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CREEP TRANSITION IN A THIN ROTATING NON-HOMOGENEOUS DISC

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Creep stresses in a thin rotating annular disc made of non-homogeneous material have been investigated, using transition theory. As a numerical example, it has been seen that the presence of non-homogeneity having lesser value at the bore, reduces the stresses and the angular velocity required for steady state of creep as compared to the homogeneous disc; however, that having higher value at the bore than at the rim, increases the magnitude of angular velocity and creep stresses significantly, and hence increases the possibility of a fracture in the vicinity of the bore.

INTRODUCTION

The design of rotating discs has been a subject of prime interest with numerous practical applications in industry, engineering and space technology etc. Solution for isotropic homogeneous discs is given by Timoshenko and Goodier[18] in the elastic range and by Blazynski[1], Chakrabarty[2] and Heyman[10] for the plastic range with the help of Tresca's yield condition. Rabotnov[12] and Wahl[19, 20] have obtained the stresses for the problem of creep by making certain simplifying assumptions, viz., elastic strains can be ignored, associated flow rule govern, shear-stress law applies, creep behaviour can be expressed by the product of a power function of stress times some function of time etc. Incompressibility of the material in creep problems is one of the most important assumption, and in fact, it is not possible to find a solution in closed form without it. Seth's transition theory[15, 16] does not require any adhoc assumptions; and thus poses and solves a more general problem from which cases pertaining to the assumptions can be worked out. It uitlizes the concept of generalized strain measure and asymptotic transition through the turning points of the governing differential equations and has been successfully applied to many problems in plasticity and creep[4-9]. Seth[16] has defined the generalized strain measure as,

$$e_{ii} = \int_0^{e_{ii}^A} [1 - 2e_{ii}^A]^{(n/2)-1}\, de_{ii}^A = \frac{1}{n}[1 - (1 - 2e_{ii}^A)^{n/2}] \quad \text{... (1)}$$

where n is the measure and e_{ii}^A are the principal Almansi finite strain components.

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For $n = -2, -1, 0, 1, 2$ it gives Cauchy, Green, Hencky, Swainger and Almansi measures respectively.

There are materials (rock masses, hot-rolled copper, aluminium and magnesium alloys etc.) where elastic constants, or moduli, of the material vary with the location of the point. For such materials, certainly some degree of non-homogeneity is present. Olszak[11] have discussed the effect of non-homogeneity in elasticity and plasticity. Chaudhuri[3] obtained the stresses in a non-homogeneous rotating annulus, allowing the Poisson's ratio to vary radially. In this paper, we have solved the problem of creep in a thin rotating annular disc made of non-homogeneous material, using the transition theory. Non-homogeneity in the disc is taken due to the radial variation of the modulus of rigidity;

$$\mu = \mu_0 r^{-k} \qquad \dots (2)$$

where μ_0 and k are the real constants.

Governing Equations

Consider an annular disc made of non-homogeneous material having a and b as internal and external radii respectively, and rotating with an angular velocity ω. Thickness of the disc is assumed sufficiently small, so that it is effectively in a state of plane stress (i.e. $\tau_{zz} = 0$). The displacement components in cylindrical polar co-ordinates are[7-9],

$$u = r(1 - \beta),\ v = 0,\ w = dz \qquad \dots (3)$$

where β is a function of r only and d is a constant.

Using eqn. (3) in eqn. (1), we get the generalized strain components as,

$$\left.\begin{aligned} e_{rr} &= \frac{1}{n}\left[1 - (r\beta' + \beta)^n\right], \\ e_{\theta\theta} &= \frac{1}{n}\left[1 - \beta^n\right], \\ e_{zz} &= \frac{1}{n}\left[1 - (1 - d)^n\right], \\ e_{r\theta} &= e_{\theta z} = e_{zr} = 0. \end{aligned}\right\} \qquad \dots (4)$$

Stress-strain relations for the problem are[17, 7-9],

$$\left.\begin{aligned} \tau_{rr} &= \left(\frac{2\lambda\mu}{\lambda+2\mu}\right)[e_{rr}+e_{\theta\theta}] + 2\mu\, e_{rr}, \\ \tau_{\theta\theta} &= \left(\frac{2\lambda\mu}{\lambda+2\mu}\right)[e_{rr}+e_{\theta\theta}] + 2\mu\, e_{\theta\theta}, \\ \tau_{zz} &= \tau_{zr} = \tau_{r\theta} = \tau_{\theta z} = 0 \end{aligned}\right\} \quad \ldots (5)$$

where μ is a function of r only and λ is a constant.
Substituting eqn. (4) in eqn. (5), the non-zero stress components become

$$\left.\begin{aligned} \tau_{rr} &= \frac{2\mu}{n}[3-2c-\beta^n\{1-c+(2-c)(P+1)^n\}], \\ \tau_{\theta\theta} &= \frac{2\mu}{n}[3-2c-\beta^n\{2-c+(1-c)(P+1)^n\}] \end{aligned}\right\} \quad \ldots (6)$$

where

$$c = 2\mu/(\lambda+2\mu), \quad r\beta' = \beta P.$$

Equation of equilibrium to be satisfied is

$$\frac{\partial(r\tau_{rr})}{\partial r} - \tau_{\theta\theta} + \rho\,\omega^2 r^2 = 0 \quad \ldots (7)$$

where ρ is the constant density of the material.
Using eqn. (6) in eqn. (7), we get a non-linear differential equation in β as,

$$\begin{aligned} &1 + rc' + (rc'-1)(P+1)^n - (nP + r\mu'/\mu)\{1-c+(2-c)(P+1)^n\} \\ &+ \frac{r}{\beta^n}\{(3-2c)\,\mu'/\mu - 2c' + n\rho\,\omega^2 r/2\mu\} - n(2-c)\,\beta P\,(P+1)^{n-1}\, dP/d\beta \\ &= 0. \end{aligned} \quad \ldots (8)$$

Transition points of β in eqn. (8) are $P = -1, \pm\infty$.
Boundary conditions require that

$$\tau_{rr} = 0 \quad \text{at } r = a \text{ and } r = b. \quad \ldots (9)$$

SOLUTION

Transitional creep stresses for stationary state of creep are obtained by taking the asymptotic solution through the stress difference at the transition point $P \to -1$ (Gupta and Rana[4], Gupta and Shukla[6]). We define the transition function R as,

$$R = \tau_{\theta\theta} - \tau_{rr} = \frac{2\mu\beta^n}{n}[(P+1)^n - 1]. \qquad \text{... (10)}$$

Taking the logarithmic differentiation of eqn. (10) with respect to r, we get

$$\frac{d(\log R)}{dr} = \frac{\mu'}{\mu} + \frac{nP}{r}\left[1 + \frac{\beta(P+1)^{n-1}\, dP/d\beta}{(P+1)^n - 1}\right]. \qquad \text{... (11)}$$

Substituting the value of $dP/d\beta$ from eqn. (8) in eqn. (11) and taking the asymptotic value as $P \to -1$, we get after integration

$$R = A(2-c)\,\mu r^{-n} \exp f_1(r) \qquad \text{... (12)}$$

where A is a constant of integration and

$$f_1(r) = \int \left[\left(\frac{1-c}{2-c}\right)[(\mu'/\mu) - (n/r)] - \frac{1}{(2-c)r} - \frac{\beta^{-n}}{2-c}\{|(3-2c)\,\mu'/\mu - 2c' + n\rho\omega^2 r/2\,\mu\}|\right] dr.$$

From eqns. (10) and (12), we have

$$\tau_{\theta\theta} - \tau_{rr} = Ar\, F_1(r) \qquad \text{... (13)}$$

where

$$F_1(r) = (2-c)\,\mu r^{-n-1} \exp f_1(r).$$

Substituting eqn. (13) in eqn. (7), we get after integration

$$\tau_{rr} = B + A\int F_1(r)\, dr - (\rho\omega^2 r^2/2) \qquad \text{... (14)}$$

where B is a constant of integration.

Applying the boundary conditions (9) in eqn. (14), we obtain

$$B = (\rho\omega^2 a^2/2) - A\int^{a} F_1(r)\, dr, \quad A = \frac{\rho\omega^2(b^2 - a^2)}{2\int_a^b F_1(r)\, dr}. \qquad \text{... (15)}$$

Consequently, eqns. (13) and (14) take the form

$$\left.\begin{aligned} \tau_{\theta\theta} &= \tau_{rr} + \frac{\rho\omega^2 (b^2 - a^2)}{2\int_a^b F_1(r)\,dr} \cdot r F_1(r) \\ \tau_{rr} &= \frac{\rho\omega^2}{2}\left[\left(\frac{b^2 - a^2}{\int_a^b F_1(r)\,dr}\right)\int_a^r F_1(r)\,dr - r^2 + a^2\right] \cdot \end{aligned}\right\} \quad \text{... (16)}$$

Equation (16) give the transitional creep stresses for a thin rotating non-homogeneous disc.

Taking the non-homogeneity in the disc due to variable modulus of rigidity as given in eqn. (2), eqns. (16) become

$$\left.\begin{aligned} \tau_{\theta\theta} &= \tau_{rr} + \frac{\rho\omega^2 (b^2 - a^2)}{2\int_a^b F_2(r)\,dr} \cdot r F_2(r) \\ \tau_{rr} &= \frac{\rho\omega^2}{2}\left[\left(\frac{b^2 - a^2}{\int_a^b F_2(r)\,dr}\right)\int_a^r F_2(r)\,dr - r^2 + a^2\right] \end{aligned}\right\} \quad \text{... (17)}$$

where

$$F_2(r) = 2\mu_0 (l_0 + r^k)^{t_1} r^{-t_2} \exp f_2(r)/(2l_0 + r^k),$$

$$f_2(r) = \frac{3k}{2n}\left(\frac{r}{D}\right)^n - \frac{\rho\omega^2}{4\mu_0 D^n}\left[(r^{t_2}/t_2) + l_0 \int \left(\frac{r^{t_2 - 1}}{l_0 + r^k}\right) dr\right]$$

$$- \frac{kl_0}{2D^n}\int \left(\frac{2l_0 + 5r^k}{2l_0 + r^k}\right)\frac{r^{n-1}}{(l_0 + r^k)}\,dr.$$

$$l_0 = \mu_0/\lambda, \quad t_1 = (1 + k - n)/2k, \quad t_2 = 2 + k + n,$$

$$\beta = D/r;$$

D being a constant.

It can be seen that the value of $|\tau_{\theta\theta} - \tau_{rr}|$ [although depends upon $F_2(r)$ i.e. l_0, t_1, t_2 etc.] is maximum at $r = a$, therefore, yielding of the disc starts at the internal surface and in this case eqns. (17) give

$$|\tau_{\theta\theta} - \tau_{rr}| = \left| \frac{\rho\omega_i^2}{2} \left\{ \frac{b^2 - a^2}{\int_a^b F_2(r)\, dr} \right\} aF_2(a) \right| = Y_1; \qquad \dots (18)$$

The angular velocity ω_i necessary for initial yielding can be obtained as,

$$\frac{\rho\omega_i^2 b^2}{Y_1} = \frac{2\int_a^b F_2(r)\, dr}{a\, F_2(a)\,(1 - R_0^2)} = W_1 \qquad \dots (19)$$

and eqns. (17) become

$$\sigma_{\theta_1} = \sigma_{r_1} + RF_2(r)/R_0 F_2(a),$$

$$\sigma_{r_1} = \frac{1}{a\, F_2(a)} \left[\int_a^r F_2(r)\, dr - \left(\frac{R^2 - R_0^2}{1 - R_0^2} \right) \int_a^b F_2(r)\, dr \right] \qquad \dots (20)$$

where

$$R_0 = a/b,\; R = r/b,\; \sigma_{\theta_1} = \tau_{\theta\theta}/Y_1,\; \sigma_{r_1} = \tau_{rr}/Y_1.$$

For an incompressible material[17], $\lambda \to \infty$ (or $I_0 \to 0$), therefore eqns. (17) yields

$$Y_2 = \left| \frac{\rho\omega_c^2 b^2 (1 - R_0^2)\, bF_3(b)}{2\int_a^b F_3(r)\, dr} \right| \qquad \dots (21)$$

and the angular velocity ω_c required by the disc is

$$\frac{\rho\, \omega_c^2 b^2}{Y_2} = \frac{2\int_a^b F_3(r)\, dr}{b\, F_3(b)\,(1 - R_0^2)} = W_2 \qquad \dots (22)$$

where

$$F_3(r) = \frac{2E}{3} r^{t_3 - 1} \exp f_3(r), \quad t_3 = -\frac{1}{2}[3(k+n) + 1],$$

$$f_3(r) = \frac{3k}{2n} \left(\frac{r}{D} \right)^n - \frac{3n\rho\omega_c^2 r^{t_2}}{4Et_2 D^n}, \quad \mu_0 \to E/3.$$

Therefore, the creep stresses (17) become

$$\left.\begin{aligned} \sigma_{\theta_2} &= \sigma_{r_2} + RF_3(r)/F_3(b) \\ \sigma_{r_2} &= \frac{1}{bF_3(b)}\left[\int_a^r F_3(r)\,dr - \left(\frac{R^2-R_0^2}{1-R_0^2}\right)\int_a^b F_3(r)\,dr\right] \end{aligned}\right\} \quad \ldots (23)$$

where

$$\sigma_{\theta_2} = \tau_{\theta\theta}/Y_2, \quad \sigma_{r_2} = \tau_{rr}/Y_2.$$

Taking the first term in the expansion of exponential (for smaller values of $f_3(r)$, eqns. (21)-(23) reduce to.

$$\left.\begin{aligned} Y_2^* &= \frac{\rho\omega_c^2 b^2}{2} t_3 \left[\frac{1-R_0^2}{1-R_0^{t_3}}\right], \\ W_2^* &= \frac{2}{t_3}\left[\frac{1-R_0^{t_3}}{1-R_0^2}\right], \\ \sigma_{\theta_2}^* &= \sigma_{r_2}^* + R^{t_3}, \\ \sigma_{r_2}^* &= \frac{1}{t^3}\left[R^{t_3} - R_0^{t_3} - \left(\frac{1-R_0^{t_3}}{1-R_0^2}\right)(R^2-R_0^2)\right]. \end{aligned}\right\} \quad \ldots (24)$$

Homogeneous Case

For a homogeneous disc ($k = 0$), eqn. (2) reduces to

$$\mu = \mu_0 \quad \text{(a constant).} \quad \ldots (25)$$

Consequently, eqn. (12) becomes

$$R = Ar\,F_4(r) \quad \ldots (26)$$

where

$$F_4(r) = r^{d_0-1}\exp\left[-\frac{n\rho\omega^2 r^{n+2}}{2\mu_0(2-c_0)(n+2)D^n}\right],$$

$$d_0 = -\left[\frac{1+n(3-2c_0)}{2-c_0}\right], \quad c_0 = 2\mu_0/(\lambda+2\mu_0)$$

and the creep transitional stresses (17) take the form

$$\left.\begin{aligned} \tau_{\theta\theta} &= \tau_{rr} + \frac{\rho\omega^2 (b^2 - a^2)\, r}{2\int_a^b F_4(r)\,dr} F_4(r), \\ \tau_{rr} &= \frac{\rho\omega^2}{2}\left[\left(\frac{b^2 - a^2}{\int_a^b F_4(r)\,dr}\right)\int_a^r F_4(r)\,dr - r^2 + a^2\right]. \end{aligned}\right\} \quad \ldots (27)$$

Yielding in the disc starts at the internal surface and we have

$$\left.\begin{aligned} Y_1 &= \left|\frac{\rho\omega_i^2}{2}\left(\frac{b^2 - a^2}{\int_a^b F_4(r)\,dr}\right) aF_4(a)\right|, \\ W_1 &= \frac{2\int_a^b F_4(r)\,dr}{a\,F_4(a)\,(1 - R_0^2)}, \\ \sigma_{\theta_1} &= \sigma_{r_1} + RF_4(r)/R_0F_4(a), \\ \sigma_{r_1} &= \frac{1}{a\,F_4(a)}\left[\int_a^r F_4(r)\,dr - \left(\frac{R^2 - R_0^2}{1 - R_0^2}\right)\int_a^b F_4(r)\,dr\right]. \end{aligned}\right\} \quad \ldots (28)$$

For an incompressible material[4, 6]; $c_0 \to -0$, eqn. (27) gives

$$\left.\begin{aligned} Y_2 &= \left|\frac{\rho\omega_c^2\, b^2 (1 - R_0^2)\, b\, F_5(b)}{2\int_a^b F_5(r)\,dr}\right|, \\ W_2 &= \frac{2\int_a^b F_5(r)\,dr}{b\,F_5(b)\,(1 - R_0^2)}, \\ \sigma_{\theta_2} &= \sigma_{r_2} + RF_5(r)/F_5(b), \\ \sigma_{r_2} &= \frac{1}{b\,F_5(b)}\left[\int_a^r F_5(r)\,dr - \left(\frac{R^2 - R_0^2}{1 - R_0^2}\right)\int_a^b F_5(r)\,dr\right] \end{aligned}\right\} \quad \ldots (29)$$

where

$$F_5(r) = r^{-3(n+1)/2} \exp\left[\frac{-3n\,\rho\omega_c^2\, r^{n+2}}{4E(n+2)D^n}\right].$$

Taking the first term in the expansion of the exponential, eqn. (29) becomes

$$\left.\begin{aligned} Y_2^* &= \frac{\rho\omega_c^2\, b^2\, d_1}{2}\left[\frac{1-R_0^2}{1-R_0^{d_1}}\right], \\ W_2^* &= \frac{2}{d_1}\left(\frac{1-R_0^{d_1}}{1-R_0^2}\right), \\ \sigma_{\theta_2}^* &= \sigma_{r_2}^* + R^{d_1}, \\ \sigma_{r_2}^* &= \frac{1}{d_1}\left[R^{d_1} - R_0^{d_1} - \left(\frac{1-R_0^{d_1}}{1-R_0^2}\right)(R^2 - R_0^2)\right] \end{aligned}\right\} \quad \ldots (30)$$

where

$$d_1 = -(3n+1)/2.$$

Numerical Discussion

To see the effect of non-homogeneity on creep transition in the analysis of a thin rotating annular disc, eqn. (24) has been considered. The stresses and the angular velocity required by a disc with radii ratio, $R_0 = 0.50$, have been calculated for various values of k and n, and the results are presented in Tables I and II.

Table I

Angular velocity (W_2^*) *required by a non-homogeneous* ($\mu = \mu_0 r^{-k}$) *disc with* $R = 0.50$, *for steady state of creep*

k/n	1/15	1/6	1/3	1.0	2.0
-1.00	1.37515	1.44141	1.56210	2.20914	4.0
-0.75	1.54941	1.62769	1.77058	2.54174	4.70160
-0.50	1.75555	1.84839	2.01821	2.94062	5.55362
-0.25	2.00032	2.11086	2.31343	3.42059	6.59120
0.00	2.29207	2.42415	2.66667	4.0	7.85806
0.25	2.64107	2.79944	3.09077	4.70160	9.40873
0.50	3.05999	3.25054	3.60165	5.55362	11.31126
0.75	3.56452	3.79451	4.21902	6.59120	13.65067
1.00	4.17409	4.45254	4.96731	7.85806	16.53333

TABLE II

Distribution of stresses in a non-homogeneous ($\mu = \mu_0 r^{-k}$) disc with R_0 = 0.50, for steady state of creep

k	σ^* \R	0.50	0.60	0.70	0.80	0.90	1.0
				n = 1/15			
−1.0	$\sigma^*_{r_2}$	0.0	0.03054	0.04557	0.04536	0.03012	0.0
	$\sigma^*_{\theta_2}$	0.53589	0.66199	0.77099	0.86341	0.93965	1.0
0.0	$\sigma^*_{r_2}$	0.0	0.13571	0.18676	0.17380	0.10899	0.0
	$\sigma^*_{\theta_2}$	1.51572	1.49436	1.42539	1.31706	1.17424	1.0
1.0	$\sigma^*_{r_2}$	0.0	0.41982	0.53348	0.46669	0.27861	0.0
	$\sigma^*_{\theta_2}$	4.28709	3.34318	2.64840	2.06445	1.52626	1.0
				n = 1/3			
−1.0	$\sigma^*_{r_2}$	0.0	0.04906	0.07165	0.07003	0.04577	0.0
	$\sigma^*_{\theta_2}$	0.70171	0.82366	0.90831	0.96446	0.99445	1.0
0.0	$\sigma^*_{r_2}$	0.0	0.18667	0.25143	0.23	0.14222	0.0
	$\sigma^*_{\theta_2}$	2.0	1.85333	1.68	1.48	1.25333	1.0
1.0	$\sigma^*_{r_2}$	0.0	0.55510	0.69097	0.59534	0.35136	0.0
	$\sigma^*_{\theta_2}$	5.65685	4.14120	3.13021	2.34227	1.65270	1.0
				n = 1.0			
−1.0	$\sigma^*_{r_2}$	0.0	0.12494	0.17287	0.16158	0.10168	0.0
	$\sigma^*_{\theta_2}$	1.41421	1.41593	1.36810	1.27961	1.15578	1.0
0.0	$\sigma^*_{r_2}$	0.0	0.39111	0.49959	0.43875	0.26272	0.0
	$\sigma^*_{\theta_2}$	4.0	3.16889	2.54041	2.00125	1.49728	1.0
1.0	$\sigma^*_{r_2}$	0.0	1.09263	1.29391	1.07626	0.61910	0.0
	$\sigma^*_{\theta_2}$	11.31371	7.06946	4.77854	3.25992	2.06505	1.0

It can be observed from Table I that a disc in which non-homogeneity varies from a lower value at the internal surface to a higher value at the outer surface (i.e. $k < 0$), requires lesser angular velocity for steady state of creep as compared to the homogeneous disc ($k = 0$), and reverse is the case for a disc in which non-homogeneity varies from a higher value at the internal surface to a lower value at the outer surface (i.e. $k > 0$). The magnitude of the velocity increases with the increase in the values of measure n; $1/15 \leq n \leq 2$.

Table II shows that the creep stresses are reduced considerably for a disc which is less non-homogeneous at the bore than at the rim ($k < 0$) as compared to the homogeneous case ($k = 0$); and their values are increased significantly for a disc which is more non-homogeneous at the bore than at the rim. The magnitude of the stresses further increases with the increase in the values of measure n. The circumferential stress is maximum at the outer surface of a disc with $k < 0$ for $1/15 \leq n \leq 1/3$. However, it becomes maximum at the internal surface of a disc with $k \geq 0$ and in such cases the magnitude of the radial stress also increases in the vicinity of the bore. Rimrott[13, 14] and Gupta[4] have shown similar results that if a material tends to fracture by cleavage, it will likely to begin as a subsurface fracture close to the bore, because it is where the largest stresses occur. This means that an increase in the values of $k > 0$, increases the possibility of a fracture in the vicinity of the bore even at a lesser values of n.

Thus, the non-homogeneity having lesser value at the bore ($k < 0$) lowers the values of stresses and angular velocity required for steady state of creep against the homogeneous case ($k = 0$); however, that having higher value at the bore ($k > 0$) increases the magnitudes of angular velocity and creep stresses significantly, and hence increases the possibility of a fracture at the bore. The circumferential creep stress is maximum at the outer surface of a disc with $k < 0$ for $1/15 \leq n \leq 1/3$, however, it becomes maximum at the bore for $k \geq 0$.

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