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THE SORET EFFECT ON THE ROTATORY THERMOSOLUTAL CONVECTION OF THE VERONIS TYPE

HARI MOHAN

Department of Mathematics, Himachal Pradesh University, Summer Hill, Shimla 171 005

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The present paper mathematically establishes that Soret[1]-driven rotatory thermosolutal convection of the Veronis[2] type cannot manifest oscillatory motion of growing amplitude if the modified thermosolutal Rayleigh number R_S', the Lewis number τ, and the Prandtl number σ satisfy the inequality $R_S' \leq \frac{27}{4}\pi^4\left(1+\frac{\tau}{\sigma}\right)$ and $\gamma < 1$, γ being the stability ratio.

1. INTRODUCTION

The hydrodynamic stability of nonreactive binary fluids has been the subject of extensive research[3, 4]. The onset of convection in binary fluids is of special interest because it may provide a sensitive method for the experimental determination of some of the transport coefficients[5]. Typically, the system considered theoretically is a horizontal layer of a two component fluid in a uniform vertical gravitational field. Such a system may become unstable against convection if it is subjected to temperature and/or chemical potential gradients. The stability properties of binary fluids are quite different from those of pure fluids because of the Soret and Dufour effects[1]. The Soret effect arises when the mass flux contains a term that depends on the temperature gradient. The analogous effect that arises from a concentration gradient-dependent term in the heat flux is called the Dufour effect. The phenomenological equations relating the heat flux J_Q and the solute flux J_c to the thermal and solute gradients presents in a binary fluid mixture may be formulated (see, for example, de Groot and Mazur[6]) as

$$J_Q = -k\,\nabla T - \rho\, TC\,\frac{\partial \mu}{\partial c}\,D'\,\nabla C, \qquad \text{... (a)}$$

$$J_c = -\,\rho D\,\,\{\nabla C + S_T C(1-C)\,\nabla T\}, \qquad \text{... (b)}$$

where T is temperature, C the concentration, ρ the density, K the thermal conductivity, D diffusivity, S_T the Soret coefficient, D' $(=S_T D)$ the Dufour coefficient and μ the chemical potential of the solute. In liquid mixture one can neglect the second term in J_Q, the Dufour effect term, but the same approximation cannot be justified in gaseous mixture. On the other hand, the second term in J_c, the Soret effect term, can be significant in both liquid and gaseous mixture. An externally imposed temperature gradient produces a chemical potential gradient in the system; the normal Soret effect occurs when the concentration of higher molecule mass is higher in the colder region. Similarly, an imposed chemical potential gradient results in a temperature gradient, and the normal Dufour effect is defined in analogy with Soret effect. The sense of migration of the molecular species is determined by the sign of the Soret coefficient[7].

Caldwell[8] pointed out that the concentration gradient set up by the Soret diffusion would lead to a situation similar to that considered by Veronis[2], if the sign of the Soret coefficient were opposite to the normal one. Veronis[2] has studied the onset of steady and oscillatory convection generated by infinitesimal perturbations, and has also done calculations on the onset of finite amplitude modes, all with free surface boundary conditions. Hurle and Jakeman assumed a salt distribution set up by thermal diffusion, and induced the Soret effect in their perturbation equations as they calculated the onset steady and oscillatory modes for both free and solid boundaries, for infinitesimal perturbation only. Veronis[9] (and Shirtcliffe[10]) used a quantity called R_s, a solute Rayleigh number and for reasonably dilute solution ($C < 1$),

$$R_s = \gamma R_T \qquad \text{or} \qquad \gamma = R_s/R_T. \qquad \text{... (c)}$$

The parameter γ is called the stability ratio when applied to thermosolutal or double diffusive phenomenon.

Rotation introduces a number of new elements into hydrodynamic problem and the key to our understanding of the consequences of rotation, some of which might appear rather intriguing and unexpected at first sight, is best provided by an analysis of its effect on certain general theorems of Helmholtz and Kelvin relating to vorticity. From a geophysical standpoint, the effect of rotation and magnetic field, acting separately or simultaneously, on the present problem is of practical interest. The case when rotation alone is present has been analyzed by Antorang and Velarde[11].

The present paper mathematically establishes that Soret-driven rotatory thermosolutal convection of Veronis' type cannot manifest as oscillatory motions of growing amplitude if the thermosolutal Rayleigh number R_S', the Lewis number τ (the ratio of mass diffusivity to heat diffusivity) and the Prandtl number σ satisfy the inequality $R_s' \leq \frac{27}{4}\pi^4\left(1+\frac{\tau}{\sigma}\right)$ and $\gamma < 1$, γ being the stability ratio.

2. Mathematical Formulation and Analysis

The relevant governing equations and boundary conditions of Soret-driven thermosolutal convection wherein a uniform rotation parallel to gravity is superimposed with slight change in notations are easily seen to be given by[11, 12]

$$(D^2 - a^2)\left(D^2 - a^2 - \frac{p}{\sigma}\right)\omega = R_T a^2 \theta - R_s a^2 \phi + (D\zeta)T, \quad \text{... (1)}$$

$$(D^2 - a^2 - p)\theta = -\omega, \quad \text{... (2)}$$

$$\{\tau(D^2 - a^2) - p\}\ \phi + \tau(D^2 - a^2)\,\theta = \omega, \quad \text{... (3)}$$

and

$$\left(D^2 - a^2 - \frac{p}{\sigma}\right)\zeta = -D\omega, \quad \text{... (4)}$$

where $R_T = \dfrac{g\alpha\beta\, d^4}{\kappa\nu}$, $\beta > 0$, and $R_S = \dfrac{g\alpha'\beta'\, d^4}{\kappa\nu}$, $\beta' = S_T C_0 (1 - C_0)\,\beta$, $\beta' > 0$,

with

$$\omega = 0 = \theta = \phi = D^2\omega = D\zeta \quad \text{at} \quad z = 0 \text{ and } z = 1 \quad \text{... (5)}$$

(both boundaries dynamically free),

or

$$\omega = 0 = \theta = \phi = D\omega = \zeta \quad \text{at} \quad z = 0 \text{ and } z = 1 \quad \text{... (6)}$$

(both boundaries rigid),

or $\omega = 0 = \theta = \phi = D^2\omega = D\zeta$ at $z = 0$

and $\omega = 0 = \theta = \phi = D\omega = \zeta$ at $z = 1$ ... (7)

(lower boundary dynamically free and upper boundary rigid),

or $\omega = 0 = \theta = \phi = D^2\omega = D\zeta$ at $z = 1$

and $\omega = 0 = \theta = \phi = D\omega = \zeta$ at $z = 0$ ... (8)

(lower boundary rigid and upper boundary dynamically free),

where z is the real independent variable such that $0 \le z \le 1$, $D = \dfrac{d}{dz}$ is the differentiation with respect to z, a^2 is a constant, $\sigma > 0$ is a constant, $\tau > 0$ is a constant, R_T and R_S are positive constants, $T > 0$ is a constant, $p = p_r + ip_i$ is a complex constant in general such that p_r and p_i real constants and as a consequence the dependent variables

$$\omega(z) = \omega_r(z) + i\omega_i(z),\ \theta(z) = \theta_r(z) + i\theta_i(z),\ \phi(z) = \phi_r(z) + i\phi_i(z)$$

and $\zeta(z) = \zeta_r(z) + i\,\zeta_i(z)$ are complex valued functions of real variable z. The meanings of symbols from the physical point of view are as follows : z is the vertical coordinate, $\dfrac{d}{dz}$ the differentiation along the vertical direction, a^2 the square of the wave number, σ the Prandtl number, τ the Lewis number, R_T the Rayleigh number, R_S the concentration Rayleigh number, T the Taylor number p the complex growth rate, ω the vertical velocity, θ the temperature, ϕ the concentration, and ζ the vertical vorticity. It may further be noted that eqns. (1)-(7) describe an eigenvalue problem for p and govern Soret-driven rotatory thermosolutal instability for any combination of dynamically free and rigid boundaries.

We prove the following theorem.

Theorem 1 — If $R_T > 0$, $R_s > 0$, $T > 0$, $\frac{\tau}{\sigma} \leq 1$, $p_r \geq 0$, $p_i \neq 0$ and $R_s' \leq \frac{27}{4}\pi^4\left(1+\frac{\tau}{\sigma}\right)$ then a necessary condition for the existence of nontrivial solution $(\omega, \theta, \phi, \zeta, p)$ of equation (1)-(4) with boundary conditions (5) or (6) is that

$$\gamma < 1. \qquad \text{... (9)}$$

PROOF : Using the transformation

$$\left.\begin{aligned} \tilde{\phi} &= \left(\frac{\tau-1}{\tau}\right)\phi + \theta \\ \tilde{\theta} &= \theta \\ \tilde{\omega} &= \omega \\ \text{and} \quad \tilde{\xi} &= \zeta \end{aligned}\right\} \qquad \text{... (10)}$$

eqns. (1)-(8) assume the following forms :

$$(D^2-a^2)\left(D^2-a^2-\frac{p}{\sigma}\right)\omega = R_T' a^2\theta - R_s' a^2 \phi + T(D\zeta) \qquad \text{... (11)}$$

$$(D^2-a^2-p)\theta = -\omega \qquad \text{... (12)}$$

$$\{\tau(D^2-a^2)-p\}\ \phi = -\frac{\omega}{\tau} \qquad \text{... (13)}$$

$$(D^2-a^2-p)\zeta = -D\omega \qquad \text{... (14)}$$

with

$$\omega = 0 = \theta = \phi = D^2\omega = D\zeta \quad \text{at} \quad z = 0 \text{ and } z = 1 \qquad \text{... (15)}$$

or

$$\omega = 0 = \theta = \phi = D\omega = \zeta \quad \text{at} \quad z = 0 \text{ and } z = 1 \qquad \text{... (16)}$$

or

$$\left.\begin{aligned} &\omega = 0 = \theta = \phi = D^2\omega = D\zeta \quad \text{at} \quad z = 0 \\ \text{and} \quad &\omega = 0 = \theta = \phi = D\omega = \zeta \quad \text{at} \quad z = 1 \end{aligned}\right\} \qquad \text{... (17)}$$

or

$$\left.\begin{aligned} &\omega = 0 = \theta = \phi = D\,\omega = \zeta \qquad \text{at} \quad z = 0 \\ &\text{and} \\ &\omega = 0 = \theta = \phi = D^2\,\omega = D\zeta \qquad \text{at} \quad z = 1 \end{aligned}\right\} \qquad \dots (18)$$

where R_T' (modified Rayleigh numbers) $= \left\{ R_T + \frac{\tau}{\tau - 1} R_s \right\}$, $R_s' = \langle \frac{\tau}{\tau - 1} \rangle R_s$ and the sign '~' has been omitted for simplicity.

Multiplying eqn. (11) by ω^* (*indicates complex cònjugation) throughout and integrating the resulting equation over the vertical range of z, we get

$$\int_0^1 \omega^* (D^2 - a^2) \left(D^2 - a^2 - \frac{p}{\sigma} \right) \omega\, dz$$

$$= R_T' a^2 \int_0^1 \omega^* \theta\, dz - R_s' a^2 \int_0^1 \omega^* \phi\, dz + T \int_0^1 \omega^* D\zeta. \qquad \dots (19)$$

Making use of eqns. (12)-(14) and the fact that $\omega(0) = 0 = \omega(1)$, we can write

$$R_T' a^2 \int_0^1 \omega^* \theta\, dz = -R_T' a^2 \int_0^1 \theta\, (D^2 - a^2 - p^*)\, \theta^*\, dz \qquad \dots (20)$$

$$-R_S' a^2 \int_0^1 \omega^* \phi\, dz = \tau^2 R_s' a^2 \int_0^1 \phi \left(D^2 - a^2 - \frac{p^*}{\tau} \right) \phi^*\, dz \qquad \dots (21)$$

and

$$T \int_0^1 \omega^* D\zeta\, dz = -T \int_0^1 \zeta\, D\omega^*\, dz = T \int_0^1 \zeta \left(D^2 - a^2 - \frac{p^*}{\sigma} \right) \zeta^*\, dz. \qquad \dots (22)$$

Combining eqns. (19)-(22), we obtain

$$\int_0^1 \omega^* (D^2 - a^2) \left(D^2 - a^2 - \frac{p}{\sigma} \right) \omega\, dz$$

$$= -R_T' a^2 \int_0^1 \theta\, (D^2 - a^2 - p^*)\, \theta^*\, dz + R_s' a^2 \tau^2 \int_0^1 \phi \left(D^2 - a^2 - \frac{p^*}{\tau} \right) \phi^*\, dz$$

$$+ T \int_0^1 \zeta \left(D^2 - a^2 - \frac{p^*}{\sigma} \right) \zeta^*\, dz. \qquad \dots (23)$$

Integrating the various terms of eqn. (23) by parts for an appropriate number of times and making use of either of the boundary conditions (15)-(16), it follows that

$$\int_0^1 (|D^2\omega|^2 + 2a^2|D\omega|^2 + a^4|\omega|^2)\,dz + \frac{p}{\sigma}\int_0^1 (|D\omega|^2 + a^2|\omega|^2)\,dz$$

$$= R_T' a^2 \int_0^1 (|D\theta|^2 + a^2|\theta|^2 + p^*|\theta|^2)\,dz$$

$$- R_s' a^2 \tau^2 \int_0^1 \left(|D\phi|^2 + a^2|\phi|^2 + \frac{p^*}{\tau}|\phi|^2\right) dz$$

$$- T\int_0^1 \left(|D\zeta|^2 + a^2|\zeta|^2 + \frac{p^*}{\sigma}|\zeta|^2\right) dz. \quad \text{... (24)}$$

Equating the real and imaginary parts of both sides of eqn. (24) and cancelling $p_i \neq 0$ throughout from the imaginary part, we get

$$\int_0^1 (|D^2\omega|^2 + 2a^2|D\omega|^2 + a^4|\omega|^2)\,dz + \frac{p_r}{\sigma}\int_0^1 (|D\omega|^2 + a^2|\omega|^2)\,dz$$

$$= R_T' a^2 \int_0^1 (|D\theta|^2 + a^2|\theta|^2 + p_r|\theta|^2)\,dz$$

$$- R_s' a^2 \tau^2 \int_0^1 \left(|D\phi|^2 + a^2|\phi|^2 + \frac{p_r}{\tau}|\phi|^2\right) dz$$

$$- T\int_0^1 \left(|D\zeta|^2 + a^2|\zeta|^2 + \frac{p_r}{\sigma}|\zeta|^2\right) dz \quad \text{... (25)}$$

and

$$\frac{1}{\sigma}\int_0^1 (|D\omega|^2 + a^2|\omega|^2)\,dz = -R_T' a^2 \int_0^1 |\theta|^2\,dz + R_s' a^2 \tau \int_0^1 |\phi|^2 + \frac{T}{\sigma}\int_0^1 |\zeta|^2\,dz. \quad \text{... (26)}$$

We write eqn. (25) in the alternative form

$$\int_0^1 (|D^2\omega|^2 + 2a^2|D\omega|^2 + a^4|\omega|^2)\,dz + \frac{p_r}{\sigma}\int_0^1 (|D\omega|^2 + a^2|\omega|^2)\,dz$$

$$= R_T' a^2 \int_0^1 (|D\theta|^2 + a^2|\theta|^2)\,dz - R_s' a^2 \tau^2 \int_0^1 |D\phi|^2 +$$

(Equation continued on p. 615)

$$+ a^2 |\phi|^2) dz - T \int_0^1 (|D\zeta|^2 + a^2 |\zeta|^2) dz$$

$$+ p_r a^2 \left(R_T' \int_0^1 |\theta|^2 dz - R_s' \tau \int_0^1 |\phi|^2 dz - T \int_0^1 |\zeta|^2 dz \right) \quad \ldots (27)$$

and derive the validity of the theorem from the resulting inequality obtained by replacing each one of the terms of this equation by its appropriate estimate.

We first note that since ω, θ and ϕ satisfy $\omega(0) = 0 = \omega(1)$, $\theta(0) = 0 = \theta(1)$ and $\phi(0) = 0 = \phi(1)$, we have by Rayleigh-Ritz inequality[13]

$$\int_0^1 |D\omega|^2 dz \geq \pi^2 \int_0^1 |\omega|^2 dz \quad \ldots (28)$$

$$\int_0^1 |D\theta|^2 dz \geq \pi^2 \int_0^1 |\theta|^2 dz \quad \ldots (29)$$

$$\int_0^1 |D\phi|^2 dz \geq \pi^2 \int_0^1 |\phi|^2 dz \quad \ldots (30)$$

and

$$\int_0^1 |D^2\omega|^2 dz \geq \pi^4 \int_0^1 |\omega|^2 dz. \quad \ldots (31)$$

Utilizing inequalities (28) and (31), we obtain

$$\int_0^1 (|D^2 \omega|^2 + 2a^2 |D\omega|^2 + a^4 |\omega|^2) dz$$

$$\geq (\pi^2 + a^2)^2 \int_0^1 |\omega|^2 dz. \quad \ldots (32)$$

Since $p_r \geq 0$, we have

$$\frac{p_r}{\sigma} \int_0^1 (|D\omega|^2 + a^2 |\omega|^2) dz \geq 0. \quad \ldots (33)$$

Next, multiplying eqn. (12) by θ^* throughout and integrating the various terms on the left hand side of resulting equation by parts for an appropriate number of times by making use of the boundary conditions on θ, namely $\theta(0) = 0 = \theta(1)$, we have from the real part of final equation

$$\int_0^1 (|D\theta|^2 + a^2|\theta|^2)\,dz + p_r \int_0^1 |\theta|^2\,dz$$

$$= \text{Real part of} \left(\int_0^1 \theta^* \omega\, dz \right)$$

$$\leq \left| \int_0^1 \theta^* \omega\, dz \right| \leq \int_0^1 |\theta^* \omega|\, dz$$

$$\leq \int_0^1 |\theta^*||\omega|\, dz$$

$$\leq \int_0^1 |\theta||\omega|\, dz \leq \left\{ \int_0^1 |\theta|^2\, dz \right\}^{1/2} \left\{ \int_0^1 |\phi|^2\, dz \right\}^{1/2}$$

(utilizing Schwartz inequality),

and combining this inequality with inequality (29) and the fact that $p_r \geq 0$, we get

$$(\pi^2 + a^2)^2 \int_0^1 |\theta|^2\, dz \leq \left\{ |\theta|^2\, dz \right\}^{1/2} \left\{ \int_0^1 |\omega|^2\, dz \right\}^{1/2}$$

which implies that

$$\left\{ \int_0^1 |\theta|^2 \right\}^{1/2} \leq \frac{1}{(\pi^2 + a^2)} \left\{ \int_0^1 |\omega|^2\, dz \right\}^{1/2}$$

and thus

$$\int_0^1 (|D\theta|^2 + a^2|\theta|^2)\, dz \leq \frac{1}{(\pi^2 + a^2)} \int_0^1 |\omega|^2\, dz. \qquad \text{... (34)}$$

Further, utilizing inequality (30), we have

$$R_s' a^2 \tau^2 \int_0^1 (|D\phi|^2 + a^2|\phi|^2)\, dz$$

$$\geq R_s' a^2 \tau^2 (\pi^2 + a^2) \int_0^1 |\phi|^2\, dz$$

$$\geq \frac{\tau}{\sigma} (\pi^2 + a^2) \left\{ \int_0^1 (|D\omega|^2 + a^2|\omega|^2)\, dz - T \int_0^1 |\zeta|^2\, dz \right\}$$

[utilizing eqn. (26)]

$$\geq \frac{\tau}{\sigma}(\pi^2 + a^2)\left\{ (\pi^2 + a^2)\int_0^1 |\omega|^2 dz - T\int_0^1 |\zeta|^2 dz \right\} \quad \text{(utilizing eqn. 28)}$$

$$\geq \frac{\tau}{\sigma}(\pi^2 + a^2)^2 \int_0^1 |\omega|^2 dz - T(\pi^2 + a^2)\int_0^1 |\zeta|^2 dz$$

(since $\frac{\tau}{\sigma} \leq 1$),

and therefore,

$$- R_s' a^2 \tau^2 \int_0^1 (|D\phi|^2 + a^2|\phi|^2)\, dz$$

$$\leq -\frac{\tau}{\sigma}(\pi^2 + a^2)^2 \int_0^1 |\omega|^2 dz + T(\pi^2 + a^2)\int_0^1 |\zeta|^2 dz \qquad \text{... (35)}$$

so that

$$- R_s' a^2 \tau^2 \int_0^1 (|D\phi|^2 + a^2|\phi|^2)\, dz$$

$$- T\int_0^1 (|D\zeta|^2 + a^2|\zeta|^2)\, dz$$

$$\leq -\frac{\tau}{\sigma}(\pi^2 + a^2)^2 \int_0^1 |\omega|^2 dz + T\int_0^1 (\pi^2|\zeta|^2 - |D\zeta|^2)\, dz. \qquad \text{... (36)}$$

Also, from eqn. (26) and the fact that $p_r \geq 0$, we obtain

$$p_r a^2 \left\{ R_T' \int_0^1 |\theta|^2 dz - R_s' \tau \int_0^1 |\phi|^2 dz - \frac{T}{a^2\sigma}\int_0^1 |\zeta|^2 dz \right\} \leq 0. \qquad \text{... (37)}$$

Now if permissible, let $R_s' \geq R_T'$ or $\gamma \geq 1$. Then in that case, we derive from equation. (27) and inequalities (32)-(34), (36) and (37) that

$$\left\{ (\pi^2 + a^2)^2 \left(1 + \frac{\tau}{\sigma}\right) - \frac{R_T' a^2}{(\pi^2 + a^2)} \right\} \int_0^1 |\omega|^2 dz$$

$$+ T\int_0^1 (|D\zeta|^2 - \pi^2|\zeta|^2)\, dz < 0. \qquad \text{... (38)}$$

The second integral on the left-hand side is always nonnegative irrespective of the nature of the bounding surfaces. For the case when these are both rigid we have by the boundary conditions (6) that

$$\zeta(0) = \zeta(1) = 0 \qquad \ldots (39)$$

and therefore by the Rayleigh-Ritz inequality it follows that

$$\int_0^1 |D\zeta|^2 \, dz \geq \pi^2 \int_0^1 |\zeta|^2 \, dz. \qquad \ldots (40)$$

Therefore inequality (38) implies that

$$R_T' > \frac{(\pi^2 + a^2)^3}{a^2} \left(1 + \frac{\tau}{\sigma} \right) \qquad \ldots (41)$$

so that we necessarily have

$$R_T' > \frac{27}{4} \pi^4 \left(1 + \frac{\tau}{\sigma} \right) \qquad \ldots (42)$$

Since the minimum value of $\frac{(\pi^2 + a^2)^3}{a^2}$ for $a^2 > 0$ is $\frac{27}{4}\pi^4$.

Hence, if $R_s' \leq R_T + R_s' = R_T' \leq \frac{27}{4} \pi^4 \left(1 + \frac{\tau}{\sigma} \right)$, then we must have

$$\gamma < 1 \qquad \ldots (43)$$

and this completes the proof of the theorem.

On the other hand, when both the bounding surfaces are dynamically free the resulting eigenvalue problem described by equations (11)-(14) together with boundary conditions (15) or (16) can be exactly be solved with

$$\zeta = \frac{A\pi}{\pi^2 + a^2 + \frac{p}{\sigma}} \cos \pi z \qquad \ldots (44)$$

where A is an arbitrary constant, and therefore

$$\int_0^1 |D\zeta|^2 \, dz = \pi^2 \int_0^1 |\zeta|^2 \, dz \qquad \ldots (45)$$

so that inequality (38) again implies the inequality (41), (42) and (43) and the theorem is thus proved.

Note : In the context of Oceanography, $\tau = 0.01$ and $\sigma = 7$ (Veronis[2]) so that the condition $\tau/\sigma \leq 1$ remains valid, while in the contexts of astrophysics and terrestrial physics $\tau << \sigma$, which again ensures the validity of the condition $\tau/\sigma \leq 1$.

Special Case — Setting $T = 0$ and proceeding exactly in a similar way as in Theorem 1 it can be easily seen that the Soret-driven thermosolutal convection of Veronis type cannot manfest as oscillatory motion of growing amplitude if $R_s' \leq \frac{27}{4}\pi^4\left(1+\frac{\tau}{\sigma}\right)$ and $\gamma < 1$ which is essentially the same result as derived in Theorem 1.

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