Indian J. pure appl. Math., 27(1) : 35-42, January 1996

ALMOST PRODUCT FINSLER STRUCTURES AND CONNECTIONS ON VECTOR BUNDLE

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(*Received* 6 *June* 1994; *after revision* 21 *August* 1995; *accepted* 5 *September* 1995)

The study of structures defined on the total space E of a vector bundle $EM = (E, \pi, M)$ yields an advantageous solution of Finsler type problems of a differentiable manifold[1, 2]. The purpose of the present paper is to study almost product Finsler structures, almost product Finsler type connections and to obtain the family of the almost product Finsler type connections relative to the almost product Finsler structure on vector bundle. Here the notations and terminology of Miron[2] are used.

1. INTRODUCTION

Let $EM = (E, \pi, M)$ be a vector bundle with the $(m + n)$-dimensional total space E and n-dimensional base space M. We denote by E_u^v the local fibre of the vertical bundle VE at $u \in E$ and N_u the complementary space of the E_u^v in the tangent space E_u at u to the total space E. Therefore

$$E_u = N_u \oplus E_u^v. \qquad \text{... (1.1)}$$

A nonlinear connection on E is a differentiable distribution $N : u \in E \to N_u \subset E_u$ with the property (1.1).

We denote by (x^i, y^a), the canonical coordinates of the point $u \in E$. The transformation of canonical coordinates $(x, y) \to (x', y')$ of a point of E are given by

$$\begin{aligned} x^{i'} &= x^{i'}(x^1, ..., x^n) \\ y^{a'} &= L^{a'}{}_b (x^1, ..., x^n)\, y^b \end{aligned} \qquad \text{... (1.2)}$$

$$\det (L^{a'}{}_b) \neq 0; \quad i = 1, 2, ..., n; \; a = 1, 2, .., m.$$

Let $(\partial/\partial x^i, \partial/\partial y^a)$, (dx^i, dy^a) be the natural basis and cobasis and $(\delta/\delta x^i, \partial/\partial y^a)$, $(dx^i, \delta y^a)$ the adapted basis and cobasis on E. These bases are related by the coefficients of non-linear connections as follows :

$$\delta/\delta x^i = \partial/\partial x^i - N^a{}_i\, \partial/\partial y^a, \;\; \delta y^a = dy^a + N^a{}_i\, dx^i. \qquad \ldots (1.3)$$

For every vector field X on E there exists the unique decomposition.

$$X = X^H + X^V\,;\; X^H{}_u \in N_u\,,\;\; X^V{}_u \in E_u\,, \qquad \forall \quad u \in E, \qquad \ldots (1.4)$$

where X^H is called the horizontal part and X^V is called the vertical part of X.

A linear connection ∇ on E is a Finsler type connection if and only if it determines a unique decomposition.

$$\nabla_X Y = \nabla^H{}_X Y + \nabla^V{}_X Y\,, \qquad \forall \quad X, Y \in \mathcal{X}(E) \qquad \ldots (1.5)$$

where $\mathcal{X}(E)$ is $\mathcal{F}$-(E) module of the vector fields on E.

The coefficients of Finsler type connection ∇ in adapted frames are denoted by $F\Gamma = (N, \underset{1}{F}, \underset{2}{F}, \underset{1}{C}, \underset{2}{C})$ and are given by (1.3) and

$$\nabla^H_{\delta/\delta x^k}\;\; \delta/\delta x^j = \underset{1}{F}{}^i{}_{jk}(x,y)\, \delta/\delta x^i, \;\; \nabla^H_{\delta/\delta x^k}\;\; \partial/\partial y^b = \underset{2}{F}{}^a{}_{bk}(x,y)\;\; \partial/\partial y^a,$$

$$\nabla^V_{\partial/\partial y^c}\;\; \delta/\delta x^j = \underset{1}{C}{}^i{}_{jc}(x,y)\, \delta/\delta x^i, \;\; \nabla^V_{\partial/\partial y^c}\;\; \partial/\partial y^b = \underset{2}{C}{}^a{}_{bc}(x,y)\;\; \partial/\partial y^a,$$

$$\ldots (1.6)$$

where $\underset{1}{F}\,(= \underset{1}{F}{}^i{}_{jk}(x,y))$ and $\underset{2}{F}\,(= \underset{2}{F}{}^a{}_{bk}(x,y))$ are called the coefficients of h-connection ∇^H and $\underset{1}{C}\,(= \underset{1}{C}{}^i{}_{jc}(x,y))$ and $\underset{2}{C}\,(= \underset{2}{C}{}^a{}_{bc}(x,y))$ are called the coefficients of v-connection ∇^V.

For a tensor field K, for instance of type (1, 1) on E, there are four Finsler tensor fields of types $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$. Their components are denoted by $K^i{}_j$, $K^a{}_j$, $K^i{}_a$, $K^a{}_j$; h- and v-covariant derivatives are given by

$$K^i{}_{j|k} = \delta K^i{}_j/\delta x^k + \underset{1}{F}{}^i{}_{hk} K^h{}_j - \underset{1}{F}{}^h{}_{jk} K^i{}_h,$$

$$K^i{}_{j\|a} = \partial K^i{}_j/\partial y^a + \underset{1}{C}{}^i{}_{ha} K^h{}_j - \underset{1}{C}{}^h{}_{ja} K^i{}_h,$$

etc.

If $F\Gamma = (N, \underset{1}{F}, \underset{2}{F}, \underset{1}{C}, \underset{2}{C})$ and $F\overline{\Gamma} = (\overline{N}, \underset{1}{\overline{F}}, \underset{2}{\overline{F}}, \underset{1}{\overline{C}}, \underset{2}{\overline{C}})$ are two Finsler type connections on E, then a unique system of Finsler tensor fields $(A^a{}_i, \underset{1}{B}{}^i{}_{jk}, \underset{2}{B}{}^a{}_{bk}, \underset{1}{D}{}^i{}_{jc}, \underset{2}{D}{}^a{}_{bc})$ is determined such that

$$\left\{\begin{array}{l} \overline{N}^a{}_i = N^a{}_i - A^a{}_i \\ \underset{1}{\overline{F}}{}^i{}_{jk} = \underset{1}{F}{}^i{}_{jk} - \underset{1}{B}{}^i{}_{jk}, \quad \underset{2}{\overline{F}}{}^a{}_{bk} = \underset{2}{F}{}^a{}_{bk} - \underset{2}{B}{}^a{}_{bk}, \\ \underset{1}{\overline{C}}{}^i{}_{jc} = \underset{1}{C}{}^i{}_{jc} - \underset{1}{D}{}^i{}_{jc}, \quad \underset{2}{\overline{C}}{}^a{}_{bc} = \underset{2}{C}{}^a{}_{bc} - \underset{2}{D}{}^a{}_{bc}. \end{array}\right\} \quad \ldots (1.7)$$

Conversely, given the Finsler type connection $F\Gamma = (N, \underset{1}{F}, \underset{2}{F}, \underset{1}{C}, \underset{2}{C})$ and a system $(A, \underset{1}{B}, \underset{2}{B}, \underset{1}{D}, \underset{2}{D})$ of Finsler tensor fields, the connection $F\overline{\Gamma}$ given by (1.7) is a Finsler type connection on E.

The essence of this paper is to apply the analogous method in paper by Miron-Hashiguchi[3] to equation (2.2).

Further in the following for clarity we use the term Finsler type connection in place of Finsler connection.

2. Almost Product Finsler Type Connection on Vector Bundle

Definition 2.1 — An almost product structure on E is defined by $J \in \tau^1{}_1(E)$, satisfying $J^2 = I$. In components form

$$J = J^i{}_j(x, y)\, \delta/\delta x^i \otimes dx^j + J^i_a(x, y)\, \delta/\delta x^i \otimes dy^a$$

$$+ J^a{}_i(x, y)\, \partial/\partial y^a \otimes dx^i + J^a{}_b(x, y)\, \partial/\partial y^a \otimes dy^b$$

which has matrix

$$\begin{pmatrix} J^i{}_j & J^i{}_b \\ J^a{}_j & J^a{}_b \end{pmatrix}$$

with

$$J^i{}_j J^j{}_k + J^i{}_b J^b{}_k = \delta^i{}_k, \quad J^a{}_j J^j{}_k + J^a{}_b J^b{}_k = 0.$$

$$J^i{}_j J^j{}_c + J^i{}_b J^b{}_c = 0, \quad J^a{}_j J^j{}_c + J^a{}_b J^b{}_c = \delta^a{}_c. \quad \ldots (2.1)$$

Definition 2.2 — A Finsler type connection ∇ on E is called almost product Finsler type connection or connection compatible with almost product Finsler structure J, if and only if

$$\left.\begin{array}{l} J^i{}_{j|k} = 0, \; J^i{}_{j\|c} = 0, \; J^i{}_{c|k} = 0, \; J^j{}_{c\|b} = 0 \\ J^c{}_{i|k} = 0, \; J^c{}_{i\|b} = 0, \; J^a{}_{b|k} = 0, \; J^a{}_{b\|c} = 0. \end{array}\right\} \quad \ldots (2.2)$$

Hereafter, for brevity, let us take $J^i{}_a = J^a{}_i = 0$.

Then the new matrix is

$$\begin{pmatrix} J^i_j & \cdot 0 \\ 0 & J^a_b \end{pmatrix}$$

with $$J^i_j \, J^j_k = \delta^i_k, \; J^a_b \, J^b_c = \delta^a_c. \qquad \text{... (2.3)}$$

We, now associate to the almost product Finsler structure J, for the decomposition $J = J^i_j \, \delta/\delta x^i \otimes dx^j + J^a_b \, \partial/\partial y^a \otimes \delta y^b$, the Finsler tensor fields $\underset{1}{\phi}, \underset{2}{\phi}, \underset{1}{\psi}, \underset{2}{\psi}$ called Obata operators[5]. There components are

$$\left\{ \begin{array}{ll} \underset{1}{\phi}{}^{ik}{}_{jl} = (\delta^i_j \, \delta^k_l + J^i_j J^k_l)/2, & \underset{2}{\phi}{}^{ik}{}_{jl} = (\delta^i_j \, \delta^k_l - J^i_j \, J^k_l)/2, \\ \underset{1}{\psi}{}^{ac}{}_{bd} = (\delta^a_b \, \delta^c_d + J^a_b J^c_d)/2, & \underset{2}{\psi}{}^{ac}{}_{bd} = (\delta^a_b \, \delta^c_d - J^a_b \, J^c_d)/2. \end{array} \right\} \qquad \text{... (2.4)}$$

The operators act, in the following ways

$$\underset{1}{\phi} \cdot t : \underset{1}{\phi}{}^{ik}{}_{jh} \, t^j_{km}$$

t being a Finsler tensor field of type (1.2).

Lemma 2.1 (Yano[6]) — A tensor a being given, in order that the linear equation

$$\underset{2}{\phi} \cdot X = A \mid \underset{1}{\psi} \cdot X = A \qquad \text{... (2.5)}$$

with unknown tensor X admit a solution, it is necessary and sufficient that A satisfies

$$\underset{1}{\phi} \cdot A = 0 \mid \underset{2}{\psi} \cdot A = 0 \qquad \text{... (2.6)}$$

and if A satisfies this condition, then the general solution of (2.3) is given by

$$X = A + \underset{1}{\phi} \cdot Y \mid X = A + \underset{2}{\psi} \cdot Y. \qquad \text{... (2.7)}$$

Y being an arbitrary tensor of the same type as X.

We can see that for any almost product Finsler type connection on vector bundle, the operators $\underset{1}{\phi}, \underset{2}{\phi}, \underset{1}{\psi}$ and $\underset{2}{\psi}$ are h- and v-covariant constants.

The family of all almost product Finsler type connections on the total space of vector bundle can be determined by a well known method[3] based on the above lemma.

Let $(A^a_i, \underset{1}{B}{}^i_{jk}, \underset{2}{B}{}^a_{bk}, \underset{1}{D}{}^i_{jc}, \underset{2}{D}{}^a_{bc})$ be the difference tensors of the pair $(F\overline{\Gamma}, F\Gamma)$. Then any $F\overline{\Gamma}$ on E can be expressed as (1.7). Requiring $F\overline{\Gamma}$ to be an almost product Finsler type connection, we obtain for the Finsler tensor fields $A, \underset{1}{B}, \underset{2}{B}, \underset{1}{D}$ and $\underset{2}{D}$ the following expressions :

$$\underset{1}{B}{}^{i}{}_{lk} = (J^{j}{}_{l}J^{i}{}_{j|k})/2 + (J^{j}{}_{l}A^{a}{}_{k}\,\partial/\partial y^{a} J^{i}{}_{j})/2 + \underset{1}{\phi}{}^{im}{}_{jl}\underset{1}{Y}{}^{j}{}_{mk}\ ,$$

$$\underset{2}{B}{}^{a}{}_{gk} = (J^{b}{}_{g}J^{a}{}_{b|k})/2 + (J^{b}{}_{g}A^{d}{}_{k}\,\partial/\partial y^{d} J^{a}{}_{b})/2 + \underset{1}{\psi}{}^{ac}{}_{bg}\underset{2}{Y}{}^{b}{}_{ck}\ ,$$

$$\underset{1}{D}{}^{i}{}_{lc} = (J^{j}{}_{l}J^{i}{}_{j\|c})/2 + \underset{1}{\phi}{}^{ik}{}_{jl}\,\underset{1}{Z}{}^{j}{}_{kc}\ ,$$

$$\underset{2}{D}{}^{a}{}_{gc} = (J^{b}{}_{g}J^{a}{}_{b\|c})/2 + \underset{1}{\psi}{}^{ad}_{bg}\,\underset{2}{Z}{}^{b}{}_{dc}\ ,$$

where $\underset{1}{Y}$, $\underset{2}{Y}$, $\underset{1}{Z}$ and $\underset{2}{Z}$ are arbitrary Finsler tensor fields.

Hence, we have :

Theorem 2.1 — The general family of the almost product Finsler type connection $F\overline{\Gamma} = (\overline{N}, \underset{1}{\overline{F}}, \underset{2}{\overline{F}}, \underset{1}{\overline{C}}, \underset{2}{\overline{C}})$ relative to the almost product Finsler structure J with the assumption that $J^{a}_{i} = J^{i}_{a} = 0$ on the total space E of vector bundle EM is given by

$$\overline{N}{}^{a}{}_{i} = N^{a}{}_{i} - A^{a}{}_{i},$$

$$\underset{1}{\overline{F}}{}^{i}{}_{jk} = \underset{1}{F}{}^{i}{}_{jk} - (J^{l}{}_{j}J^{i}{}_{l|k})/2 - (J^{l}{}_{j}A^{a}{}_{k}\,\partial/\partial y^{a} J^{i}{}_{l})/2 - \underset{1}{\phi}{}^{im}{}_{lj}\,\underset{1}{Y}{}^{l}{}_{mk}\ ,$$

$$\underset{2}{\overline{F}}{}^{a}{}_{bk} = \underset{2}{F}{}^{a}{}_{bk} - (J^{g}{}_{b}J^{a}{}_{g|k})/2 - (J^{g}{}_{b}A^{d}{}_{k}\,\partial/\partial y^{d} J^{a}{}_{g})/2 - \underset{1}{\psi}{}^{ac}{}_{gb}\,\underset{2}{Y}{}^{g}{}_{ck}\ ,$$

$$\underset{1}{\overline{C}}{}^{i}{}_{jc} = \underset{1}{C}{}^{i}{}_{jc} - (J^{l}{}_{j}J^{i}{}_{l\|c})/2 - \underset{1}{\phi}{}^{ik}{}_{lj}\underset{1}{Z}{}^{l}{}_{kc}\ ,$$

$$\underset{2}{\overline{C}}{}^{a}{}_{bc} = \underset{2}{C}{}^{a}{}_{bc} - (J^{g}{}_{b}J^{a}{}_{g\|c})/2 - \underset{1}{\psi}{}^{ad}{}_{gb}\underset{2}{Z}{}^{g}{}_{dc}\ , \qquad \text{... (2.8)}$$

where | (resp . ||) is the h-(resp. v-) covariant derivative with respect to an arbitrary initial Finsler type connection $F\Gamma$ on E and X, $\underset{1}{Y}$, $\underset{2}{Y}$, $\underset{1}{Z}$ and $\underset{2}{Z}$ are arbitrary Finsler tensor fields.

If we take $A^{a}{}_{i} = 0$, $\underset{1}{Y}{}^{i}{}_{jk} = 0$, $\underset{2}{Y}{}^{a}{}_{bk} = 0$, $\underset{1}{Z}{}^{i}{}_{jc} = 0$ and $\underset{2}{Z}{}^{a}{}_{bc} = 0$, then we have

Theorem 2.2 — If the initial connection is $F\Gamma$ then the following Finsler type connection

$$\overset{k}{N}{}^{a}{}_{i} = N^{a}{}_{i}\ ,$$

$$\underset{1}{\overset{k}{F}}{}^{i}{}_{jk} = \underset{1}{F}{}^{i}{}_{jk} - (J^{l}{}_{j}J^{i}{}_{l|k})/2,\ \underset{2}{\overset{k}{F}}{}^{a}{}_{bk} = \underset{2}{F}{}^{a}{}_{bk} - (J^{g}{}_{b}J^{a}{}_{g|k})/2,$$

$$\underset{1}{\overset{k}{C}}{}^{i}{}_{jc} = \underset{1}{C}{}^{i}{}_{jc} - (J^{l}{}_{j}J^{i}{}_{l\|c})/2,\ \underset{2}{\overset{k}{C}}{}^{a}{}_{bc} = \underset{2}{C}{}^{a}{}_{bc} - (J^{g}{}_{b}J^{a}{}_{g\|c})/2 \qquad \text{... (2.9)}$$

is an almost product Finsler type connection for the given decomposition of J.

Definition 2.3 — The Finsler type connection $K\Gamma = (\overset{k}{N}, \underset{1}{\overset{k}{F}}, \underset{2}{\overset{k}{F}}, \underset{1}{\overset{k}{C}}, \underset{2}{\overset{k}{C}})$ given by (2.9) is called the almost product Kawaguchi type connection on E derived from $F\Gamma$.

Next we find the interesting particular case, if we take $A^a{}_i = 0$, $\underset{1}{Y}{}^i{}_{jk} = \underset{2}{Y}{}^a{}_{bk} = \underset{1}{Z}{}^i{}_{jc} = \underset{2}{Y}{}^a{}_{bc} = 0$, and the initial connection $F\Gamma$ as follows :

$$\overset{m}{N}{}^a{}_i = \overset{0}{N}{}^a{}_i, \quad \underset{1}{\overset{m}{F}}{}^i{}_{jk} = \underset{1}{\overset{0}{F}}{}^i{}_{jk}, \quad \underset{2}{\overset{m}{F}}{}^a{}_{bk} = \partial \overset{0}{N}{}^a{}_k / \partial y^b$$

$$\underset{1}{\overset{m}{C}}{}^i{}_{jc} = \underset{1}{\overset{0}{C}}{}^i{}_{jc} \quad \text{and} \quad \underset{2}{\overset{m}{C}}{}^a{}_{bc} = \underset{2}{\overset{0}{C}}{}^a{}_{bc}$$

where $\overset{0}{N}$ is a fixed non-linear connection. Then the following Finsler type connection $F\overset{q}{\Gamma} = (\overset{q}{N}, \underset{1}{\overset{q}{F}}, \underset{2}{\overset{q}{F}}, \underset{1}{\overset{q}{C}}, \underset{2}{\overset{q}{C}})$

$$\left.\begin{aligned} &\overset{q}{N}{}^a{}_i = \overset{m}{N}{}^a{}_i \\ &\underset{1}{\overset{q}{F}}{}^i{}_{jk} = \underset{1}{\overset{m}{F}}{}^i{}_{jk} - (J^l{}_j J^i{}_{l \overset{m}{|} k})/2, \quad \underset{2}{\overset{q}{F}}{}^a{}_{bk} = \underset{2}{\overset{m}{F}}{}^a{}_{bk} - (J^g{}_b J^a{}_{g \overset{m}{|} k})/2, \\ &\underset{1}{\overset{q}{C}}{}^i{}_{jc} = \underset{1}{\overset{m}{C}}{}^i{}_{jc} - (J^l{}_j J^i{}_{l \overset{m}{\|} c})/2, \quad \underset{2}{\overset{q}{C}}{}^a{}_{bc} = \underset{2}{\overset{m}{C}}{}^a{}_{bc} - (J^g{}_b J^a{}_{g \overset{m}{\|} c})/2 \end{aligned}\right\} \quad \dots (2.10)$$

is an almost product Finsler type connection for the given decomposition of J, where $\overset{m}{|}$ (resp. $\overset{m}{\|}$) is h- (resp. v-) covariant derivative with respect to $F\overset{m}{\Gamma}$ on E. The connection $F\overset{q}{\Gamma}$ will be called canonical almost product Finsler type connection derived from $F\overset{m}{\Gamma}$ on E.

We can obtain other results as follows.

Theorem 2.3 — If the initial Finsler type connection $F\Gamma$ is an almost product Finsler type connection, then the general family of the almost product Finsler type connection for the given decomposition of J is given by

$$\bar{N}{}^a{}_i = N^a{}_i - A^a{}_i,$$

$$\underset{1}{\bar{F}}{}^i{}_{jk} = \underset{1}{F}{}^i{}_{jk} - (J^l{}_j A^a{}_k \, \partial/\partial y^a J^i{}_l)/2 - \underset{1}{\phi}{}^{im}{}_{lj} \underset{1}{Y}{}^l{}_{mk},$$

$$\underset{2}{\bar{F}}{}^a{}_{bk} = \underset{2}{F}{}^a{}_{bk} - (J^g{}_b A^d{}_k \, \partial/\partial y^d J^a{}_g)/2 - \underset{1}{\psi}{}^{ac}{}_{gb} \underset{2}{Y}{}^g{}_{ck},$$

$$\underset{1}{\bar{C}}{}^i{}_{jc} = \underset{1}{C}{}^i{}_{jc} - \underset{1}{\phi}{}^{ik}{}_{lj} \underset{1}{Z}{}^l{}_{kc}, \quad \underset{2}{\bar{C}}{}^a{}_{bc} = \underset{2}{C}{}^a{}_{bc} - \underset{1}{\psi}{}^{ad}{}_{gb} \underset{2}{Z}{}^g{}_{dc}. \quad \dots (2.11)$$

where A^a_i, $\underset{1}{Y}{}^i_{jk}$, $\underset{2}{Y}{}^a_{bk}$, $\underset{1}{Z}{}^i_{jc}$ and $\underset{2}{Z}{}^a_{bc}$ are arbitrary Finsler tensor fields.

Since A^a_i in Theorem (2.3) is arbitrary, so any non-linear connection N may become the non-linear connection of an almost product Finsler type connection. We denote by $F\Gamma(N)$, a Finsler type connection having N as non-linear connection. Now we have

Theorem 2.4 — The general family of almost product Finsler type connections $F\bar{\Gamma}(N)$ for the given decomposition of J is given by

$$\left.\begin{aligned} \underset{1}{\bar F}{}^i_{jk} = \underset{1}{F}{}^i_{jk} - \underset{1}{\phi}{}^{im}_{lj}\,\underset{1}{Y}{}^l_{mk}, \quad \underset{2}{\bar F}{}^a_{bk} = \underset{2}{F}{}^a_{bk} - \underset{1}{\psi}{}^{ac}_{gb}\,\underset{2}{Y}{}^g_{ck} \\ \underset{1}{\bar C}{}^i_{jc} = \underset{1}{C}{}^i_{jc} - \underset{1}{\phi}{}^{ik}_{lj}\,\underset{1}{Z}{}^l_{kc}, \quad \underset{2}{\bar C}{}^a_{bc} = \underset{2}{C}{}^a_{bc} - \underset{1}{\psi}{}^{ad}_{gb}\,\underset{2}{Z}{}^g_{dc} \end{aligned}\right\} \quad \ldots (2.12)$$

where $F\Gamma(N)$ is an initial almost product Finsler type connection and $\underset{1}{Y}$, $\underset{2}{Y}$, $\underset{1}{Z}$ and $\underset{2}{Z}$ are arbitrary Finsler type tensor fields.

3. A Family of Finsler Type Connections

Here we determine the family of Finsler type connections $F\Gamma$ for the given decomposition of J with the property $K\Gamma = \overset{q}{F\Gamma}$, so we consider the following set

$$\mathcal{F} = \{F\Gamma : K\Gamma = \overset{q}{F\Gamma}\}.$$

Let $(A^a{}_i, \underset{1}{B}{}^i_{jk}, \underset{2}{B}{}^a_{bk}, \underset{1}{D}{}^i_{jc}, \underset{2}{D}{}^a_{bc})$ be the difference tensors of the pair $(F\Gamma, \overset{q}{F\Gamma})$. Then

$$\left.\begin{aligned} & N^a{}_i = \overset{q}{N}{}^a{}_i - A^a{}_i \\ & \underset{1}{F}{}^i_{jk} = \underset{1}{\overset{q}{F}}{}^i_{jk} - \underset{1}{B}{}^i_{jk}, \quad \underset{2}{F}{}^a_{bk} = \underset{2}{\overset{q}{F}}{}^a_{bk} - \underset{2}{B}{}^a_{bk} \\ & \underset{1}{C}{}^i_{jc} = \underset{1}{\overset{q}{C}}{}^i_{jc} - \underset{1}{D}{}^i_{jc}, \quad \underset{2}{C}{}^a_{bc} = \underset{2}{\overset{q}{C}}{}^a_{bc} - \underset{2}{D}{}^a_{bc}. \end{aligned}\right\} \quad \ldots (3.1)$$

Writing the h- and v-covariant derivatives of both $J^i{}_j$ and $J^a{}_b$ with respect to $F\Gamma$ noticing that $\overset{q}{F\Gamma}$ is almost product Finsler type connection and proceeding in the same way as in Singh[5], we get

Theorem 3.1 — All Finsler type connections $F\Gamma$ for the given decomposition of J from the set $\mathcal{F}$ are given by

$$\left.\begin{array}{ll} N^a{}_i = \overset{q}{N}{}^a{}_i & \\ \underset{1}{F}{}^i{}_{jk} = \underset{1}{\overset{q}{F}}{}^i{}_{jk} - \underset{1}{\phi}{}^{ih}{}_{mj}\, \underset{1}{Y}{}^m{}_{hk}, & \underset{2}{F}{}^a{}_{bk} = \underset{2}{\overset{q}{F}}{}^a{}_{bk} - \underset{1}{\psi}{}^{ad}{}_{cb}\, \underset{2}{Y}{}^c{}_{dk} \\ \underset{1}{C}{}^i{}_{jc} = \underset{1}{\overset{q}{C}}{}^i{}_{jc} - \underset{1}{\phi}{}^{ih}{}_{mj}\, \underset{1}{Z}{}^m{}_{hc}, & \underset{2}{C}{}^a{}_{bc} = \underset{2}{\overset{q}{C}}{}^a{}_{bc} - \underset{1}{\psi}{}^{ag}{}_{db}\, \underset{2}{Z}{}^d{}_{gc} \end{array}\right\} \quad \ldots (3.2)$$

where $\underset{1}{Y}{}^i_{jk}$, $\underset{2}{Y}{}^a_{bk}$, $\underset{1}{Z}{}^i_{jc}$ and $\underset{2}{Z}{}^a_{bc}$ are arbitrary tensor fields.

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