta^2\lambda(s)}{3}[C(\beta - 1)\eta - h\gamma(3 - 2\eta)]T^2$$

$$+ \frac{\lambda(s)}{2}[h\eta^2 + C\alpha\beta(-\gamma)^{\beta-1}\eta^2 + \pi(1 - \eta)^2]T^2 - A = 0 \qquad \ldots(3.24)$$

and

$$\frac{\partial P(T, s, \eta)}{\partial s} = 0$$

$$\Rightarrow s = \frac{\lambda(s)}{\lambda'(s)} + C + \frac{2h\alpha(-\gamma)^{\beta+1}(1 - \eta)}{(\beta + 1)} + \frac{h\alpha\eta^{\beta+1}(\beta - 2\eta + 2)}{(\beta + 1)(\beta + 2)}T^{\beta+1}$$

$$+ \frac{\alpha\eta^{\beta}}{\beta + 1}[C\eta - h\gamma(\beta - 2\eta + 1)]T^{\beta} - \alpha\gamma\eta^{\beta-1}$$

$$\times \left[C\eta - \frac{h\gamma(\beta - 2\eta)}{2}\right]T^{\beta-1} + \ldots$$

$$- \frac{\alpha\beta(-\gamma)^{\beta-2}\eta^2}{6}[C(\beta - 1)\eta - h\gamma(3 - 2\eta)]T^2$$

$$+ \tfrac{1}{2}[h\eta^2 + C\alpha\beta(-\gamma)^{\beta-1}\eta^2 + \pi(1 - \eta)^2]T \qquad \ldots(3.25)$$

(3.24) and (3.25) determine the optimal approximate values of $T$ and $s$ as studied in the previous model. (3.19) will determine the optimal value of $Q$.

Also

$$\begin{aligned}\frac{\partial P(T,s,\eta)}{\partial \eta} = &- C\alpha\lambda(s) + \frac{2h\alpha(-\gamma)^{\beta+1}\lambda(s)}{(\beta+1)} \\ &- \frac{h\alpha\eta^{\beta}\lambda(s)}{(\beta+1)(\beta+2)}[(\beta+1) - 2\eta]\,T^{\beta+1} \\ &- \frac{\alpha\eta^{\beta-1}\lambda(s)}{(\beta+1)}[C\beta\eta - h\beta\gamma(\beta - 2\eta + 1) + \eta(C + 2h\gamma)]\,T^{\beta} \\ &+ \alpha\gamma\lambda(s)\,\eta^{\beta-2}\left[(\beta-1)\left(C\eta - \frac{h\gamma(\beta-2\eta)}{2}\right).\right. \\ &\left. + \eta(C + h\gamma\eta)\right] T^{\beta-1} + \ldots \\ &+ \frac{\alpha\beta(-\gamma)^{\beta-2}\lambda(s)\,\eta}{6}[2C(\beta-1)\,\eta - 2h\gamma(3 - 2\eta) \\ &+ \eta C(\beta-1) + 2h\eta\gamma] - \lambda(s)\,\eta[h + C\alpha\beta(-\gamma)^{\beta-1}]\,T \\ &+ \lambda(s)\,\pi(1-\eta)\,T = 0. \qquad \ldots(3.26)\end{aligned}$$

Dividing (3.26) by $\pi$ throughout and taking limits as $\pi \to \infty$, $\eta \to 1$ then $T = T_1$, the model is reduced to one without shortages.

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