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A STABLE ITERATION PROCEDURE FOR QUASI-CONTRACTIVE MAPS

M. O. OSILIKE

Department of Mathematics, University of Nigeria, Nsukka, Nigeria

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It is proved that a certain Mann iteration procedure is T-stable for quasi-contractive maps in Banach spaces which are either q-uniformly smooth or p-uniformly convex, $1 < p, q < \infty$.

INTRODUCTION

Let E be a Banach space, and T a selfmap of E. Let $x_0 \in E$ and let $x_{n+1} = f(T, x_n)$ define an iteration procedure which yields a sequence of points $\{x_n\}_{n=0}^{\infty}$. For example, the function iteration, $x_{n+1} = f(T, x_n) = Tx_n$. Suppose that $\{x_n\}_{n=0}^{\infty}$ converges to a fixed point x^* of T. Let $\{y_n\}_{n=0}^{\infty} \subseteq E$, and let $\varepsilon_n = \| y_{n+1} - f(T, y_n) \|$. If $\lim \varepsilon_n = 0$ implies that $\lim y_n = x^*$, then the iteration procedure defined by $x_{n+1} = f(T, x_n)$ is said to be T-stable. Stability results for several iteration procedures for certain contractive definitions have been established in recent papers by several researchers[4, 5, 9, 10, 15-17]. Harder and Hicks[5] showed that function iteration, for mappings T satisfying various contractive definitions is T-stable, as well as for several iteration schemes other than function iteration. Rhoades[15] (see also Rhoades[16]) extended most of the results of Harder and Hicks[5] to an independent contractive definition, and also proved stability results for some additional iteration procedures. Recently, Rhoades[17] considered the following class of mappings : there exists $c \in [0, 1)$ such that

$$\| Tx - Ty \| \le c \max \{\| x - y \|, \frac{1}{2} [\| x - Tx \| + \| y - Ty \|], \| x - Ty \|, \| y - Tx \|\}. \quad \ldots (1)$$

For this contractive definition, which is more general than those of Harder and Hicks[5], and Rhoades[15, 16], he proved several stability results which are generalizations and extensions of most of the results of Harder and Hicks[5] and Rhoades[15, 16]. The author (see Osilike[9]) showed that the Ishikawa iteration procedure is T-stable for the class of mappings satisfying condition (1). More recently, the author (Osilike[10]) established stability results for mappings satisfying a contractive definition which is more general than (1), and consequently more general than the contractive definitions of Harder and Hicks[5], and Rhoades[15, 16]. We considered the following contractive definition : there exist constants $a \in (0, 1)$ and $L \geq 0$ such that

$$\| Tx - Ty \| \leq a \|x-y\| + L \|x - Tx\| \qquad \text{... (2)}$$

for all $x, y \in E$. For the contractive definition (2) we obtained results which are generalizations and extensions of most of the results of Harder and Hicks[5], Rhoades[15-17] and the author (Osilike[9]).

For some of the applications of stability results, the reader may consult Harder[4]. In the sequel, we consider a contractive definition which is more general than the contractive definition (1), and independent of the contractive definition (2). A mapping T is said to be quasi-contractive (see for example Ćirić[3] if there exists a $k \in [0, 1)$ such that

$$\| Tx - Ty \| \leq k \max \{\| x - y \|, \| x - Tx \|, \| x - Ty \|, \| y - Tx \|, \| y - Ty \|\} \qquad \text{... (3)}$$

for all $x, y \in E$. It is clear that condition (1) implies (3). The following example shows that the class of mappings satisfying (1) is a proper subset of the class of mappings satisfying (3).

Example — Let $E = \mathcal{R}$ (with the usual norm), and $K = [0, 1]$. Define $T : K \to K$ by

$$Tx = \begin{cases} \frac{1}{2}, & 0 \leq x < 1 \\ 0, & x = 1. \end{cases}$$

Then T is quasi-contractive with $k = 1/2$. However, T does not satisfy condition (1) since if $x = 1/2$, and $y = 1$, then

$$| Tx - Ty | = \frac{1}{2} = \max \left\{ |x-y|, \frac{1}{2} [|x - Tx|, |y - Ty|], |x - Ty|, |y - Tx| \right\}.$$

The example above and the example in Osilike[10] show that contractive definitions (2) and (3) are independent. Furthermore, it is shown in Rhoades[14] that the contractive definition (3) is one of the most general contractive-type definitions for which Picards iteration yields a unique fixed point.

It is our purpose in this paper to prove that certain Mann iteration procedure is T-stable for quasi-contractive maps in Banach spaces which are either q-uniformly smooth or p-uniformly convex (see definition below). These Banach spaces include all Hilbert spaces, L_p (or l_p) spaces, $1 < p < \infty$, and the Sobolev spaces, W_m^p, $1 < p < \infty$.

PRELIMINARIES

Let E be a Banach space. The modulus of smoothness of E is the function

$$\rho_E : [0, \infty) \rightarrow [0, \infty)$$

defined by

$$\rho_E(t) = \sup \left\{ \frac{1}{2}(\| x+y \| + \| x-y \|) - 1 : \| x \| \leq 1, \ \| y \| \leq t \right\}.$$

E is uniformly smooth if $\lim_{t \to \infty} \rho_E(t) = 0$.

Let $q > 1$. E is said to be q-uniformly smooth (or to have a modulus of smoothness of power type $q > 1$) if there exists a constant $c > 0$ such that $\rho_E(t) \leq ct^q$. All Hilbert spaces, L_p (or l_p) spaces, $p \geq 2$, and the Sobolev spaces, W_m^p, $p \geq 2$ are 2-uniformly smooth, while for $1 < p \leq 2$, L_p (or l_p), and W_m^p spaces are p-uniformly smooth.

In the sequel we shall need the following :

Lemma 1 (Chidume and Osilike[2]) — Let $q > 1$, and let E be a q-uniformly smooth Banach space. Then for all $x, y, z \in E$ and $\lambda \in [0, 1]$ the inequality

$$\| \lambda x + (1-\lambda)y - z \|^q \leq [1 - \lambda(q-1)] \| y - z \|^q + \lambda c_q \| x - z \|^q$$
$$- \lambda [1 - \lambda^{q-1} c_q] \| x - y \|^q \qquad \ldots (4)$$

holds.

Without loss of generality we may assume $c_q \geq 1$.

The modulus of convexity of E is the function

$$\delta_E : [0, 2] \rightarrow [0, 1]$$

defined by

$$\delta_E(\varepsilon) = \inf \left\{ 1 - \| \frac{1}{2}(x+y) \| : \| x \| \leq 1, \ \| y \| \leq 1, \ \| x - y \| \geq \varepsilon \right\}.$$

E is uniformly convex if and only if $\delta_E(\varepsilon) > 0$ for all ε in $(0, 2]$.

Let $p > 1$, then E is said to be p-uniformly convex (or to have a modulus of convexity of power type $p > 1$) if there exists a constant $c > 0$ such that $\delta_E \geq c\varepsilon^p$ for all ε in $(0, 2]$.

Hilbert spaces, L_p (or l_p) and W_m^p spaces, $1 < p \leq 2$ are 2-uniformly convex, while for $p \geq 2$, L_p (or l_p) and W_m^p spaces are p-uniformly convex.

It follows from Theorem 1 of Xu[21] (see also Corollary 1 of Xu[20]) that for $p > 1$, E is p-uniformly convex if and only if there exists a constant $c_p > 0$ such that

$$\| \lambda x + (1-\lambda) y \|^p \le \lambda \| x \|^p + (1-\lambda) \| y \|^p - \omega_p(\lambda) c_p \| x - y \|^p \qquad \dots (5)$$

for all x, $y \in E$ and $\lambda \in (0, 1)$, where

$$\omega_p(\lambda) = \lambda(1-\lambda)^p + (1-\lambda)\lambda^p.$$

We shall also need the following :

Lemma 2 (Qihou[11]) — Let $\{x_n\}_{n=1}^{\infty}$ satisfy

$$x_{n+1} \le \omega x_n + \sigma_n$$

where $x_n \ge 0$, $\sigma_n \ge 0$, $\lim_{n \to \infty} \sigma_n = 0$, and $0 \le \omega \le 1$. Then $x_n \to 0$ as $n \to \infty$.

Theorem 1 (Xu[19]) — Let C be a nonempty closed convex subset of a Banach space E and $T : C \to C$ a quasi-contractive mapping. Suppose $\alpha_n > 0$ for all $n \ge 0$ and $\Sigma_{n=0}^{\infty} \alpha_n = \infty$. Then the sequence $\{x_n\}_{n=0}^{\infty}$ defined from any $x_0 \in C$ by

$$y_n \in C_0\left(\{x_i\}_{i=0}^{n} \cup \{Tx_i\}_{i=0}^{n} \right), \quad n \ge 0$$

$$x_{n+1} = (1-\alpha_n) x_n + \alpha_n T y_n, \qquad n \ge 0$$

converges strongly to the unique fixed point of T, where $C_0(A)$ denotes the convex hull of the subset A of E.

Remark 1 : Rhoades[18] has recently extended the above Theorem of Xu to a more general class of mappings.

MAIN RESULTS

For the rest of this paper, k is the constant appearing in the definition of a quasi-contractive map, x^* will denote the fixed point of T, and c_q and c_p are the constants appearing in inequalities (4) and (5) respectively. We now prove the following :

Theorem 2 — Let $q > 1$ and let E be a q-uniformly smooth Banach space. Let $T : E \to E$ be a quasi-contractive mapping with $c_q k^q < \min\{(q - 1), 1\}$. Let $\{c_n\}_{n=0}^{\infty}$ be a real sequence satisfying

$$\frac{1}{2}\left[\frac{1}{c_q}(1 - c_q k^q) \right]^{\frac{1}{q-1}} \le c_n \le \left[\frac{1}{c_q}(1 - c_q k^q) \right]^{\frac{1}{q-1}} \qquad \dots (6)$$

Let $\{x_n\}_{n=0}^{\infty}$ be the sequence generated from an arbitrary $x_0 \in E$ by

$$x_{n+1} = (1 - c_n) x_n + c_n T x_n, \quad n \ge 0. \qquad \dots (7)$$

Let $\{y_n\}_{n=0}^{\infty} \subseteq E$ and define $\{\varepsilon_n\}_{n=0}^{\infty}$ by

$$\varepsilon_n = \| y_{n+1} - (1 - c_n)y_n - c_n Ty_n \|. \quad \text{... (8)}$$

Then

$$\| y_{n+1} - x^* \| \le \varepsilon_n + \left[1 - \frac{1}{2} \left[\frac{1}{c_q} (1 - c_q k^q) \right]^{\frac{1}{q-1}} (q - 1 - c_q k^q) \right]^{\frac{1}{q}} \| y_n - x^* \| \quad \text{... (9)}$$

and $\lim y_n = x^*$ if and only if $\lim \varepsilon_n = 0$.

PROOF : Since $c_n > 0$ for all $n \ge 0$ and $\Sigma_{n=0}^{\infty} c_n = \infty$, it follows from Theorem 1 that $\lim x_n = x^*$.

Observe that

$$\| y_{n+1} - x^* \| \le \varepsilon_n + \| (1 - c_n) (y_n - x^*) + c_n (Ty_n - x^*) \|. \quad \text{... (10)}$$

Using Lemma 1 we obtain

$$\| (1 - c_n) (y_n - x^*) + c_n (Ty_n - x^*) \|^q$$
$$\le [1 - c_n (q - 1)] \| y_n - x^* \|^q + c_n c_q \| Ty_n - x^* \|^q$$
$$- c_n [1 - c_n^{q-1} c_q] \| y_n - Ty_n \|^q. \quad \text{... (11)}$$

Using (3) we obtain,

$$\| Ty_n - x^* \| \le k \max \{\| y_n - x^* \|, \| y_n - Ty_n \|\},$$

so that

$$\| Ty_n - x^* \|^q \le k^q \| y_n - x^* \|^q + k^q \| y_n - Ty_n \|^q. \quad \text{... (12)}$$

Using (12) in (11) yields

$$\| (1 - c_n) (y_n - x^*) + c_n (Ty_n - x^*) \|^q$$
$$\le [1 - c_n (q - 1 - c_q k^q)] \| y_n - x^* \|^q$$
$$- c_n [1 - c_n^{q-1} c_q - c_q k^q] \| y_n - Ty_n \|^q. \quad \text{... (13)}$$

Condition (6) implies that

$$1 - c_n (q - 1 - c_q k^q) \le 1 - \frac{1}{2} \left[\frac{1}{c_q} (1 - c_q k^q) \right]^{\frac{1}{q-1}} (q - 1 - c_q k^q)$$

and $\quad 1 - c_n^{q-1} c_q - c_q k^q \ge 0.$

Thus (13) reduces to

$$\| (1-c_n)(y_n - x^*) + c_n (Ty_n - x^*) \|$$

$$\leq \left[1 - \frac{1}{2} \left[\frac{1}{c_q} (1 - c_q k^q) \right]^{\frac{1}{q-1}} (q - 1 - c_q k^q) \right]^{\frac{1}{q}} \| y_n - x^* \|. \quad \dots (14)$$

Using (14) in (13) now yields (9).

Suppose $\lim \varepsilon_n = 0$. Set $\rho_n = \| y_n - x^* \|$, and

$$h_q = \left[1 - \frac{1}{2} \left[\frac{1}{c_q} (1 - c_q k^q) \right]^{\frac{1}{q-1}} (q - 1 - c_q k^q) \right]^{\frac{1}{q}} < 1.$$

Then $\rho_{n+1} \leq h_q \| y_n - x^* \| + \varepsilon_n$, and it follows from Lemma 2 that $\rho_n \to 0$ as $n \to \infty$.

Suppose $\lim y_n = x^*$, then

$$\varepsilon_n = \| y_{n+1} - (1 - c_n) y_n - c_n Ty_n \|$$

$$\leq \| y_{n+1} - x^* \| + \| (1 - c_n)(y_n - x^*) + c_n (Ty_n - x^*) \|$$

$$\leq \| y_{n+1} - x^* \| + h_q \| y_n - x^* \| \to 0 \text{ as } n \to \infty,$$

completing the proof of Theorem 2.

Theorem 3 — Let $1 < p \leq 2$, and let E be a p-uniformly convex Banach space. Let $T : E \to E$ be a quasi-contractive map with $k^p \leq c_p$, and let $\{c_n\}_{n=0}^{\infty}$ be a real sequence satisfying

$$\frac{1}{2}\left(1 - \frac{k^p}{c_p}\right) \leq c_n \leq 1 - \frac{k^p}{c_p}. \quad \dots (15)$$

Let $\{x_n\}_{n=0}^{\infty}$ be the sequence generated from an arbitrary $x_0 \in E$ by (7). Let $\{y_n\}_{n=0}^{\infty} \subseteq E$ and define $\{\varepsilon_n\}_{n=0}^{\infty}$ by (8). Then

$$\| y_{n+1} - x^* \| \leq \varepsilon_n + \left[1 - \frac{1}{2}\left(1 - \frac{k^p}{c_p}\right)(1 - k^p) \right]^{\frac{1}{p}} \| y_n - x^* \|, \quad \dots (16)$$

and $\lim y_n = x^*$ if and only if $\lim \varepsilon_n = 0$.

PROOF : It follows from Theorem 1 that $\lim x_n = x^*$. Using inequality (5) we obtain

$$\| (1-c_n)(y_n - x^*) + c_n (Ty_n - x^*) \|^p \leq (1 - c_n) \| y_n - x^* \|^p + c_n \| Ty_n - x^* \|^p$$

$$- \omega_p (c_n) c_p \| y_n - Ty_n \|^p. \quad \dots (17)$$

Since $1 < p \leq 2$, then

$$\omega_p(c_n) = c_n (1 - c_n)^p + (1 - c_n) c_n^p \geq c_n (1 - c_n)^2 + (1 - c_n) c_n^2 = c_n (1 - c_n)$$

and using this in (17) yields

$$\| (1 - c_n) (y_n - x^*) + c_n (Ty_n - x^*) \|^p \leq [1 - c_n (1 - k^p)] \| y_n - x^* \|^p$$
$$- c_n [(1 - c_n) c_p - k^p] \| y_n - Ty_n \|^p. \quad \text{... (18)}$$

Condition (15) implies that

$$1 - c_n (1 - k^p) \leq 1 - \frac{1}{2} \left(1 - \frac{k^p}{c_p} \right) (1 - k^p),$$

and $(1 - c_n) c_p - k^p \geq 0$. Thus (18) reduces to

$$\| (1 - c_n) (y_n - x^*) + c_n (Ty_n - x^*) \|$$
$$\leq \left[1 - \frac{1}{2} \left(1 - \frac{k^p}{c_p} \right) (1 - k^p) \right]^{\frac{1}{p}} \| y_n - x^* \|,$$

and hence

$$\| y_{n+1} - x^* \| \leq \varepsilon_n + \left[1 - \frac{1}{2} \left(1 - \frac{k^p}{c_p} \right) (1 - k^p) \right]^{\frac{1}{p}} \| y_n - x^* \|.$$

It follows as in the proof of Theorem 2 that $\lim y_n = x^*$ if and only if $\lim \varepsilon_n = 0$.

Theorem 4 — Let $p \geq 2$, and let E be a p-uniformly convex Banach space. Let $T : E \to E$ be a quasi-contractive map with $2^{p-2} k^p < c_p$. Let $\{c_n\}_{n=0}^{\infty}$ be a real sequence satisfying

$$\frac{1}{2} \left(1 - \frac{2^{p-2} k^p}{c_p} \right) \leq c_n \leq 1 - \frac{2^{p-2} k^p}{c_p}. \quad \text{... (19)}$$

Let $\{x_n\}_{n=0}^{\infty}$ be a sequence generated from an arbitrary $x_0 \in E$ by (7). Let $\{y_n\}_{n=0}^{\infty} \subseteq E$ and define $\{\varepsilon_n\}_{n=0}^{\infty}$ by (8). Then

$$\| y_{n+1} - x^* \| \leq \varepsilon_n + \left[1 - \frac{1}{2} \left(1 - \frac{2^{p-2} k^p}{c_p} \right) (1 - k^p) \right]^{\frac{1}{p}} \| y_n - x^* \|,$$

and $\lim y_n = x^*$ if and only if $\lim \varepsilon_n = 0$.

PROOF : It follows from Theorem 1 that $\lim x_n = x^*$. Using inequality (5) we obtain

$$\| (1-c_n)(y_n - x^*) + c_n (Ty_n - x^*) \|^p \le (1-c_n) \| y_n - x^* \|^p + c_n \| Ty_n - x^* \|^p - \omega_p (c_n) c_p \| y_n - Ty_n \|^p$$

$$= (1-c_n) \| y_n - x^* \|^p + c_n \| Ty_n - x^* \|^p - c_n (1-c_n) c_p [(1-c_n)^{p-1} + c_n^{p-1}] \| y_n - Ty_n \|^p$$

$$\le [1 - c_n (1-k^p)] \| y_n - x^* \|^p - c_n [(1-c_n) c_p ((1-c_n)^{p-1} + c_n^{p-1}) - k^p] \| y_n - Ty_n \|^p. \quad \text{... (20)}$$

Since $p \ge 2$, it follows that $(1-c_n)^{p-1} + c_n^{p-1} \ge 2^{-(p-2)}$. Furthermore, condition (19) implies that

$$1 - c_n (1-k^p) \le 1 - \frac{1}{2}\left(1 - \frac{2^{p-2} k^p}{c_p}\right)(1-k^p) \text{ and}$$

$(1-c_n) c_p [(1-c_n)^{p-1} + c_n^{p-1}] - k^p \ge 0$. Thus, (20) reduces to

$$\| (1-c_n)(y_n - x^*) + c_n (Ty_n - x^*) \| \le \left[1 - \frac{1}{2}\left(1 - \frac{2^{p-2} k^p}{c_p}\right)(1-k^p) \right]^{\frac{1}{p}} \| y_n - x^* \|,$$

and hence

$$\| y_{n+1} - x^* \| \le \varepsilon_n + \left[1 - \frac{1}{2}\left(1 - \frac{2^{p-2} k^p}{c_p}\right)(1-k^p) \right]^{\frac{1}{p}} \| y_n - x^* \|.$$

The rest of the argument now follows as in the proof of Theorem 2 to yield that $\lim y_n = x^*$ if and only if $\lim \varepsilon_n = 0$.

Corollary — Let H be a Hilbert space and $T : E \to E$ a quasi-contractive map. Let $\{c_n\}_{n=0}^{\infty}$ be a real sequence satisfying

$$\frac{1}{2}(1-k^2) \le c_n \le 1 - k^2.$$

Let $\{x_n\}_{n=0}^{\infty}$ be the real sequence generated from an arbitrary $x_0 \in E$ by (7). Let $\{y_n\}_{n=0}^{\infty} \subseteq H$ and define $\{\varepsilon_n\}_{n=0}^{\infty}$ by (8). Then

$$\| y_{n+1} - x^* \| \le \varepsilon_n + \left[1 - \frac{1}{2}(1-k^2)^2 \right]^{\frac{1}{2}} \| y_n - x^* \|,$$

and $\lim y_n = x^*$ if and only if $\lim \varepsilon_n = 0$.

PROOF : Hilbert spaces are 2-uniformly smooth and 2-uniformly convex, and satisfy (4) and (5) with $c_q = 1$ and $c_p = 1$ respectively. Hence the result follows from any of the Theorems 2, 3, or 4.

Remark 2 : Harder and Hicks[5] have shown that, if $0 \le k \le 1/2$, then definition (3) is contained in definition (19) of that paper, which is included in definition (1) of this paper. Consequently, in Theorem 2, if $1 < q < 2$ and is such that $(q-1)^{1/q} \le 1/2$, then Theorem 1 is a special case of the corresponding result in Harder and Hicks[5] or Rhoades[17]. In Lim[7] it is shows that $c_p > 2^{2-p}$. Therefore, for $p > 4$, it is possible for k to be $\le 1/2$.

Remark 3 : Theorem 1 shows that both the Mann and Ishikawa iteration procedures can be used to approximate the fixed point of a quasi-contractive map in any Banach space. Theorems 2, 3 and 4 show that certain Mann iteration procedures are T-stable for quasi-contractive maps in Banach spaces which are either q-uniformly smooth or p-uniformly convex, $1 < p < \infty$. It is of interest to know whether or not an Ishikawa-type iteration procedure is T-stable for quasi-contractive maps in these Banach spaces. Furthermore, we do not know whether or not any of Mann-type or the Ishikawa-type iteration methods is T-stable for quasi-contractive maps in Banach spaces more general than those considered here.

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