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AN ANISOTROPIC MAGNETOHYDRODYNAMIC UNIVERSE IN GENERAL RELATIVITY

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Considering the cylindrically symmetric metric of Marder, we have constructed a non-static cylindrically symmetric cosmological model which is spatially homogeneous nondegenerate Petrov type I. The energy momentum tensor has been assumed to be that of a perfect fluid with an electromagnetic field and the 4-current is either zero or space-like. The model represents an expanding and shearing but non-rotating fluid flow which is also geodetic. The requirement of positive conductivity for a physically realistic model imposes an additional restriction on the metric potentials. Various physical and geometrical properties of the model have been discussed.

1. INTRODUCTION

In recent years there has been a lot of interest in magnetohydrodynamic cosmologies in general relativity. Cosmological models in the presence of a magnetic field have been studied by Zel'dovich (1965) and Thorne (1967). Galaxies and interstellar spaces exhibit the presence of strong magnetic fields (Zel'dovich and Novikov 1971). Monoghan (1966) and Seymour (1966) have discussed the magnetic field in stellar bodies and Ginzburg (1965) has studied the gravitational collapse of the magnetic star.

Jacobs (1967) has studied the behaviour of the general Bianchi type I cosmological model in the presence of the spatially homogeneous magnetic field. This problem has been studied again by De (1975) with a different approach. This work has been further extended by Tupper (1977a) to include Einstein-Maxwell fields in which the electric field is non-zero. He has also interpreted certain type VI cosmologies with electromagnetic field (Tupper 1977b).

Recently Roy and Prakash (1978) taking the cylindrically symmetric metric of Marder (1958) have constructed a spatially homogeneous cosmological model in the presence of an incident magnetic field which is also anisotropic and nondegenerate Petrov type I. In this paper the energy momentum tensor has been assumed to be that of a perfect fluid with an electromagnetic field and a spatially homogeneous cosmological model has been obtained. It is found that the model represents an expanding and shearing but non-rotating fluid flow which is also geodetic. We have also shown that the model has a 4-current which is either zero or space-like. The latter corresponds to the case of magnetohydrodynamics (MHD). The requirement that the conductivity be positive imposes an additional restriction on the metric potentials. It is found that the electromagnetic field gives positive contributions to the expansion, shear and free gravitational field which die out for large values of time at a slower rate than the corresponding quantities in the absence of the electromagnetic field. When the cosmological constant $\Lambda = 0$, it is found that in the absence of electromagnetic field pressure and density become equal and conversely if pressure and density are equal (stiff matter) there is no electromagnetic field.

2. Solution of the Field Equations

We consider here the cylindrically symmetric metric in the form given by Marder (1958)

$$ds^2 = A^2(dt^2 - dx^2) - B^2dy^2 - C^2dz^2 \qquad \text{...(2.1)}$$

where A, B, C are functions of t only. The distribution consists of a perfect fluid and an electromagnetic field. Thus

$$G_{ij} + \Lambda g_{ij} = -K\,[(\rho + p)\,\lambda_i\lambda_j - pg_{ij} + E_{ij}] \qquad \text{...(2.2)}$$

$$g_{ij}\lambda^i\lambda^j = 1 \qquad \text{...(2.3)}$$

$$E_{ij} = g^{\alpha\beta}F_{i\alpha}F_{j\beta} - \tfrac{1}{4}g_{ij}F_{\alpha\beta}F^{\alpha\beta} \qquad \text{...(2.4)}$$

$$F_{[ij,k]} = 0 \qquad \text{...(2.5)}$$

$$F^{ij}_{\ ;j} = J^i \qquad \text{...(2.6)}$$

where E_{ij} is the electromagnetic energy-momentum tensor, F_{ij} the electromagnetic field tensor, Λ cosmological constant, J^i the current 4-vector and ρ and p are the density and pressure of the distribution. The coordinates are chosen to be comoving so that

$$\lambda^1 = \lambda^2 = \lambda^3 = 0,\ \lambda^4 = \frac{1}{A}\cdot \qquad \text{...(2.7)}$$

We label the coordinates $(x, y, z, t) = (x^1, x^2, x^3, x^4)$.

The off-diagonal components of (2.2) are

$$\left.\begin{array}{ll}
\text{(a)} & F_{12}F_{24}B^{-2} + F_{13}F_{34}C^{-2} = 0 \\
\text{(b)} & F_{12}F_{14}A^{-2} - F_{23}F_{34}C^{-2} = 0 \\
\text{(c)} & F_{13}F_{14}A^{-2} + F_{23}F_{24}B^{-2} = 0 \\
\text{(d)} & F_{14}F_{24}A^{-2} - F_{13}F_{23}C^{-2} = 0 \\
\text{(e)} & F_{14}F_{34}A^{-2} + F_{12}F_{23}B^{-2} = 0 \\
\text{(f)} & F_{24}F_{34} - F_{12}F_{13} = 0
\end{array}\right\} \qquad \text{...(2.8)}$$

which lead to three possible cases :

(i) $F_{24} = F_{34} = F_{12} = F_{13} = 0$ at least one of F_{14}, F_{23} non-zero i.e. when the field F_{ij} is in x-direction only.

(ii) $F_{14} = F_{34} = F_{12} = F_{23} = 0$ at least one of F_{24}, F_{13} non-zero i.e. when the field is in y-direction only.

(iii) $F_{14} = F_{24} = F_{13} = F_{23} = 0$ at least one of F_{34}, F_{12} non-zero i.e. when the field is in z-direction only.

Hence the electromagnetic field is non-null and consists of an electric and/or magnetic field both of which are in the direction of same space axis. Without loss of generality we may consider only case (i) in which the fields are in the x-direction. We write

$$F_{14}^2 A^{-4} + F_{23}^2 B^{-2}C^{-2} = L^2. \qquad \text{...(2.9)}$$

The diagoal components of the eqn. (2.2) may be written as

$$\frac{2}{A^2}\left[\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4C_4}{AC} - \frac{A_4B_4}{AB} - \frac{A_4^2}{A^2}\right] - 2\Lambda$$
$$= -K[-L^2 + (\rho + 3p)] \qquad \text{...(2.10)}$$

$$-\frac{2}{A^2}\left[\frac{A_{44}}{A} + \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} - \frac{A_4^2}{A^2}\right] + 2\Lambda = -K[L^2 + (\rho - p)] \qquad \text{...(2.11)}$$

$$-\frac{2}{A^2}\left[\frac{B_{44}}{B} + \frac{B_4C_4}{BC}\right] + 2\Lambda = -K[-L^2 + (\rho - p)] \qquad \text{...(2.12)}$$

$$-\frac{2}{A^2}\left[\frac{C_{44}}{C} + \frac{B_4C_4}{BC}\right] + 2\Lambda = -K[-L^2 + (\rho - p)] \qquad \text{...(2.13)}$$

where the suffix 4 after the symbols A, B, C stands for ordinary differentiation with respect to time. It is evident from these equations that L^2, ρ and p are each functions of time alone. From eqns. (2.5) and (2.9) it follows that F_{23} is a constant and F_{14} is a function of t only i.e.

$$F_{23} = k, \; F_{14} = \pm A^2(L^2 - k^2B^{-2}C^{-2})^{1/2} \qquad \text{...(2.14)}$$

where k is a constant.

The case when there is no electric field i.e. when $F_{14} = 0$, we have $J^i = 0$. It is the case considered by Roy and Prakash (1978). Here we assume that $F_{14} \neq 0$ and find the only non-zero component of J^i to be

$$J^1 = \pm \frac{1}{A^2BC}\frac{\partial}{\partial t}[BC(L^2 - k^2B^{-2}C^{-2})^{1/2}]. \qquad \text{...(2.15)}$$

Equation (2.15) shows that J^i is space-like, unless $L^2 = fB^{-2}C^{-2}$ where f is a constant in which case $J^i = 0$. The 4-current J^i is in general the sum of the convection current and the conduction current (Lichnerowicz 1967 and Greenberg 1971) :

$$J^i = \epsilon_0 \lambda^i + \zeta \lambda_j F^{ij} \qquad ...(2.16)$$

where ϵ_0 is the rest charge density and ζ is the conductivity. In the case considered here we have $\epsilon_0 = 0$ i.e. magnetohydrodynamics. Thus

$$\zeta = -\frac{1}{A} I_4 I^{-1}, \qquad ...(2.17)$$

where

$$I = BC(L^2 - k^2B^{-2}C^{-2})^{1/2}.$$

The requirement of positive conductivity in (2.17) puts further restrictions on A, B, C. Hence in magnetohydrodynamic case metric potentials are restricted not only by the field equations and energy conditions (Hawking and Penrose 1973) they are also restricted by the requirement that the conductivity be positive for a realistic model.

Equations (2.10) - (2.13) are four equations in six unknowns A, B, C, ρ, p and L. For complete determinacy of this system of equations, we make two assumptions viz.,

(i) F_{14} is such that

$$L^2 = l^2 B^{-4} C^{-4} \qquad ...(2.18)$$

where l is a constant.

(ii) The space time is Petrov type I degenerate (the degeneracy being in y and z directions) which requires that

$$C^{12}_{12} = C^{13}_{13} \quad \text{with } B \neq C. \qquad ...(2.19)$$

Thus from (2.19) we have

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{2A_4}{A}\left(\frac{C_4}{C} - \frac{B_4}{B}\right) = 0. \qquad ...(2.20)$$

Equations (2.12) and (2.13) yield

$$\frac{B_{44}}{B} = \frac{C_{44}}{C} \qquad ...(2.21)$$

which on integration gives

$$B_4 C - BC_4 = k_1 \qquad ...(2.22)$$

k_1 being an arbitrary constant.

Further form (2.20) and (2.21) we get

$$\frac{A_4}{A}\left(\frac{C_4}{C}-\frac{B_4}{B}\right)=0. \qquad ...(2.23)$$

Since $B \neq C$, eqn. (2.23) gives

$$A = N \text{(a constant)}. \qquad ...(2.24)$$

From eqns. (2.11), (2.12) and (2.24) we have

$$\frac{B_{44}}{B}+\frac{B_4C_4}{BC}=-\,KN^2L^2. \qquad ...(2.25)$$

Putting $B/C = \alpha$ and $BC = \beta$, eqn. (2.22) reduces to

$$\left(\frac{\alpha_4}{\alpha}\right)\beta = k_1 \qquad ...(2.26)$$

and eqn. (2.25) turns into

$$\frac{1}{\beta}\left[\left(\frac{\alpha_4}{\alpha}+\frac{\beta_4}{\beta}\right)\beta\right]_4=-2KN^2L^2. \qquad ...(2.27)$$

From eqns. (2.26) and (2.27) we have

$$\frac{\beta_{44}}{\beta}=-2KN^2L^2 \qquad ...(2.28)$$

which after the use of eqn. (2.18) goes to the from

$$\beta_{44}=-\frac{2KN^2l^2}{\beta^3}. \qquad ...(2.29)$$

Equation (2.29) on integration, gives

$$[\beta_4]^2=\frac{2KN^2l^2}{\beta^2}+k_2^2 \qquad ...(2.30)$$

where k_2^2 is an arbitrary constant.

From eqns. (2.26) and (2.30) we get

$$\frac{d\alpha}{\alpha}=\frac{k_1}{k_2}\frac{d\beta}{(\beta^2+k_3^2)^{1/2}} \qquad ...(2.31)$$

where

$$k_3^2=\frac{2KN^2l^2}{k_2^2}. \qquad ...(2.32)$$

Integration of eqn. (2.31) gives

$$\alpha = k_4\,[\beta+(\beta^2+k_3^2)^{1/2}]^{k_1/k_2} \qquad ...(2.33)$$

k_4 being a constant of integration. Therefore

$$B^2 = k_4\beta\,[\beta + (\beta^2 + k_3^2)^{1/2}]^{k_1/k_2} \qquad ...(2.34)$$

and

$$C^2 = \frac{\beta}{k_4}\,[\beta + (\beta^2 + k_3^2)^{1/2}]^{-k_1/k_2}. \qquad ...(2.35)$$

Hence the metric (2.1) can be written as

$$ds^2 = A^2\left[\frac{d\beta^2}{(d\beta/dt)^2} - dx^2\right] - B^2dy^2 - C^2dz^2 \qquad ...(2.36)$$

which by use of eqns. (2.24), (2.30), (2.34) and (2.35) takes the form

$$ds^2 = N^2\left[-dx^2 + \frac{d\beta^2}{(k_2^2/\beta^2)(\beta^2 + k_3^2)}\right] - k_4\beta\,[\beta + (\beta^2 + k_3^2)^{1/2}]^{k_1/k_2}\,dy^2 - \frac{\beta}{k_4}\,[\beta + (\beta^2 + k_3^2)^{1/2}]^{-k_1/k_2}\,dz^2. \qquad ...(2.37)$$

The transformation

$$Nx \to X,\ k_4y \to Y,\ k_4^{-1}z \to Z,\ \beta \to \sqrt{(T^2 - k_3^2)} \qquad ...(2.38)$$

reduces (2.37) to the form

$$ds^2 = \frac{N^2dT^2}{k_2^2} - dX^2 - (T^2 - k_3^2)^{1/2}\,[T + (T^2 - k_3^2)^{1/2}]^{k_1/k_2}\,dY^2 - (T^2 - k_3^2)^{1/2}\,[T + (T^2 - k_3^2)^{1/2}]^{-k_1/k_2}\,dZ^2 \qquad ...(2.39)$$

which can be further transformed to the metric

$$ds^2 = dT^2 - dX^2 - (T^2 - P^2)^{1/2}\,[T + (T^2 - P^2)^{1/2}]^q\,dY^2 - (T^2 - P^2)^{1/2}\,[T + (T^2 - P^2)^{1/2}]^{-q}\,dZ^2. \qquad ...(2.40)$$

This metric has no singularity and will be real only when $T^2 > P^2$.

3. Some Physical Features

(a) *The Distribution in the Model*

For the model (2.40) pressure p and density ρ are given by

$$Kp = \frac{T^2}{4}(T^2 - P^2)^{-2} - \frac{q^2}{4}(T^2 - P^2)^{-1} + \frac{3Kl^2}{2}(T^2 - P^2)^{-2} + \Lambda \qquad ...(3.1)$$

$$K\rho = \frac{T^2}{4}(T^2 - P^2)^{-2} - \frac{q^2}{4}(T^2 - P^2)^{-1} + \frac{Kl^2}{2}(T^2 - P^2)^{-2} - \Lambda. \quad ...(3.2)$$

The model has to satisfy the reality conditions (Ellis 1971)

(i) $\rho + p > 0$

(ii) $\rho + 3p > 0$

which requires that

$$P^2 < T^2 < \frac{P^2q^2 + 4Kl^2}{(1 - q^2)} \quad ...(3.3)$$

and

$$\Lambda > \frac{-1}{2(T^2 - P^2)^2}[(1 - q^2)\,T^2 + P^2q^2 + 5Kl^2]. \quad ...(3.4)$$

The condition (3.3) holds only when $q^2 < 1$.

In the case of disordered radiation ($\rho = 3p$) we have

$$\Lambda = \frac{-1}{8(T^2 - P^2)^2}[(1 - q^2)\,T^2 + P^2q^2 + 8Kl^2] \quad ...(3.5)$$

and

$$K\rho = 3Kp = \frac{1}{8(T^2 - P^2)^2}[5(1 - q^2)\,T^2 + 5P^2q^2 + 4Kl^2] \quad ...(3.6)$$

and in the case of stiff matter ($\rho = p$)

$$\Lambda = \frac{-Kl^2}{2(T^2 - P^2)^2} \quad ...(3.7)$$

and

$$Kp = K\rho = \left[\frac{T^2}{4}(T^2 - P^2)^{-2} - \frac{q^2}{4}(T^2 - P^2)^{-1}\right] + \frac{Kl^2}{2}(T^2 - P^2)^{-2}. \quad ...(3.8)$$

The flow vector λ^i is given by

$$\lambda^1 = \lambda^2 = \lambda^3 = 0, \; \lambda^4 = 1. \quad ...(3.9)$$

The flow vector λ^i satisfies $\lambda^i_{;j}\lambda^j = 0$. Thus the lines of flow are geodesics. Tensor of rotation W_{ij} defined by

$$W_{ij} = \lambda_{i;j} - \lambda_{j;i} \quad ...(3.10)$$

is identically zero. Thus the fluid filling the universe is non-rotational.

The scalar of expansion $\Theta = \lambda^i_{;i}$ is given by

$$\Theta = \frac{T}{(T^2 - P^2)^{3/2}} \qquad ...(3.11)$$

which tends to zero when $T \to \infty$.

The components of the shear tensor defined by

$$\sigma_{ij} = \tfrac{1}{2}(\lambda_{i;j} + \lambda_{j;i}) - \tfrac{1}{3}\Theta(g_{ij} - \lambda_i\lambda_j) \qquad ...(3.12)$$

are

$$\sigma_{11} = \frac{1}{3T}(T^2 - P^2)^{-3/2},$$

$$\sigma_{22} = [T + (T^2 - P^2)^{1/2}]^q \{-\tfrac{1}{2}[T(T^2 - P^2)^{-1/2} + q] + \tfrac{1}{3}T(T^2 - P^2)^{-1}\},$$

$$\sigma_{33} = [T + (T^2 - P^2)^{1/2}]^{-q} \{-\tfrac{1}{2}[T(T^2 - P^2)^{-1/2} - q] + \tfrac{1}{3}T(T^2 - P^2)^{-1}\},$$

$$\sigma_{44} = 0; \qquad ...(3.13)$$

the other components of σ_{ij} being zero.

The non-vanishing components of the conformal curvature tensor C_{ijhi} are

$$C^{12}_{12} = C^{13}_{13} = -\tfrac{1}{2}C^{23}_{23} = -\tfrac{1}{6}[\tfrac{1}{2}(T^2 - P^2)^{-1/2} - 3T^2(T^2 - P^2)^{-3/2} + \tfrac{1}{2}q^2(T^2 - P^2)^{-1} - \tfrac{1}{4}T^2(T^2 - P^2)^{-2}]. \qquad ...(3.14)$$

The non-vanishing component of the charge current 4-vector is given by

$$J^1 = l^2T(T^2 - P^2)^{-2}[l^2 - k^2(T^2 - P^2)]^{-1/2}. \qquad ...(3\ 15)$$

The conductivity is given by

$$\zeta = l^2T(T^2 - P^2)^{-1}[l^2 - k^2(T^2 - P^2)]^{-1}. \qquad ...(3.16)$$

For a physically realistic MHD model ζ has to be positive which requires that

$$0 < T < (k^2P^2 + l^2)^{1/2}/k.$$

(b) The Doppler Effect in the Model

The track of a light pulse in the model (2.40) is obtained by setting

$$ds^2 = 0 \text{ i.e.}$$

$$\left(\frac{dX}{dT}\right)^2 + (T^2 - P^2)^{1/2}[T + (T^2 - P^2)^{1/2}]^q\left(\frac{dY}{dT}\right)^2 + (T^2 - P^2)^{1/2}[T + (T^2 - P^2)^{1/2}]^{-q}\left(\frac{dZ}{dT}\right)^2 = 1. \qquad ...(3.17)$$

For the case when velocity is along z-axis, eqn. (3.17) gives

$$\frac{dZ}{dT} = \pm (T^2 - P^2)^{-1/4} [T + (T^2 - P^2)^{1/2}]^{q/2}$$
$$= \pm \psi(T). \qquad \ldots(3.18)$$

Hence the light pulse leaving a particle at (0, 0, z) at time T_1 would arrive at a later time T_2 given by

$$\int_{T_1}^{T_2} \psi(T)\, dT = \int_0^{Z} dZ. \qquad \ldots(3.19)$$

Hence

$$\psi_2(T)\, \delta T_2 = \psi_1(T)\, \delta T_1 + \frac{dZ}{dT} \delta T_1$$
$$= \psi_1(T)\, \delta T_1 + U_Z \delta T_1 \qquad \ldots(3.20)$$

where $(dZ/dT) = U_Z$ is the z-component of the velocity of the particle at the time of emission and $\psi_1(T)$ and $\psi_2(T)$ are the values of $\psi(T)$ for $T = T_1$ and $T = T_2$ respectively. From the above equation we get

$$\delta T_2 = \left\{\frac{\psi_1(T) + U_Z}{\psi_2(T)}\right\} \delta T_1. \qquad \ldots(3.21)$$

The proper time interval δT_1^0 between successive wave crests as measured by the local observer moving with the source is given by

$$\delta T_1^0 = \left\{1 - \left(\frac{dX}{dT}\right)^2 - (T^2 - P^2)^{1/2} [T + (T^2 - P^2)^{1/2}]^q \left(\frac{dY}{dT}\right)^2 \right.$$
$$\left. - (T^2 - P^2)^{1/2} [T + (T^2 - P^2)^{1/2}]^{-q} \left(\frac{dZ}{dT}\right)^2\right\}^{1/2} \delta T_1. \qquad \ldots(3.22)$$

This can be written as

$$\delta T_1^0 = \{1 - U^2\}^{1/2}\, \delta T_1 \qquad \ldots(3.23)$$

where U is the velocity of the source at the time of emission. Similarly we may write

$$\delta T_2^0 = \delta T_2 \qquad \ldots(3.24)$$

as the proper time interval between the reception of two successive wave crests by an observer at rest at the origin. Hence following Tolman (1962), the red shift in this case is given by

$$\frac{\lambda + \delta\lambda}{\lambda} = \frac{\delta T_2^0}{\delta T_1^0}$$
$$= \frac{\{(T_1^2 - P^2)^{-1/4} [T_1 + (T_1^2 - P^2)^{1/2}]^{q/2} + U_Z\}}{\{(T_2^2 - P^2)^{-1/4} [T_2 + (T_2^2 - P^2)^{1/2}]^{q/2}\} \{1 - U^2\}^{1/2}}. \qquad \ldots(3.25)$$

(c) Newtonian Analogue of Force in the Model

Here we study the effect of electromagnetic field in the force terms R_i and S_i (Narlikar and Singh 1951). The vector R_i and S_i are defined as follows (Narlikar and Singh 1951) :

$$R_i = \Delta^i_{ji} = H_{,i}/H \qquad ...(3.26)$$

$$S_i = \Delta^l_{jk}\, g^{jk} g_{li}$$
$$= g^{jk} g_{ji,k} - H_{,i}/H \qquad ...(3.27)$$

where

$$H = \sqrt{g/\gamma}.$$

For the line element (2.40) we have

$$\left.\begin{aligned} g_{11} &= -1 \\ g_{22} &= -(T^2 - P^2)^{1/2}\,[T + (T^2 - P^2)^{1/2}]^q \\ g_{33} &= -(T^2 - P^2)^{1/2}\,[T + (T^2 - P^2)^{1/2}]^{-q} \\ g_{44} &= 1 \end{aligned}\right\} \qquad ...(3.28)$$

and

$$\left.\begin{aligned} g^{11} &= -1 \\ g^{22} &= -\,(T^2 - P^2)^{-1/2}\,[T + (T^2 - P^2)^{1/2}]^{-q} \\ g^{33} &= -\,(T^2 - P^2)^{-1/2}\,[T + (T^2 - P^2)^{1/2}]^q \\ g^{44} &= 1 \end{aligned}\right\} \qquad ...(3.29)$$

$$g = -(T^2 - P^2). \qquad ...(3.30)$$

The corresponding flat metric $\gamma_{\mu\nu}$ is taken to be that of special relativity

$$ds^2 = dT^2 - dX^2 - dY^2 - dZ^2. \qquad ...(3.31)$$

Thus

$$\gamma_{ij} = [-1, -1, -1, +1] \qquad ...(3.32)$$

and

$$\gamma = -1. \qquad ...(3.33)$$

From (3.30) and (3.33)

$$H = \sqrt{g/\gamma} = (T^2 - P^2)^{1/2}. \qquad ...(3.34)$$

From (3.26) and (3.27) we get

$$R_i = [0, 0, 0, T(T^2 - P^2)^{-1}] \quad \text{...(3.35)}$$

and

$$S_i = [0, 0, 0, -T(T^2 - P^2)^{-1}]. \quad \text{...(3.36)}$$

Thus we find that Newtonian analogue of R_i and S_i both are null force vectors. R_4 and S_4 have no Newtonian analogues.

In the absence of electromagnetic field the model is given by the metric

$$ds^2 = dT^2 - dX^2 - \tfrac{1}{2}(2T)^{q+1}\, dY^2 - \tfrac{1}{2}(2T)^{-q+1}\, dZ^2 \quad \text{...(3.37)}$$

for which the pressure p_0 and density δ_0 are given by

$$Kp_0 = \frac{1 - q^2}{4T^2} + \Lambda \quad \text{...(3.38)}$$

$$K\rho_0 = \frac{1 - q^2}{4T^2} - \Lambda. \quad \text{...(3.39)}$$

The reality conditions (Ellis 1971) require that

$$q^2 < 1 \quad \text{and} \quad \Lambda > \frac{q^2 - 1}{2T^2}. \quad \text{...(3.40)}$$

Therefore vectors R_i and S_i reduce to

$$R_i = \left[0, 0, 0, \frac{1}{T}\right] \quad \text{...(3.41)}$$

and

$$S_i = \left[0, 0, 0, -\frac{1}{T}\right]. \quad \text{...(3.42)}$$

The flow vector λ^i satisfies $\lambda^i_{;j}\, \lambda^j = 0$. Thus the lines of flow are geodesics. The tensor of rotation is identically zero. The scalar of expansion is given by

$$\Theta = 1/T^2. \quad \text{...(3.43)}$$

The non-zero components of the shear tensor are

$$\left.\begin{aligned} \sigma_{11} &= \frac{1}{3T^4} \\ \sigma_{22} &= \tfrac{1}{3}(2T)^{q-1}\,[-3T(1 + q) + 2] \\ \sigma_{33} &= \tfrac{1}{3}(2T)^{-q-1}\,[-3T(1 - q) + 2]. \end{aligned}\right\} \quad \text{...(3.44)}$$

The red shift in the model is given by

$$\frac{\lambda + \delta\lambda}{\lambda} = \frac{[(2)^{q/2}\, T_1^{(q-1)/2} + Uz]}{[(2)^{q/2}\, T_2^{(q-1)/2}]\,(1 - U^2)^{1/2}} \qquad \ldots(3\cdot45)$$

where

$$Uz = \frac{dZ}{dT} = (2)^{q/2}\,(T)^{(q-1)/2}$$

is the velocity at the time of emission.

The non-vanishing components of the conformal curvature tensor are

$$C^{12}_{12} = C^{13}_{13} = -\tfrac{1}{2}C^{23}_{23} = -\frac{1}{24}\left[-\frac{10}{T} + \frac{(2q^2-1)}{T^2}\right]. \qquad \ldots(3.46)$$

As $T \to \infty$, shear, expansion and free gravitational fields vanish.

Thus the electromagnetic field gives positive contributions to expansion, shear and free gravitational field which die out for large values of T at a slower rate than the corresponding quantities in the absence of the electromagnetic field.

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APPENDIX

The non-vanishing components of the mixed Ricci tensor R^i_j for the metric (2.1) are given by

(A.1) $$R^1_1 = \frac{1}{A^2}\left[\frac{A_{44}}{A} + \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} - \frac{A_4^2}{A^2}\right]$$

(A.2) $$R^2_2 = \frac{1}{A^2}\left[\frac{B_{44}}{B} + \frac{B_4C_4}{BC}\right]$$

(A.3) $$R^3_3 = \frac{1}{A^2}\left[\frac{C_{44}}{C} + \frac{B_4C_4}{BC}\right]$$

(A.4) $$R^4_4 = \frac{1}{A^2}\left[\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4C_4}{AC} - \frac{A_4B_4}{AB} - \frac{A_4^2}{A^2}\right]$$

and

(A.5) $$R = \frac{2}{A^2}\left[\frac{A_{44}}{A} - \frac{A_4^2}{A^2} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC}\right]$$

The non-vanishing components of the Weyl conformal curvature tensor C^{hi}_{jk} or the metric (2.1) are given by

(A.6) $$C^{14}_{14} = C^{23}_{23} = \frac{1}{6A^2}\left[\frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{2A_{44}}{A} + \frac{2A_4^2}{A^2} - \frac{2B_4C_4}{BC}\right]$$

(A.7) $$C^{12}_{12} = C^{34}_{34} = \frac{1}{6A^2}\left[\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{2C_{44}}{C} + \frac{3A_4C_4}{AC} - \frac{A_4^2}{A^2} - \frac{3A_4B_4}{AB} + \frac{B_4C_4}{BC}\right]$$

(A.8) $$C^{13}_{13} = C^{24}_{24} = \frac{1}{6A^2}\left[\frac{A_{44}}{A} + \frac{C_{44}}{C} - \frac{2B_{44}}{B} + \frac{3A_4B_4}{AB} - \frac{A_4^2}{A^2} - \frac{3A_4C_4}{AC} + \frac{B_4C_4}{BC}\right].$$