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VIBRATIONS IN A TRANSVERSELY ISOTROPIC PLATE DUE TO SUDDENLY PUNCHED HOLE

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The thermoelastic vibrations generated in a homogeneous transversely isotropic stretched elastic plate due to sudden punching by a flat nose projectile have been studied in the context of generalized theory of thermoelasticity developed by Dhaliwal and Sherief. The basic governing equations have been solved by using Laplace transform technique with respect to time. As the thermal relaxation effects are of short duration, the discussion is confined to small time approximations. The results have also been discussed on the wavefronts. It is observed that the jumps in stress and temperatrue decay exponentially with time and decay is more rapid at the thermal wavefront that at the elastic wave-front. The results obtained theoretically have been verified numerically and are represented graphically for a single crystal of Zinc.

1. INTRODUCTION

Inspite of increased usage of transversely isotropic materials (e.g, graphite, laminates, and fibre composites etc.) in structures operating at severe environments such as nuclear reactors and supersonic aircrafts, the thermoelastic problems in these materials have been solved in few papers. Hence the study of thermal stresses and distributions of temperature is of great importance. Recently, the conventional coupled theory of thermoelasticity has been extended by including the thermal relaxation times in the constitutive equations by Lord and Shulman[1] and Green and Lindsay[2] in order to remove the paradox of infinite velocity of heat propagation. The generalized theory of thermoelasticity[1] has been extended to anisotropic solids by Dhaliwal and Sherief[3]. Sharma[4] studied the transient thermoelastic wave in a homogeneous, transversely isotropic, infinite medium with a cylindrical hole. Kumar[5] studied the coupled thermoelastic waves in an infinitely extended plate resulting from a suddenly punched cylindrical hole. Sharma and Chand[6, 7] studied the thermoelastic vibrations in a homogeneous isotropic plate due to a suddenly punched hole in the context of generalized theories of thermoelasticity[1, 2].

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The objective of the present paper is to study and generalise the problem of distribution of temperature and stresses in an homogeneous isotropic elastic plate resulting from a suddenly punched hole, to a transversely isotropic solid, in the context of generalised theory of thermoelasticity developed in Dhaliwal and Sherief[3].

2. Formulation of the Problem

We consider an infinitely long homogeneous transversely isotopic thermoelastic, stretched plate of thickness d, initially at uniform temperature T_0, consider a flat nose cylindrical projectile of radius a, moving with velocity v strike the plate and begin to punch out a hole of radius equal to its own. The following assumptions are taken into considerations :

(i) The plastic flow due to punching is localized in the neighbourhood of punching section (experiment also support for $v \geq 2000$ ft s^{-1} see Kumar[5]).

(ii) The punching begins instantaneously at $t = 0$ over the whole punched section, based on a small value of plate thickness d and a large value of impact speed v.

(iii) The punching action starts at an average velocity $v/2$, the projectile's velocity in the compressional wave develops in both projectile and plate on impact, i.e. the plate material below projectile is removed as a plug at $v/2$, thus the corresponding punching time, $(2d/v) = l'$ is based on large ratio diameter of projectile to plate thickness.

Consider the origin of the cylindrical coordinate system (r, θ, z) at the axis of the cylindrical hole. Considering the case of radial symmetry so that nonzero displacement component $u = u\ (r, t)$.

The basic equations[3] of motion and heat conduction in the absence of body forces and heat sources are

$$c_{11}\,[u_{,rr} + (r^{-1}u)_{,r}] - \beta_1 T_{,r} = \rho\,\ddot{u} \qquad \text{... (2.1)}$$

$$K(T_{,rr} + r^{-1}T_{,r}) - \rho\,C_e\,(\dot{T} + \tau_0\,\ddot{T}) = \beta_1 T_0\,[\dot{u}_{,r} + r^{-1}\dot{u} + \tau_0\,(\ddot{u}_{,r} + r^{-1}\ddot{u})] \qquad \text{... (2.2)}$$

where $\beta_1 = (c_{11} + c_{12})\,\alpha_1 + c_{13}\,\alpha_3$; α_1, α_3 are coefficients of linear expansion, c_{ij} are isothermal elastic parameters, K is the thermal conductivity, ρ, C_e and τ_0 are respectively the density, specific heat at constant strain and thermal relaxation time. The comma notation is used for spatial derivatives and superposed dot represents the time derivatives. We define the quantities

$$\left.\begin{array}{l} R = \omega^* r/v_p\,, \quad \tau = \omega^* t, \quad U = \rho\omega^* v_p u/T_0\beta_1\,, \quad z = T/T_0 \\ \tau_0^* = \omega^*\tau_0, \quad \varepsilon = \beta_1 T_0/\rho\,C_e c_{11}\,, \quad \omega^* = c_{11}C_e/K\,, \quad v_p^2 = c_{11}/\rho, \end{array}\right\} \qquad \text{... (2.3)}$$

where ω^* is the characteristic frequency of the plate, v_p is the velocity of longitudinal wave and ε is the coupling constant. Introducing quantities (2.3) in equations (2.1) and (2.2), we obtain

$$U_{,RR} + R^{-1} U_{,R} - R^{-2} U - \ddot{U} = Z_{,R} \qquad \text{... (2.4)}$$

$$Z_{,RR} + R^{-1} Z_{,R} - (\dot{Z} + \tau_0^* \ddot{Z}) = \varepsilon\, [\dot{U}_{,R} + R^{-1} \dot{U} + \tau_0^* (\ddot{U}_{,R} + R^{-1} \ddot{U})] \qquad \text{... (2.5)}$$

The boundary of the cylindrical hole, $R = a$, is given by

$$R = (\omega^* a/v_p) = \eta. \qquad \text{... (2.6)}$$

Assuming the plate is the rest and initially undisturbed. The initial and regularity conditions can be written as

$$\left.\begin{aligned} &U = 0 = Z, \quad \text{at} \quad \tau = 0, \quad R \geq \eta \\ &\frac{\partial U}{\partial \tau} = 0, \quad \text{at} \quad \tau = 0 \end{aligned}\right\} \qquad \text{... (2.7)}$$

$$U = 0 = Z, \quad \text{at} \quad \tau = 0, \quad R \to \infty. \qquad \text{... (2.8)}$$

The boundary conditions are given by

$$S_{RR} = \left\{\begin{array}{ll} 0, & \tau < 0 \\ -\sigma \tau/l, & 0 < \tau < l \\ -\sigma, & \tau > l \end{array}\right\} R = \eta \text{ at } l = 2d\omega^*/v \qquad \text{... (2.9)}$$

and

$$Z(\eta, \tau) = 0 \qquad \text{... (2.10)}$$

where

$$S_{RR} = U_{,R} + bR^{-1} U - Z, \quad b = c_{12}/c_{11},$$

is dimensionless form of stress in the radial direction.

3. Solution of the Problem

Applying the Laplace transform defined by

$$\overline{\psi}(R, p) = \int_0^\infty \psi(R, \tau)\, e^{-p\tau}\, d\tau, \qquad \text{... (3.1)}$$

w.r.t. time, to equations (2.4) and (2.5), we obtain

$$[D(D + R^{-1}) - p^2]\, \overline{U} = D\, \overline{Z}, \qquad \text{... (3.2)}$$

$$[(D + R^{-1})D - \tau^* p^2]\, \overline{z} = \varepsilon\, \tau^* p^2 (D + R^{-1})\, \overline{U}, \qquad \text{... (3.3)}$$

where

$$D = d/dR \text{ and } \tau^* = \tau_0^* + p^{-1}.$$

Simplifying eqns. (3.2) and (3.3), we get

$$\{[D(D+R^{-1})]^2 - (m_1^2 + m_2^2)\, D(D+R^{-1}) + m_1^2 m_2^2\}\overline{U} = 0, \quad \text{... (3.4)}$$

$$\{[(D+R^{-1})\, D]^2 - (m_1^2 + m_2^2)\, (D+R^{-1})D + m_1^2 m_2^2\}\overline{Z} = 0, \quad \text{... (3.5)}$$

where m_1^2 and m_2^2 are the roots of the equation

$$m^4 - p(\lambda_1 + \lambda_2 p)m^2 + \tau^* p^4 = 0, \quad \text{... (3.6)}$$

and

$$\lambda_1 = 1 + \varepsilon, \quad \lambda_2 = 1 + \tau_0^* + \varepsilon\, \tau_0^*. \quad \text{... (3.7)}$$

Now solving eqns. (3.4) and (3.5), by using eqn. (2.8), we obtain

$$\overline{U} = E_1 K_1 (m_1 R) + E_2 K_1 (m_2 R), \quad \text{... (3.8)}$$

$$\overline{Z} = F_1 K_0 (m_1 R) + F_2 K_0 (m_2 R), \quad \text{... (3.9)}$$

where $K_1 (m_i R)$ and $K_0 (m_i R)$ are modified Bessel's functions of order 1 and 0, respectively. Substituting eqns. (3.8) and (3.9) into eqns. (3.2) and (3.3) we obtain

$$F_i = (p^2 - m_i^2)\, E_i / m_i, \quad i = 1,\ 2. \quad \text{... (3.10)}$$

Now

$$\overline{S}_{RR} = E_1 M_1 (m_1 R) + E_2 M_2 (m_2 R), \quad \text{... (3.11)}$$

$$\overline{Z} = E_1 N_1 (m_1 R) + E_2 N_2 (m_2 R), \quad \text{... (3.12)}$$

where

$$\left.\begin{aligned} M_1(m_1R) &= [p^2 K_0(m_1R) + m_1 \beta_1^2 K_1 (m_1R)/R]/m_1 \\ M_2(m_2R) &= [p^2 K_0(m_2R) + m_2 \beta_2^2 K_1 (m_2R)]/m_2, \\ N_1(m_1R) &= (p^2 - m_1^2)\, K_0 (m_1R)/m_1, \\ N_2(m_2R) &= (p^2 - m_2^2)\, K_0 (m_2R)/m_2, \\ \beta_2^2 &= (c_{11} - c_{12})/c_{11}. \end{aligned}\right\} \quad \text{... (3.13)}$$

Applying boundary conditions (2.9) and (2.10) to eqns. (3.11) and (3.12), we obtain

$$E_1 = -\sigma(1-e^{-lp})\, N_2(m_2\eta)/lp^2 A, \qquad \dots (3.14)$$

$$E_2 = \sigma(1-e^{-lp})\, N_1(m_1\eta)/lp^2 A,$$

where

$$A = M_1(m_1\eta)\, N_2(m_2\eta) - M_2(m_2\eta)\, N_1(m_1\eta). \qquad \dots (3.15)$$

Substituting eqns. (3.14) into eqns. (3.8) and (3.9), we obtain

$$\bar{U} = \sigma(1-e^{-lp})\, [K_1(m_2R)\, N_1(m_1\eta) - K_1(m_1R)\, N_2(m_2\eta)]/lp^2 A, \dots (3.16)$$

$$\bar{Z} = \sigma(1-e^{-lp})\, [m_1(p^2-m_2^2)\, K_0(m_2R)\, N_1(m_1\eta)$$
$$- m_2(p^2-m_1^2)\, K_0(m_1 R)\, N_2(m_2\eta)]/m_1 m_2\, lp^2 A, \qquad \dots (3.17)$$

$$\bar{S}_{RR} = \sigma(1-e^{-lp})\, [N_1(m_1\eta)\, M_2(m_2R) - N_2(m_2\eta)\, M_1(m_1R)]/lp^2 A. \dots (3.18)$$

4. Small Time Approximations

Because of the presence of the damping term in the energy equation (2.2), the dependence of roots m_i on p is complicated and hence the inversion of the Laplace transform is difficult because the isolation of p is impossible. These difficulties, however, are reduced if we use some approximate methods. As the 'second sound' effects are of short duration (Green[8]), i.e. we take p large. The roots m_i (i = 1, 2) of eqn. (3.6) may be expanded binomially after we retain only the positive sign. Thus we obtain

$$m_i = p v_i^{-1} + \phi_i + o(p^{-1}),\ i = 1,\ 2 \qquad \dots (4.1)$$

where

$$v_{1,2} = \sqrt{2}\, [\lambda_2 \pm (\lambda_2^2 - 4\tau_0^*)^{1/2}]^{-1/2}, \qquad \dots (4.2)$$

$$\phi_{1,2} = [\lambda_1 \pm (\lambda_1\lambda_2 - 2)/(\lambda_2^2 - 4\tau_0^*)^{1/2}]/2\sqrt{2}\, [\lambda_2 \pm (\lambda_2^2 - 4\tau_0^*)^{1/2}]^{1/2}. \dots (4.3)$$

From the above analysis it can easily be proved that there exist two waves, namely the elastic and thermal waves, and the fromer follows the latter one. The modified Bessel's function $K_n(z)$ has asymptotic expansion[9]

$$K_n(z) = \left(\frac{\pi}{2z}\right)^{1/2} e^{-z}\left[1 + \frac{(4n^2-1^2)}{(8z)\lfloor 1} + \frac{(4n^2-1^2)(4n^2-3^2)}{(8z)^2\lfloor 2} + \dots\right] \dots (4.4)$$

Putting eqns. (3.13), (4.1) and (4.4) into eqns (3.16) to (3.18), we obtain

$$\bar{U}(R,\ p) = \sigma(\eta/R)^{1/2}\, \{v_1(v_2^2-1)\,(p^{-2} + A_1 p^{-3} + \dots)\, e^{-m_1 R_1}$$
$$- v_2(v_1^2-1)\,(p^{-2} + A_1' p^{-3} + \dots)\, e^{-m_2 R_1}\}/(v_2^2 - v_1^2), \qquad \dots (4.5)$$

$$\overline{Z}(R,\ p) = -\sigma(\eta/R)^{1/2}(v_1^2-1)(v_2^2-1)\ \{(p^{-1}+A_2 p^{-2}+\ldots)\, e^{-m_1 R_1} - (p^{-1}+A_2' p^{-2}+\ldots)\, e^{-m_2 R_1}\}/(v_2^2-v_1^2), \qquad \ldots (4.6)$$

$$\overline{S}_{RR}\ (R,\ p) = -\sigma(\eta/R)^{1/2}\{(A_3 p^{-1}+A_4 p^{-2}+\ldots.)\, e^{-m_1 R_1} - (A_3' p^{-1}+A_4' p^{-2}+\ldots.)\, e^{-m_2 R_1}\}/(v_2^2-v_1^2), \qquad \ldots (4.7)$$

where

$$A_1 = [3v_1\,\eta\,(v_2^2-1) - v_2 R\,(v_2^2-1) - 8R\eta\,(v_2^2-1)\,A_5 - 8R\eta(\phi_2\, v_2+\phi_2\, v_2^3)]/8\,\eta\, R\,(v_2^2-1),$$

$$A_1' = [3v_2\,\eta\,(v_1^2-1) - v_1 R\,(v_1^2-1) - 8R\,\eta(\phi_1\, v_1+\phi_1\, v_1^3) - 8\,\eta R\,(v_1^2-1)\,A_5]/8\,\eta\, R\,(v_1^2-1),$$

$$A_2 = [8\eta\, R\,(v_1^2-1)(v_2^2-1)\,A_5 - (v_1^2-1)(v_2^2-1)(v_1\,\eta+v_2 R) - 8\,\eta\, R\,\{(v_2^2-1)(\phi_1\, v_1+\phi_1\, v_1^3) + (v_1^2-1)(\phi_2\, v_2+\phi_2\, v_2^3)\}]/8\,\eta\, R\,(v_1^2-1)(v_2^2-1),$$

$$A'_2 = [8\eta\, R\,(v_2^2-1)(v_1^2-1)\,A_5 - (v_2^2-1)(v_1^2-1)(v_2\,\eta+v_1 R) - 8\,\eta\, R\,\{(v_1^2-1)(\phi_2\, v_2+\phi_2\, v_2^3) + (v_2^2-1)(\phi_1\, v_1+\phi_1\, v_1^3)\}]/8\,\eta\, R\,(v_2^2-1)(v_1^2-1),$$

$$A_3 = v_1^2\,(v_2^2-1),$$

$$A'_3 = v_2^2\,(v_1^2-1),$$

$$A_4 = v_1^2\ \{8\eta\,\beta_2^2\,(v_2^2-1) - 8\eta\, R\,(v_2^2-1)\,A_5 - 8\,(v_2^2-1)(v_1\,\eta+v_2\, R) - 8\,\eta\, R\,(\phi_2\, v_2+\phi_2\, v_2^3)\}/8\,\eta\, R\,(v_2^2-v_1^2),$$

$$A'_4 = v_2^2\ \{8\eta\,\beta_2^2\,(v_1^2-1) - 8\eta\, R\,(v_1^2-1)\,A_5 - 8\,(v_1^2-1)(v_2\,\eta+v_1\, R) - 8\,\eta\, R\,(\phi_1\, v_1+\phi_1\, v_1^3)\}/8\,\eta\, R\,(v_2^2-v_1^2),$$

$$A_5 = [(v_2^2\,\{(v_1+v_2)(v_1^2-1) + 8\,\eta\,(v_1\,\phi_1+\phi_1\, v_1^3)\} /8\,\eta - (\beta_2^2\ \{v_2\,(v_1^2-1) - v_1\,(v_2^2-1)\}/\eta) + (v_1^2\ \{(v_1+v_2)(v_2^2-1) + 8\eta\,(\phi_2\, v_2+\phi_2\, v^3)\}/8\eta)]/(v_2^2-v_1^2),$$

and

$$R_1 = (R - \eta).$$

Now inverting the Laplace transforms of eqns. (4.5)-(4.7), we obtain

$$U(R,\tau) = \sigma(\eta/R)^{1/2}\,[v_2\,(v_1^2-1)\ \{A'_1\,(\tau - R_1/v_2) + 1\}$$
$$(\tau - R_1/v_2)\,H(\tau - R_1/v_2)\,e^{-\phi_2 R_1} - v_1\,(v_2^2-1)\,\{1 + A_1\,(\tau - R_1/v_1)\}$$
$$(\tau - R_1/v_1)\,H(\tau - R_1/v_1)\,e^{-\phi_1 R_1}]/(v_2^2 - v_1^2), \qquad \text{... (4.8)}$$

$$Z(R,\tau) = \sigma\,(\eta/R)^{1/2}\,(v_1^2-1)\,(v_2^2-1)\,[\{1 + (\tau - R_1/v_2)\,A'_2\}$$
$$H(\tau - R_1/v_2)\,e^{-\phi_2 R_1} - \{1 + (\tau - R_1/v_1)\,A_2\}$$
$$H(\tau - R_1/v_1)\,e^{-\phi_1 R_1}]/(v_2^2 - v_1^2), \qquad \text{... (4.9)}$$

$$S_{RR}\,(R,\tau) = \sigma\,(\eta/R)^{1/2}\,[\{A'_3 + A'_4\,(\tau - R_1/v_2)\}\ H(\tau - R/v_2)\,e^{-\phi_2 R_1}$$
$$-\ \{A_3 + A_4\,(\tau - R_1/v_1)\}\ H(\tau - R_1/v_1)\,e^{-\phi_1 R_1}]/(v_2^2 - v_1^2).$$
$$\text{... (4.10)}$$

5. Long Time Solutions

The long time solutions can be obtained by expanding the roots m_i^2 $(i = 1, 2)$ of eqn (3.6) for a small value of p in the Taylor series. We get

$$m_1 = (1+\varepsilon)^{1/2}\sqrt{p} + o(p^{3/2}), \qquad \text{... (5.1)}$$

$$m_2 = (1+\varepsilon)^{-1/2}\,p + o(p^2). \qquad \text{... (5.2)}$$

Putting the values of m_i into eqns. (4.5)-(4.7), we obtain the transformed values of deformation, temperature and stress. It is observed that m_i do not involve the thermal relaxation time τ_0^*, which ascertains that the 'second sound' effects are of short duration.

6. Discussion of the Results at the Wavefronts

The short-time solutions obtained in the previous section show that they consist of two waves, i.e., the dilatational wave and the thermal wave travelling with velocities v_1 and v_2, respectively. The terms containing $H(\tau - R/v_1)$ and $H(\tau - R/v_2)$ represent the contributions of the elastic wave and the thermal wave in the vicinity of their wavefronts $R = v_1\tau$ and $R = v_2\tau$, respectively.

The deformation is found to be continuous but the temperature and stress are found to be discontinuous. The discontinuities are given by

$$(Z^+ - Z^-)_{R_1 = \nu_1\tau} = \sigma\,(\eta/R)^{1/2}\,[(\nu_1^2 - 1)\,(\nu_2^2 - 1)\exp\,(-\,\phi_1\,\nu_1\,\tau)]/(\nu_2^2 - \nu_1^2), \qquad \text{... (6.1)}$$

$$(Z^+ - Z^-)_{R_1 = \nu_2\tau} = -\,\sigma\,(\eta/R)^{1/2}\,[(\nu_1^2 - 1)\,(\nu_2^2 - 1)\exp\,(-\,\phi_2\,\nu_2\,\tau)]/(\nu_2^2 - \nu_1^2), \qquad \text{... (6.2)}$$

$$(S^+_{RR} - S^-_{RR})_{R_1 = \nu_1\tau} = \sigma\,(\eta/R)^{1/2}\,[\nu_1^2\,(\nu_2^2 - 1)\exp\,(-\,\phi_1\,\nu_1\,\tau)]/(\nu_2^2 - \nu_1^2), \qquad \text{... (6.3)}$$

$$(S^+_{RR} - S^-_{RR})_{R_1 = \nu_2\tau} = -\,\sigma\,(\eta/R)^{1/2}\,[\nu_2^2\,(\nu_1^2 - 1)\exp\,(-\,\phi_2\,\nu_2\,\tau)]/(\nu_2^2 - \nu_1^2). \qquad \text{... (6.4)}$$

The discontinuities decay exponentially with time.

7. Particular Cases

(i) If $\tau_0 = 0$, then we have the case of coupled thermoelasticity and thus $\lambda_1 = 1 + \varepsilon$, $\lambda_2 = 1$, $\nu_1 = 1$, $\nu_2 \to \infty$, $\phi_1 = \varepsilon/2$, $\phi_2 \to \infty$. In this case the temperature at both the wavefronts and stress at the thermal wave-front become continuous, whereas the stress experiences a finite jump at the elastic wavefront, given by

$$(S^+_{RR} - S^-_{RR})_{R_1 = \nu_1\tau} = -\,\sigma(\eta/R)^{1/2}\exp\,[-\,\varepsilon\,\tau/2]. \qquad \text{... (7.1)}$$

(ii) If there is no coupling between the strain and the thermal fields, i.e., $\varepsilon = 0$, then we have $\lambda_1 = 1$, $\lambda_2 = 1 + \tau_0^*$, $\nu_1 = 1$, $\nu_2 = (\tau_0^*)^{-1/2}$, $\phi_1 = 0$, $\phi_2 = \nu_2/2$. In this case the temperature at both the wave-fronts and stress at the thermal wavefront become continuous, whereas the stress experiences a finite jump at the elastic wavefront, given by

$$(S^+_{RR} - S^-_{RR})_{R_1 = \nu_1\tau} = -\,\sigma(\eta/R)^{1/2}. \qquad \text{... (7.2)}$$

Here the jump in stress vanishes as the radial distance increases. This result agrees with Sharma[4].

(iii) If $\varepsilon = 0$ and $\tau_0 = 0$ i.e. if both the coupling and relaxation effects are ignored, we have $\lambda_1 = 1, \lambda_2 = 1, \nu_1 = 1$, $\nu_2 \to \infty$, $\phi_1 = 0$, $\phi_2 \to \infty$. In this case the results obtained agree with case (ii) above.

(iv) If we take $c_{11} = (\lambda + 2\mu)$, $c_{12} = \lambda = c_{13}$, $\alpha_1 = \alpha_3 = \alpha_T$, $\beta_1 = \beta$, then the above results will reduce to those obtained in Sharma and Chand[6].

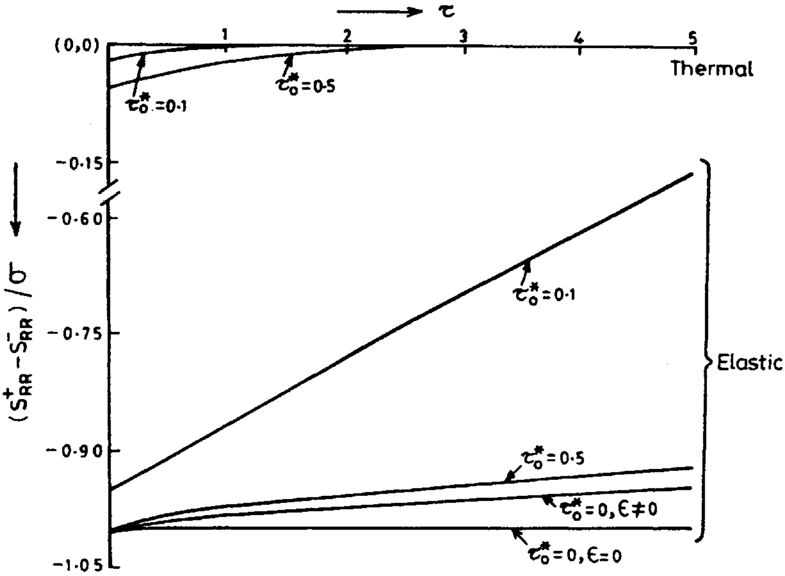

FIG. 1. The variations of jump in stress w.r.t. time, at the wavefronts.

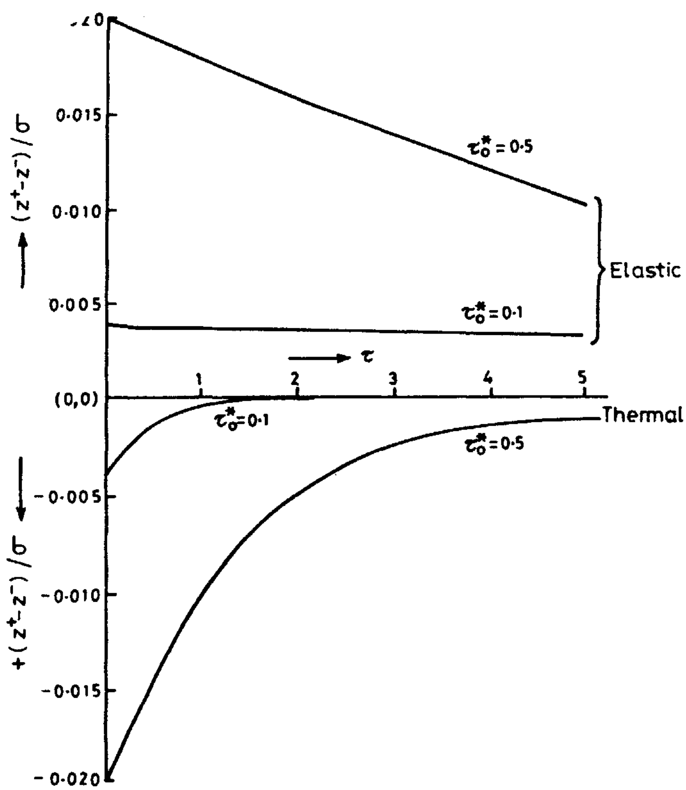

FIG. 2. The variations of jump in temperature w.r.t. time at the wavefronts.

8. Numerical Results and Discussion

In this section, the jumps obtained theoretically, in the previous sections, for temperature and stress are computed numerically for a single crystal of Zinc[10] for which the physical data is as under $c_{11} = 1.628 \times 10^{-11}$ Nm^{-2}, $c_{12} = 0.36 \times 10^{11}$ Nm^{-2}, $\varepsilon = 0.022$, $\rho = 7.14 \times 10^{3}$ Kgm^{-3}, $c_{13} = 0.508 \times 10^{11}$ Nm^{-2}, $\beta_1 = 5.75 \times 10^{6}$ Nm^{-1} deg^{-1}, $C_e = 3.9 \times 10^{2}$ J Kg^{-1} deg^{-1}, $T_0 = 296^{\circ}$ K.

The variations of the jumps are plotted w.r.t. time for three values of relaxation times $\tau_0^* = 0.0, 0.1, 0.5$ as shown in Figs. 1 and 2. The jumps decay exponentially with time. It is also observed that the decay of these jumps is more rapid at the thermal wavefront that at the elastic wave-front.

The jumps in the values of stress have been observed to be of larger magnitude at the thermal wavefront than at the elastic one, however, the jumps in case of temperature follows the reverse trend. The jumps in stress and temperature tends to zero with the decrease in the values of thermal relaxation time except in case of uncoupled thermoelasticity in which stress experience a finite jump at the elastic wavefront. The temperature and stress vanish after some time on the thermal wavefront which shows that these functions become continuous after words. The identical vanishing of the jumps at the thermal wavefront after certain time clearly demonstrates the difference between the coupled and the generalized theories of thermoelasticity. In the former theory the heat wave propagates with infinite speed, so the value of any of the considered functions is not identically zero (though may be very small) for any large values of time. In generalized theory the response to the thermal and mechanical effects does not reach infinity instantaneously but remains in a bounded region of the space.

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