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ON (F, g, r, S)-STRUCTURE IN GENERAL RELATIVITY

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The projective, conformal, conharmonic and concircular curvature tensors for (F, g, r, S)-structure in general relativity have been studied in the index-free notation. The corresponding Nijenhuis tensor for (F, g, r, S)-structure has also been given and it is shown that for conharmonically- and concircularly-flat space-times endowed with an (F, g, r, S)-structure the divergence of the Nijenhuis tensor vanishes.

1. INTRODUCTION

Recently, Mishra (1976) has studied the electromagnetic fields in an invariant (index-free) manner by classifying the (1, 1) tensor field F into four classes according to its eigenvalues and has worked out various interesting geomertic properties for the different classes.

In this paper, a study of the projective, conformal, conharmonic and concircular curvature tensors (in index-free notation) have been made for an (F, g, r, S)-structure of null kind (or a null electromagnetic field). The Nijenhuis tensor for (F, g, r, S)-structure has also been given and it is seen that for conharmonically- and concircularly-flat space-times endowed with an (F, g, r, S)-structure, the divergence of the Nijenhuis tensor vanishes.

We shall first develop an (F, g, r, S)-structure of null kind on a V_4.

Let V_4 be the space-time of general relativity with the gravitational symmetric tensor field g. Let F be a tensor field of type (1, 1) such that the corresponding (0, 2) type antisymmetric tensor field $'F$ is given by (Mishra 1976)

$$'F(X, Y) \overset{def}{=} g(F(X), Y) \qquad ...(1.1)$$

and let

$$4K \overset{def}{=} -C_1^1 F(F) \qquad ...(1.2)$$

where C_s^r is the operation of contraction with respect to the rth contravariant and sth covariant slot. The stress tensor $\mathcal{F}$ of type (1, 1) associated with the given Maxwell field F is then given by

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$$\mathcal{F}(X) = \bar{\bar{X}} + KX, \quad \text{where} \quad \bar{X} = F(X). \qquad \ldots(1.3)$$

The characteristic equation for F is given by

$$F^4 + 2KF^2 + |F| I_4 = 0 \qquad \ldots(1.4)$$

where $F^p = F^{(p-1)}(F)$, $F^0 = I_4$. Then eigenvalues for F are

$$\alpha_1 = -\alpha_2 = i(\sqrt{D} + K)^{1/2} = M_i \sin \sigma,$$

$$\alpha_3 = -\alpha_4 = (\sqrt{D} - K)^{1/2} = M \cos \sigma,$$

where $D \overset{def}{=} K^2 - |F|$ and $M^2 = \sqrt{D}$.

The electromagnetic field is said to be of

First class if $K |F| \neq 0$,

Second class if $|F| = 0, K \neq 0$,

Third class if $|F| = 0, K = 0, \bar{\bar{X}} \neq 0$,

Fourth class if $\bar{\bar{X}} = 0$.

An electromagnetic field of third class is the null eletromagnetic field.

For the third class, all the four eigenvalues of F vanish. Let S be an eigenvector of F. It has been shown by Hlavaty (1958) and Mishra (1976) that we can then choose a set of linearly independent null vectors $\{P, Q, R, S\}$, P, Q being the complex conjugate, and the set $\{p, q, r, s\}$ of 1-forms dual to $\{P, Q, R, S\}$ such that

$$X = p(X)\, P + q(X)\, Q + r(X)\, R + s(X)\, S \qquad \ldots(1.5)$$

and

$$F(X) = \bar{X} = r(X)\, T + t(X)\, S \qquad \ldots(1.6)$$

where $t = (p + q)/\sqrt{2}$ and $T = (P + Q)/\sqrt{2}$, the vectors T and S are called the polarization and propagation vectors, respectively. In this case, without any loss of generality, we can assume that

$$g(X, Y) = (p \otimes q + q \otimes p - r \otimes s - s \otimes r)\,(X, Y). \qquad \ldots(1.7)$$

It may easily be seen that the following relations are valid for the third class (cf. Mishra 1976).

$$F(P) = \frac{1}{\sqrt{2}} S, \quad F^2(P) = F(S) = 0,$$

$$F(Q) = \frac{1}{\sqrt{2}} S, \quad F^2(Q) = 0,$$

$$F(R) = T, \quad F^2(R) = \frac{1}{\sqrt{2}} S, \quad F^2(S) = 0,$$

$$F(T) = \frac{1}{\sqrt{2}} S, \quad F^2(T) = 0,$$

$$F^3(P) = F^3(Q) = F^3(R) = F^3(T) = 0$$

$$r(X) = \sqrt{2}\ p(\bar{X}) = \sqrt{2}\ q(\bar{X}), \quad r(\bar{X}) = \bar{S} = 0,$$

$$t(\bar{X}) = r(X), \quad t(\bar{\bar{X}}) = r(\bar{X}) = 0,$$

$$F^2(X) = \bar{\bar{X}} = r(X)\, S, \quad r(\bar{X}) = s(\bar{\bar{X}}) = 0,$$

$$p(\bar{\bar{X}}) = q(\bar{\bar{X}}) = 0, \quad g(S, X) = -r(X), \quad g(R, X) = -s(X) \qquad \text{...(1.8a)}$$

and

$$g(\bar{X}, Y) + g(X, \bar{Y}) = 0,$$

$$g(\bar{X}, \bar{Y}) = -\, g(\bar{\bar{X}}, Y) = -g(X, \bar{\bar{Y}}),$$

$$g(\bar{\bar{X}}, \bar{Y}) = g(\bar{X}, \bar{\bar{Y}}) = 0, \ g(\bar{\bar{X}}, \bar{\bar{Y}}) = 0. \qquad \text{...(1.8b)}$$

From eqns. (1.6) and (1.8*a*, *b*) it may be shown that

$$F^3 = 0. \qquad \text{...(1.9)}$$

Definition 1.1 — A space-time V_4 satisfying (1.5) - (1.9) is said to be endowed with an (F, g, r, S)-structure of null kind.

For (F, g, r, S)-structure of null kind*, it has been shown by Ahsan and Husain 1980) that

$$'F(X, Y) = r(X)\, t(Y) - t(X)\, r(Y) \qquad \text{...(1.10)}$$

and the Ricci tensor is related by the relations that

$$\text{Ric}\,(X, Y) = -\, r(X)\, r(Y) \qquad \text{...(1.11)}$$

$$\text{Ric}\,(\bar{X}, Y) = \text{Ric}\,(X, \bar{Y}) = \text{Ric}\,(\bar{X}, \bar{Y}) = 0 \qquad \text{...(1.12)}$$

and

$$R(\bar{X}) = 0. \qquad \text{...(1.13)}$$

2. Curvature Tensor for (F, g, r, S)-Structure

The projective curvature rensor $W(X, Y, Z)$ for a V_4 is defined by (Mishra 1965)

$$W(X, Y, Z) = R(X, Y, Z) - \tfrac{1}{3}\, [X \,\text{Ric}\,(Y, Z) - Y \,\text{Ric}\,(X, Z)] \qquad \text{...(2.1)}$$

where $R(X, Y, Z)$ is the Riemanian curvature tensor.

*For the sake of brevity, we only metion hence onward (F, g, r, S)-structure.

For projectively flat space-time, we have

$$R(X, Y, Z) = \tfrac{1}{3}\,[X \operatorname{Ric}(Y, Z) - Y \operatorname{Ric}(X, Z)]. \qquad \ldots(2.2)$$

Remarks: (a) For a space-time V_4 to be of constant curvature it is necessary and sufficient that it is projectively-flat.

(b) For projectively-flat empty space-time, there is no gravitational radiation.

We now state the following theorems. The proofs can easily be obtained by using (1.5) – (1.13).

Theorem 2.1 — For a space-time V_4 endowed with an (F, g, r, S)-structure, we have

(a) $W(\bar{X}, \bar{Y}, Z) = R(\bar{X}, \bar{Y}, Z)$

(b) $W(X, \bar{Y}, \bar{Z}) = R(X, \bar{Y}, \bar{Z})$

(c) $W(X, Y, \bar{Z}) = R(X, Y, \bar{Z})$

(d) $$W(\bar{X}, Y, Z) + W(X, \bar{Y}, Z) + W(X, Y, \bar{Z})$$
$$= R(\bar{X}, Y, Z) + R(X, \bar{Y}, Z) + R(X, Y, \bar{Z})$$
$$- \tfrac{1}{3}\,[\bar{X} \operatorname{Ric}(Y, Z) - \bar{Y} \operatorname{Ric}(X, Z)]$$

(e) $$W(\bar{X}, \bar{Y}, Z) + W(X, \bar{Y}, \bar{Z}) + W(\bar{X}, Y, \bar{Z})$$
$$= R(\bar{X}, \bar{Y}, Z) + R(X, \bar{Y}, \bar{Z}) + R(\bar{X}, Y, \bar{Z}).$$

Theorem 2.2 — For an empty space-time V_4 endowed with an (F, g, r, S)-stucture, we have

(a) $W(\bar{X}, \bar{Y}, Z) = R(\bar{X}, \bar{Y}, Z)$,

(b) $W(X, \bar{Y}, \bar{Z}) = R(X, \bar{Y}, \bar{Z})$,

(c) $W(X, Y, \bar{Z}) = R(X, Y, \bar{Z})$,

(d) $$W(\bar{X}, Y, Z) + W(X, \bar{Y}, Z) + W(X, Y, \bar{Z})$$
$$= R(\bar{X}, Y, Z) + R(X, \bar{Y}, Z) + R(X, Y, \bar{Z}).$$

(e) $$W(\bar{X}, \bar{Y}, Z) + W(X, \bar{Y}, \bar{Z}) + W(\bar{X}, Y, \bar{Z})$$
$$= R(\bar{X}, \bar{Y}, Z) + R(X, \bar{Y}, \bar{Z}) + R(\bar{X}, Y, \bar{Z}).$$

Theorem 2.3 — For a projectively-flat empty space-time V_4 endowed with an (F, g, r, S)-structure, we have

(a) $R(\bar{X}, \bar{Y}, Z) = R(X, \bar{Y}, \bar{Z}) = R(\bar{X}, Y, \bar{Z}) = R(X, Y, \bar{Z}) = 0,$

(b) $R(\bar{X}, Y, Z) + R(X, \bar{Y}, Z) = 0.$

The conformal curvature tensor $C(X, Y, Z)$ for a V_4 is defined by (Mishra 1965)

$$C(X, Y, Z) = R(X, Y, Z) - \tfrac{1}{2}\,[X \operatorname{Ric}(Y, Z) - Y \operatorname{Ric}(X, Z) - g(X, Z)\, R(Y) + g(Y, Z)\, R(X)] + \frac{\mathrm{R}}{6}\,[Xg(Y, Z) - Yg(X, Z)]. \quad ...(2.3)$$

For conformally-flat space-time, we have

$$R(X, Y, Z) = \tfrac{1}{2}\,[X \operatorname{Ric}(Y, Z) - Y \operatorname{Ric}(X, Z) - g(X, Z)\, R(Y) + g(Y, Z)\, R(X)] + \frac{\mathrm{R}}{6}\,[Xg(Y, Z) - Yg(X, Z)]. \quad ...(2.4)$$

Remark : From (2.3) it may be noted that for conformally-flat empty space-time gravitational field is absent.

We state the following theorems:

Theorem 2.4 — For a space-time V_4 endowed with an (F, g, r, S)-structure, we have

(a) $C(\bar{X}, \bar{Y}, Z) = R(\bar{X}, \bar{Y}, Z)$, (b) $C(\bar{X}, \bar{Y}, \bar{Z}) = R(\bar{X}, \bar{Y}, \bar{Z})$,

(c) $C(\bar{X}, Y, Z) + C(X, \bar{Y}, Z) + C(X, Y, \bar{Z})$

$$= R(\bar{X}, Y, Z) + R(X, \bar{Y}, Z) + R(X, Y, \bar{Z}) + \tfrac{1}{2}\,[\bar{Y} \operatorname{Ric}(X, Z) - \bar{X} \operatorname{Ric}(Y, Z)].$$

Theorem 2.5 — For an empty space-time V_4 endowed with an (F, g, r, s)-structure, we have

(a) $C(\bar{X}, \bar{Y}, Z) = R(\bar{X}, \bar{Y}, Z)$, (b) $C(\bar{X}, \bar{Y}, \bar{Z}) = R(\bar{X}, \bar{Y}, \bar{Z})$,

(c) $C(\bar{X}, Y, Z) + C(X, \bar{Y}, Z) + C(X, Y, \bar{Z})$

$$= R(\bar{X}, Y, Z) + R(X, \bar{Y}, Z) + R(X, Y, \bar{Z}).$$

Theorem 2.6 — For a conformally-flat matter-free space-time V_4 endowed with an (F, g, r, S)-structure, we have

(a) $R(\bar{X}, \bar{Y}, Z) = R(\bar{X}, \bar{Y}, \bar{Z}) = 0,$

(b) $R(\bar{X}, Y, Z) + R(X, \bar{Y}, Z) + R(X, Y, \bar{Z}) = 0.$

Proofs of these theorems follow from (1.5) - (1.13).

The projective and conformal curvature tensors are related by (Mishra and Pokhariyal 1971)

$$W(X, Y, Z) = C(X, Y, Z) + \tfrac{1}{6}\,[X \,\mathrm{Ric}\,(Y, Z) - Y \,\mathrm{Ric}\,(X, Z)] - \tfrac{1}{2}\,[g(X, Z)\, R(Y) - g(Y, Z)\, R(X)]. - \frac{\mathrm{R}}{6}\,[Xg(Y, Z) - Yg(X, Z)]. \qquad \ldots(2.5)$$

It may be noted from eqn. (2.5) that if a matter-free space-time V_4 is conformally-flat then it is necessarily projectively-flat and conversely.

We now state the following theorems.

Theorem 2.7 — For a space-time V_4 endowed with an (F, g, r, S)-structure, we have

(a) $W(\bar{X}, \bar{Y}, \bar{\bar{Z}}) = C(\bar{X}, \bar{Y}, \bar{\bar{Z}})$, (b) $W(\bar{\bar{X}}, \bar{Y}, \bar{Z}) = C(\bar{\bar{X}}, \bar{Y}, \bar{Z})$,

(c) $W(\bar{X}, \bar{\bar{Y}}, \bar{Z}) = C(\bar{X}, \bar{\bar{Y}}, \bar{Z})$, (d) $W(\bar{\bar{X}}, \bar{Y}, \bar{\bar{Z}}) = C(\bar{\bar{X}}, \bar{Y}, \bar{\bar{Z}})$,

(e) $W(\bar{\bar{X}}, \bar{\bar{Y}}, \bar{Z}) = C(\bar{\bar{X}}, \bar{\bar{Y}}, \bar{Z})$, (f) $W(\bar{X}, \bar{\bar{Y}}, \bar{\bar{Z}}) = C(\bar{X}, \bar{\bar{Y}}, \bar{\bar{Z}})$.

Theorem 2.8 — For a conformally-flat space-time V_4 endowed with an (F, g, r, S)-structure, we have

(a) $W(\bar{X}, Y, Z) + W(X, \bar{Y}, Z) + W(X, Y, \bar{Z}) = \tfrac{1}{6}\,[\bar{X} \,\mathrm{Ric}\,(Y, Z) - \bar{Y} \,\mathrm{Ric}\,(X, Z)]$,

(b) $W(\bar{\bar{\bar{X}}}, Y, Z) + W(X, \bar{\bar{\bar{Y}}}, Z) + W(X, Y, \bar{\bar{\bar{Z}}}) = 0$,

(c) $W(\bar{X}, \bar{Y}, \bar{Z}) + W(\bar{\bar{X}}, \bar{\bar{Y}}, \bar{\bar{Z}}) = 0$,

(d) $W(\bar{X}, \bar{Y}, \bar{\bar{Z}}) = W(\bar{\bar{X}}, \bar{Y}, \bar{Z}) = W(\bar{X}, \bar{\bar{Y}}, \bar{Z}) = W(\bar{\bar{X}}, \bar{Y}, \bar{\bar{Z}}) = W(\bar{X}, \bar{\bar{Y}}, \bar{\bar{Z}}) = W(\bar{\bar{X}}, \bar{\bar{Y}}, \bar{Z}) = 0.$

Theorem 2.9 — For a projectively-flat space-time V_4 endowed with an (F, g, r, S)-structure, we have

$$C(\bar{\bar{X}}, \bar{Y}, \bar{Z}) = C(\bar{X}, \bar{\bar{Y}}, \bar{Z}) = C(\bar{X}, \bar{Y}, \bar{\bar{Z}}) = C(\bar{\bar{X}}, \bar{Y}, \bar{\bar{Z}}) = C(\bar{X}, \bar{\bar{Y}}, \bar{\bar{Z}}) = C(\bar{\bar{X}}, \bar{\bar{Y}}, \bar{Z}) = 0.$$

Proofs of these theorems can easily be obtained by using (1.5) - (1.13).

The conharmonic curvature tensor $L(X, Y, Z)$ for a V_4 is defined by (Mishra 1970)

$$L(X, Y, Z) = R(X, Y, Z) - \tfrac{1}{2}[g(Y, Z)\, R(X) - g(X, Z)\, R(Y) + X \operatorname{Ric}(Y, Z) - Y \operatorname{Ric}(X, Z)]. \quad ...(2.6)$$

For conharmonically-flat space-time V_4, we have

$$R(X, Y, Z) = \tfrac{1}{2}[X \operatorname{Ric}(Y, Z) - Y \operatorname{Ric}(X, Z) + g(Y, Z)\, R(X) - g(X, Z)\, R(Y)]. \quad ...(2.7)$$

The concircular curvature tensor $M(X, Y, Z)$ for a V_4 is defined by (Mishra 1970)

$$M(X, Y, Z) = R(X, Y, Z) - \frac{\mathbf{R}}{12}[Xg(Y, Z) - Yg(X, Z)]. \quad ...(2.8)$$

For concircularly-flat space-time, we have

$$R(X, Y, Z) = \frac{\mathbf{R}}{12}[Xg(Y, Z) - Yg(X, Z)]. \quad ...(2.9)$$

Remarks : (a) For an empty conharmonically (or concircularly) flat space-time the gravitational field is absent.

(b) $M(X, Y, Z) = R(X, Y, Z)$, for a space-time V_4 endowed with an (F, g, r, S)-structure.

(c) $R(X, Y, Z) = 0$, for a concircularly-flat space-time V_4 endowed with an (F, g, r, S)-structure.

We state the following theorems. The proofs can easily be obtained using (1.5) - (1.13).

Theorem 2.10 — For a space-time V_4 endowed with an (F, g, r, S)-structure, we have

(a) $L(\bar{X}, \bar{Y}, Z) = R(\bar{X}, \bar{Y}, Z)$

(b) $L(X, \bar{Y}, \bar{Z}) = R(X, \bar{Y}, \bar{Z}) - \tfrac{1}{2}[g(\bar{Y}, \bar{Z})\, R(X)]$

(c) $L(\bar{X}, Y, \bar{Z}) = R(\bar{X}, Y, \bar{Z}) - \tfrac{1}{2}[g(\bar{X}, \bar{Z})\, R(Y)]$

(d) $L(X, \bar{\bar{Y}}, \bar{\bar{Z}}) = R(X, \bar{\bar{Y}}, \bar{\bar{Z}})$

(e) $L(X, Y, \bar{\bar{Z}}) = R(X, Y, \bar{\bar{Z}}) - \tfrac{1}{2}[g(Y, \bar{\bar{Z}})\, R(X) - g(X, \bar{\bar{Z}})\, R(Y)]$

(f) $L(\bar{X}, Y, \bar{\bar{Z}}) = R(\bar{X}, Y, \bar{\bar{Z}})$

(g) $L(\bar{X}, Y, Z) + L(X, \bar{Y}, Z) + L(X, Y, \bar{Z})$

$$= R(\bar{X}, Y, Z) + R(X, \bar{Y}, Z) + R(X, Y, \bar{Z}) - \tfrac{1}{2}[\bar{X} \operatorname{Ric}(Y, Z) - \bar{Y} \operatorname{Ric}(X, Z)].$$

Theorem 2.11 — For an empty space-time V_4 endowed with an (F, g, r, S)-structure, we have

(a) $L(\bar{X}, \bar{Y}, Z) = R(\bar{X}, \bar{Y}, Z)$, (b) $L(X, \bar{Y}, \bar{Z}) = R(X, \bar{Y}, \bar{Z})$,

(c) $L(\bar{X}, Y, \bar{\bar{Z}}) = R(\bar{X}, Y, \bar{\bar{Z}})$, (d) $L(X, Y, \bar{\bar{\bar{Z}}}) = R(X, Y, \bar{\bar{\bar{Z}}})$,

(e) $L(\bar{X}, Y, \bar{\bar{\bar{Z}}}) = R(\bar{X}, Y, \bar{\bar{\bar{Z}}})$, (f) $L(\bar{X}, Y, Z) + L(X, \bar{Y}, Z) + L(X, Y, \bar{Z})$

$$= R(\bar{X}, Y, Z) + R(X, \bar{Y}, Z) + R(X, Y, \bar{Z}).$$

Theorem 2.12 — For a conharmonically-flat space-time V_4 endowed with an (F, g, r, S)-structure, we have

(a) $$R(\bar{X}, \bar{Y}, Z) = R(X, \bar{Y}, \bar{Z}) = R(\bar{X}, Y, \bar{\bar{Z}}) = R(X, Y, \bar{\bar{\bar{Z}}}) = R(\bar{X}, Y, \bar{\bar{\bar{Z}}}) = 0$$

(b) $$R(\bar{X}, Y, Z) + R(X, \bar{Y}, Z) + R(X, Y, \bar{Z}) = 0.$$

3. Nijenhuis tensor for (F, g, r, S)-structure

The Nijenhuis tensor $N(X, Y, Z)$ for the (F, g, r, S)-structure is defined by Ahsan and Husain (1980)

$$N(X, Y, Z) = -2(\nabla_Z' F)(X, Y). \quad ...(3.1)$$

Recently, it has been shown by Ahsan (1978) that for (F, g, r, S)-structure

$$\text{div}\, N(X, Y) = R(\bar{X}, Y, \bar{Z}, Z) + R(X, \bar{Y}, \bar{Z}, Z) \quad ...(3.2)$$

where

$$R(X, Y, Z, W) \overset{def}{=} g(R(X, Y, Z), W).$$

It was also shown (Ahsan 1978) that for projectively- and conformally-flat (F, g, r, S)-structure, the divergence of the Nijenhuis tensor vanishes. Here we shall prove the following theorems.

Theorem 3.1 — For a conharmonically-flat space-time V_4 endowed with an (F, g, r, S)-structure

$$\text{div}\, N(X, Y) = 0.$$

Theorem 3.2 — For a concircularly-flat space-time V_4 endowed with an (F, g, r, S)-structure

$$\text{div}\, N(X, Y) = 0.$$

PROOF OF THEOREM 3.1 : From (2.7), we have

$$R(X, Y, Z, W) = \tfrac{1}{2}\,[g(Y, Z)\,\text{Ric}\,(X, W) - g(X, Z)\,\text{Ric}\,(Y, W) + g(X, W)\,\text{Ric}\,(Y, Z) - g(Y, W)\,\text{Ric}\,(X, Z)]. \quad \ldots(3.3)$$

For (*F*, *g*, *r*, *S*)-structure, we also have (cf. Ahsan 1978).

$$\text{Ric}\,(X, Y) = 2'F(X, \bar{Y}). \quad \ldots(3.4)$$

Thus (3.3) reduces to

$$R(X, Y, Z, W) = g(Y, Z)\,'F(X, \bar{W}) - g(X, Z)\,'F(Y, \bar{W}) + g(X, W)\,'F(Y, \bar{Z}) - g(Y, W)\,'F(X, \bar{Z}). \quad \ldots(3.5)$$

Now, using (1.7), (1.8), (1.10) and (3.5) in (3.2), we get

$$\text{div}\, N(X, Y) = 0.$$

Hence the result.

PROOF OF THEOREM 3.2 : Since for a concircularly-flat space-time V_4 endowed with an (*F*, *g*, *r*, *S*)-structure, $R(X, Y, Z, W) = 0$ and therefore, from (3.2) the result follows.

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