Indian J. pure appl. Math., 27(10) : 1029-1042, October 1996

THE REFLECTION OF *SV*-WAVES IN A MONOCLINIC MEDIUM

A. CHATTOPADHYAY, S. SAHA* AND M. CHAKRABORTY

Department of Applied Mathematics, Indian School of Mines, Dhanbad 826 004

(*Received* 13 *February* 1995; *after revision* 30 *January* 1996; *accepted* 30 *April* 1996)

In this paper the reflection phenomena has been discussed when *SV*-wave is incident in a medium of monoclinic type. The reflection coefficients for *P*- and *SV*-waves at the free boundary and also at the rigid boundary have been computed. The effects due to the crystalline nature of the medium have been distinctly marked. The results are presented graphically and compared with isotropic case.

1. INTRODUCTION

The study of ıeflection and refraction of plane harmonic elastic waves in anisotropic layered media is of considerable interest in the field of seismology. A large number of papers related to reflection and refraction of elastic waves in anisotropic media have appeared in the literature. Without going into details of the research work in this field we mention few of the papers. Some representative papers in this field may be cited as Musgrave[1], Thapliyal[2], Daley and Hron[3], Kieth and Crampin[4], Tolstoy[5]. The phenomena of reflection and refraction of plane elastic waves at a plane boundary between aelotropic media was investigated by Musgrave[1]. Thapliyal[2] studied the problem of reflection of *SH*-waves from an anisotropic transition layer. Daley and Hron[3] considered the problem of reflection and transmission in transversely isotropic media and computed the reflection and transmission coefficients. Keith and Crampin[4] discussed the problem of reflection and refraction in anisotropic media. Chattopadhyay and Choudhury[6] studied the propagation and reflection of magnetoelastic shear waves in two self-reinforced media and compared the results with reinforced-free medium whereas in an another paper (Chattopadhyay *et al.*[7]) they considered the problem of reflection of *P*-waves at free and rigid boundaries in a medium of monoclinic type.

Study of wave propagation in pre-stressed media is also of considerable interest as pre-stressed media behave like anisotropic media in nature. Several authors

*Central Road Research Institute, Delhi-Mathura Road, CRRI, New Delhi 110 020.

investigated the wave propagation in pre-stressed media. Tolstoy[5] studied elastic waves in pre-stressed solids. Norris[8] disccused the propagation of plane waves in pre-stressed elastic media. Pal and Chattopadhyay[9] studied the reflection phenomena of plane waves at a free boundary in a pre-stressed elastic half-space.

Propagation of waves in crystalline media plays an interesting role in Geophysics and also in ultrasonics and signal processing. But till now no one has studied the problem of reflection of *SV*-waves in monoclinic media. The present paper deals with the investigation of reflection of *SV*-waves in monoclinic media. In the first part of the paper we have studied reflection with free boundary and in the second part that of with rigid boundary.

2. Formulation of the Problem

The equation of motion for the propagation of *SV*-waves in monoclinic media are (Chattopadhyay *et al.*[7])

$$C_{22}\frac{\partial^2 u_2}{\partial x_2^2}+C_{44}\frac{\partial^2 u_2}{\partial x_3^2}+C_{24}\frac{\partial^2 u_3}{\partial x_2^2}+C_{43}\frac{\partial^2 u_3}{\partial x_2^2}+(C_{42}+C_{24})\frac{\partial^2 u_2}{\partial x_2\,\partial x_3}$$

$$+\ (C_{23}+C_{44})\frac{\partial^2 u_3}{\partial x_2\,\partial x_3}=\rho\frac{\partial^2 u_2}{\partial t^2} \qquad \text{... (2.1)}$$

$$C_{42}\frac{\partial^2 u_2}{\partial x_2^2}+C_{34}\frac{\partial^2 u_2}{\partial x_3^2}+C_{44}\frac{\partial^2 u_3}{\partial x_2^2}+C_{33}\frac{\partial^2 u_3}{\partial x_3^2}+(C_{44}+C_{32})\frac{\partial^2 u_2}{\partial x_2\,\partial x_3}$$

$$+\ (C_{43}+C_{34})\frac{\partial^2 u_3}{\partial x_2\,\partial x_3}=\rho\frac{\partial^2 u_3}{\partial t^2} \qquad \text{... (2.2)}$$

where u_1, u_2 and u_3 are diaplacement components in the directions of x_1, x_2 and x_3 respectively, $C_{ij}\,(i, j = 1, 2, ..., 6)$ are elastic constants and ρ is the density.

Let us consider the solution of (2.1) and (2.2) as (Achenbach[10])

$$\bar{u}^{(n)}=A_n\,\bar{d}^{\,(n)}\,e^{\eta}\,n \qquad \text{... (2.3)}$$

where the index n assigns the direction of waves, $\bar{d}$ is the unit displacement vector and

$$\eta_n=k_n\,(\bar{x}\cdot\bar{p}^{(n)}-c_n\,t), \qquad \text{... (2.4)}$$

$\bar{p}^{(n)}$ being the unit propagation vector, c_n the velocity of propagation and k_n the corresponding wave number.

For the two dimensions (Fig. 1)

$$\bar{u}^{(n)}=(u_2^{(n)},u_3^{(n)}),\ \bar{d}^{\,(n)}=(d_2^{(n)},d_3^{(n)}),\ \bar{p}^{(n)}=(p_2^{(n)},p_3^{(n)})$$

such that $\{p_2^{(n)}\}^2+\{p_3^{(n)}\}^2=1$ ... (2.5)

and

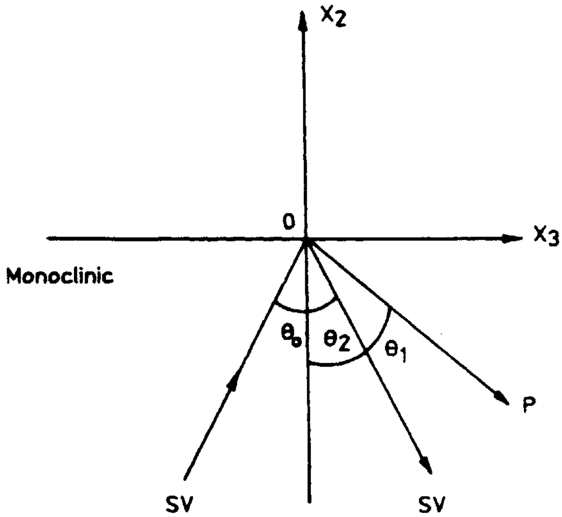

FIG. 1. Geometry of the problem.

$$\begin{bmatrix} u_2^{(n)} \\ u_3^{(n)} \end{bmatrix} = \begin{bmatrix} A_n d_2^{(n)} \\ A_n d_3^{(n)} \end{bmatrix} \exp\left[ik_n\left(x_3 p_3^{(n)} + x_2 p_2^{(n)} - c_n t\right]\right. \qquad \ldots (2.6)$$

To examine the reflection of SV-waves, we assume $n = 0$ for incident SV-waves. Therefore,

$$\left.\begin{array}{l} p_3^{(0)} = \sin\theta_0, \quad p_2^{(0)} = \cos\theta_0 \\ d_3^{(0)} = -\cos\theta_0, \quad d_2^{(0)} = \sin\theta_0 \\ c_0 = c_T \end{array}\right] \qquad \ldots (2.7)$$

$\bar{d}^{(0)}$ is obtained for SV-wave by the relationship

$$\begin{aligned} \bar{d}^{(0)} &= \bar{p}^{(0)} \times \hat{i}_1 \\ &= (\hat{i}_3 \sin\theta_0 + \hat{i}_2 \cos\theta_0) \times \hat{i}_1 \\ &= -\hat{i}_3 \cos\theta_0 + \hat{i}_2 \sin\theta_0. \end{aligned}$$

Then in the plane $x_2 = 0$, the displacements and the stresses of the incident wave are of the forms

$$u_2^{(0)} = A_0 \sin\theta_0 \exp(i\eta_0),$$

$$u_3^{(0)} = -A_0 \cos\theta_0 \exp(i\eta_0).$$

$$T_2^{(0)} = C_{22}\frac{\partial u_2}{\partial x_2} + C_{23}\frac{\partial u_3}{\partial x_3} + C_{24}\left(\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}\right)$$

$$= ik_0 A_0 (C_{22} - C_{33}) \frac{\sin 2\theta_0}{2} \exp\left[ik_0 (x_3 \sin\theta_0 - c_n t)\right]$$

$$+ ik_0 A_0 (\sin^2\theta_0 - \cos^2\theta_0) C_{24} \exp\left[ik_0 (x_3 \sin\theta_0 - c_n t)\right],$$

$$T_4^{(0)} = C_{42}\frac{\partial u_2}{\partial x_2} + C_{43}\frac{\partial u_3}{\partial x_3} + C_{44}\left(\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}\right)$$

$$= ik_0 A_0 (C_{42} - C_{43}) \frac{\sin 2\theta_0}{2} \exp\left[ik_0 (x_3 \sin\theta_0 - c_n t)\right]$$

$$- C_{44}\cos 2\theta_0\, ik_0 A_0 \exp\left[ik_0 (x_3 \sin\theta_0 - c_n t)\right], \ldots \quad (2.8)$$

where $c_n = c_T$, $\eta_0 = k_0 (x_3 \sin\theta_0 - c_T t)$, $T_2^{(n)}$ and $T_4^{(n)}$ are normal and shearing stresses (Chattopadhyay *et al.*[7]).

The incident SV-wave will give rise to a reflected P-wave and a reflected SV-wave. We denote n = 1 and n = 2 for reflected P-and SV- waves respectively.

For reflected P-wave

$$\bar{p}^{(1)} = \sin\theta_1\, \hat{i}_3 - \cos\theta_1\, \hat{i}_2,$$

$$\bar{d}^{(1)} = \bar{p}^{(1)}, \quad c_1 = c_L. \qquad \ldots (2.8a)$$

In the plane x_2 = 0, the displacement components and stress component due to reflected P-wave may be written as

$$u_2^{(1)} = -A_1 \cos\theta_1 \exp\left[ik_1 (x_3 \sin\theta_1 - c_L t)\right],$$

$$u_3^{(1)} = A_1 \sin\theta_1 \exp\left[ik_1 (x_3 \sin\theta_1 - c_L t)\right],$$

$$T_2^{(1)} = ik_1 A_1 \left[C_{22}\cos^2\theta_1 + C_{23}\sin^2\theta_1 - 2C_{24}\sin\theta_1\cos\theta_1\right]$$

$$\times \exp\left[ik_1 (x_3 \sin\theta_1 - c_L t)\right],$$

$$T_4^{(1)} = ik_1 A_1 \left[C_{42}\cos^2\theta_1 + C_{43}\sin^2\theta_1 - 2C_{44}\sin\theta_1\cos\theta_1\right]$$

$$\times \exp\left[ik_1 (x_3 \sin\theta_1 - c_L t)\right]. \quad \ldots (2.9)$$

If the reflected SV-wave makes an angle θ_2 with x_2-axis, we get

$$\bar{p}^{(2)} = \sin\theta_2\, \hat{i}_3 - \cos\theta_2\, \hat{i}_2,$$

for SV-waves $\quad c_2 = c_T$.

Therefore,

$$\bar{d}^{(2)} = \bar{p}^{(2)} \times \hat{i}_1 = \cos\theta_2\, \hat{i}_3 + \sin\theta_2\, \hat{i}_2. \qquad \ldots (2.10)$$

In the plane $x_2 = 0$, the displacement components and stress components due to reflected *SV* wave may be written as

$$u_2^{(2)} = A_2 \sin\theta_2 \exp[ik_2(-x_2\cos\theta_2 + x_3\sin\theta_2 - c_T t]$$

$$u_3^{(2)} = A_2 \cos\theta_2 \exp[ik_2(-x_2\cos\theta_2 + x_3\sin\theta_2 - c_T t]$$

$$T_2^{(2)} = [(C_{23} - C_{22})\sin\theta_2\cos\theta_2 + C_{24}(\sin^2\theta_2 - \cos^2\theta_2)]\, i_2\, k_2\, A_2 \times \exp[ik_2(x_3\sin\theta_2 - c_T t)],$$

$$T_4^{(2)} = [(C_{43} - C_{42})\sin\theta_2\cos\theta_2 + C_{44}(\sin^2\theta_2 - \cos^2\theta_2)]\, i_2\, k_2 \times \exp[ik_2(x_3\sin\theta_2 - c_T t)]. \quad \ldots (2.11)$$

If we want to study the propagation of pure modes of plane waves, then the non-zero components of the unit displacement vector and the unit propagation vector must satisfy the following relations obtained from equations (2.1)-(2.2)

$$[C_{24}\{d_3^{(n)}\}^2 + (C_{22} - C_{44})\, d_2^{(n)} d_3^{(n)} - C_{42}\{d_2^{(n)}\}^2]\ \{p_2^{(n)}\}^2$$
$$+ [C_{43}\{d_3^{(n)}\}^2 + (C_{44} - C_{33})\, d_2^{(n)} d_3^{(n)} - C_{34}\{d_2^{(n)}\}^2]\ \{p_3^{(n)}\}^2$$
$$+ [(C_{23} + C_{44})\{d_3^{(n)}\}^2 + (C_{24} + C_{42} - C_{43} - C_{34})\, d_2^{(n)} d_3^{(n)}$$
$$- (C_{44} + C_{32})\{d_2^{(n)}\}^2]\ p_2^{(n)} p_3^{(n)} = 0.$$

This equation determines definite direction of the propagation of pure mode of *SV*-waves depending upon the material constants. Now using (2.7a), (2.8a) and (2.10) we find from the above equation, the specific direction of propagation of pure modes of incident waves whose directions are given by

$$\tan\theta = \frac{M + \sqrt{M^2 - 4q}}{2} - \frac{a_1}{a_0},$$

where M satisfies the equation

$$M^6 + 12\,HM^4 + (48H^2 - 4a_0^2 I)\,M^2 - 16G^2 = 0,$$

and
$$H = \frac{a_2}{a_0} - \frac{a_1^2}{a_0^2},$$

$$G = \frac{a_3}{a_0} - 3\left(\frac{a_1}{a_0}\right)\left(\frac{a_2}{a_0}\right) + 2\left(\frac{a_1}{a_0}\right)^3,$$

$$I = \frac{a_4}{a_0} - 4\left(\frac{a_1}{a_0}\right)\left(\frac{a_3}{a_0}\right),$$

$$q' = (M^2 + 6H + 4G/M)/2,$$

$$\frac{a_1}{a_0} = \frac{1}{4}\left(\frac{C_{32}}{C_{34}} + 2\frac{C_{44}}{C_{34}} - \frac{C_{33}}{C_{34}} \right),$$

$$\frac{a_2}{a_0} = \frac{1}{6}\left(2\frac{C_{42}}{C_{34}} - 2\frac{C_{43}}{C_{34}} - \frac{C_{24}}{C_{34}} - 1 \right),$$

$$\frac{a_3}{a_0} = \frac{1}{4}\left(\frac{C_{22}}{C_{34}} - \frac{C_{23}}{C_{34}} - 2\frac{C_{44}}{C_{34}} \right),$$

$$\frac{a_4}{a_0} = -\frac{C_{24}}{C_{34}}.$$

The resulting quadratic equation in c_n^2 obtained by substituting (2.3) in eqns. (2.1) and (2.2) can be factorised into a quasi-longitudinal and a quasi-shear. These velocities are given in ratio form in eqn. (3.11).

3. Boundary Conditions and Solution of the Problem

Case I : *Reflection of SV-waves at a free boundary*

When $x_2 = 0$ is a free surface, the sum of three tractions must vanish at $x_2 = 0$ and we obtain from (2.8), (2.9) and (2.11)

$$T_2^{(0)} + T_2^{(1)} + T_2^{(2)} = 0 \quad \text{and} \quad T_4^{(0)} + T_4^{(1)} + T_4^{(2)} = 0. \qquad \text{... (3.1)}$$

Substituting in (3.1) the values of $T_2^{(n)}$, $T_4^{(n)}$, $(n = 0, 1, 2)$ from (2.8), (2.9) and (2.11) we obtain

$$\begin{aligned} & ik_0\,[C_{22}\sin\theta_0\cos\theta_0 - C_{23}\sin\theta_0\cos\theta_0 + C_{24}(\sin^2\theta_0 - \cos^2\theta_0]\,A_0 \\ & \quad \times \exp\,[ik_0\,(x_3\sin\theta_0 - c_T\,t)] \;+\; ik_1\,[C_{22}\cos^2\theta_1 + C_{23}\sin^2\theta_1 \\ & \quad - 2C_{24}\sin\theta_1\cos\theta_1]\,A_1 \;\times \exp\,[ik_1\,(x_3\sin\theta_1 - c_L\,t)] \\ & \quad + ik_2\,[-C_{22}\sin\theta_2\cos\theta_2 + C_{23}\sin\theta_2\cos\theta_2 + C_{24}(\sin^2\theta_2 - \cos^2\theta_2)]\,A_2 \\ & \quad \times \exp\,[ik_2\,(x_3\sin\theta_2 - c_T\,t)] = 0 \qquad \text{... (3.2)} \end{aligned}$$

$$\begin{aligned} & ik_0\,[C_{42}\sin\theta_0\cos\theta_0 - C_{43}\sin\theta_0\cos\theta_0 + C_{44}(\sin^2\theta_0 - \cos^2\theta_0)]\,A_0 \\ & \quad \times \exp\,[ik_0\,(x_3\sin\theta_0 - c_T\,t)] \;+\; ik_1\,[C_{42}\cos^2\theta_1 + C_{43}\sin^2\theta_1 \\ & \quad - 2C_{44}\sin\theta_1\cos\theta_1]\,A_1 \;\times \exp\,[ik_1\,(x_3\sin\theta_1 - c_L\,t)] \\ & \quad + ik_2\,[-C_{42}\sin\theta_2\cos\theta_2 + C_{43}\sin\theta_2\cos\theta_2 + C_{44}(\sin^2\theta_2 - \cos^2\theta_2)]\,A_2 \\ & \quad \times \exp\,[ik_2\,(x_3\sin\theta_2 - c_T\,t)] = 0. \qquad \text{... (3.3)} \end{aligned}$$

Equations (3.2) and (3.3) must be valid for all values of x_3 and t, hence,

$$k_0 (x_3 \sin\theta_0 - c_T t) = k_1 (x_3 \sin\theta_1 - c_L t) = k_2 (x_3 \sin\theta_2 - c_T t). \qquad \text{... (3.4)}$$

This gives

$$k_0 \sin\theta_0 = k_1 \sin\theta_1 = k_2 \sin\theta_2 = \phi, \qquad \text{... (3.5)}$$

where ϕ is the apparent wave number, $k_0 c_T = k_1 c_L = k_2 c_T = \omega$ and ω is the circular frequency. These results yield

$$k_0 = k_2,$$

$$\frac{k_0}{k_1} = \frac{c_L}{c_T}. \qquad \text{... (3.6)}$$

Also $\theta_2 = \theta_0$

$$\sin\theta_1 = \frac{k_0}{k_1}\sin\theta_0 = \bar{k}\sin\theta_0. \qquad \text{... (3.6a)}$$

With the aid of eqns. (3.2)-(3.6), the algebraic equations for A_1/A_0 and A_2/A_0 can now be simplified to

$$1 + \frac{k_1}{k_0}\left(\frac{A_1}{A_0}\right)P_1 + \left(\frac{A_2}{A_0}\right)P_2 = 0 \qquad \text{... (3.7)}$$

$$1 + \frac{k_1}{k_0}\left(\frac{A_1}{A_0}\right)P_3 + \left(\frac{A_2}{A_0}\right)P_4 = 0. \qquad \text{... (3.8)}$$

Solving (3.7) and (3.8), the coefficient of reflection of *P*-wave is

$$\frac{A_1}{A_0} = \frac{\bar{k}(P_2 - P_4)}{(P_1 P_4 - P_2 P_3)} \qquad \text{... (3.9)}$$

and, the coefficient of reflection of *SV*-waves is

$$\frac{A_2}{A_0} = \frac{(P_3 - P_1)}{(P_1 P_4 - P_2 P_3)} \qquad \text{... (3.10)}$$

where

$$P_1 = \frac{(C_{22}\cos^2\theta_1 + C_{23}\sin^2\theta_1 - 2C_{24}\sin\theta_1\cos\theta_1)}{(C_{22} - C_{23})\sin\theta_0\cos\theta_0 + C_{24}(\sin^2\theta_0 - \cos^2\theta_0)}$$

$$P_2 = \frac{(C_{23} - C_{22})\sin\theta_0\cos\theta_0 + C_{24}(\sin^2\theta_0 - \cos^2\theta_0)}{(C_{22} - C_{23})\sin\theta_0\cos\theta_0 + C_{24}(\sin^2\theta_0 - \cos^2\theta_0)}$$

$$P_3 = \frac{(C_{42}\cos^2\theta_1 + C_{43}\sin^2\theta_1 - 2C_{44}\sin\theta_1\cos\theta_1)}{(C_{42} - C_{43})\sin\theta_0\cos\theta_0 + C_{44}(\sin^2\theta_0 - \cos^2\theta_0)}$$

$$P_4 = \frac{(C_{43} - C_{42}) \sin\theta_0 \cos\theta_0 + C_{44} (\sin^2\theta_0 - \cos^2\theta_0)}{(C_{42} - C_{43}) \sin\theta_0 \cos\theta_0 + C_{44} (\sin^2\theta_0 - \cos^2\theta_0)}$$

$$\bar{k}^2 = \frac{c_L^2}{c_T^2} = \frac{(P+S) + \sqrt{(P-S) + 4QR}}{(U+Z) - \sqrt{(U-Z) + 4VW}}, \qquad \text{... (3.11)}$$

where

$$P = C_{22} \cos^2\theta_1 + C_{44} \sin^2\theta_1 - (C_{42} + C_{24}) \sin\theta_1 \cos\theta_1$$

$$Q = C_{24} \cos^2\theta_1 + C_{43} \sin^2\theta_1 - (C_{23} + C_{44}) \sin\theta_1 \cos\theta_1$$

$$R = C_{42} \cos^2\theta_1 + C_{34} \sin^2\theta_1 - (C_{44} + C_{32}) \sin\theta_1 \cos\theta_1$$

$$S = C_{44} \cos^2\theta_1 + C_{33} \sin^2\theta_1 - (C_{43} + C_{34}) \sin\theta_1 \cos\theta_1$$

$$U = C_{22} \cos^2\theta_0 + C_{44} \sin^2\theta_0 + (C_{42} + C_{24}) \sin\theta_0 \cos\theta_0$$

$$V = C_{24} \cos^2\theta_0 + C_{43} \sin^2\theta_0 + (C_{23} + C_{44}) \sin\theta_0 \cos\theta_0$$

$$W = C_{42} \cos^2\theta_0 + C_{34} \sin^2\theta_0 + (C_{44} + C_{32}) \sin\theta_0 \cos\theta_0$$

$$Z = C_{44} \cos^2\theta_0 + C_{33} \sin^2\theta_0 + (C_{43} + C_{34}) \sin\theta_0 \cos\theta_0.$$

Using the following values of C_{ij} for isotropic medium

$$\left\{ \begin{array}{l} C_{33} = C_{22} = \lambda + 2\mu, \quad C_{32} = C_{23} = \lambda, \quad C_{24} = 0, \quad C_{44} = \mu, \\ C_{34} = 0, \quad C_{42} = 0, \quad C_{43} = 0, \end{array} \right\} \qquad \text{... (3.11a)}$$

eqns. (3.9), (3.10) and (3.11) reduce to

$$\frac{A_1}{A_0} = -\frac{k \sin 4\theta_0}{\sin 2\theta_0 \sin 2\theta_1 + k^2 \cos^2 2\theta_0},$$

$$\frac{A_2}{A_0} = \frac{\sin 2\theta_0 \sin 2\theta_1 - k^2 \cos^2 2\theta_0}{\sin 2\theta_0 \sin 2\theta_1 + k^2 \cos^2 2\theta_0},$$

and $$k = \bar{k} = \sqrt{\frac{\lambda + 2\mu}{\mu}}$$

which are the reflection coefficients of reflected *P*-waves and *SV*-waves respectively for free boundary in isotropic case (cf. Achenbach[10], p-179).

Case II : *Reflection of SV-waves at a rigid boundary*

Since the boundary $x_2 = 0$ is bounded by a rigid layer, the boundary conditions may be taken as

$$u_2^{(0)} + u_2^{(1)} + u_2^{(2)} = 0,$$

and $$u_3^{(0)} + u_3^{(1)} + u_3^{(2)} = 0 \quad \text{at} \quad x_2 = 0. \qquad \text{... (3.12)}$$

Substituting the values of u_2 and u_3, $n = 0, 1, 2$ from eqns. (2.8), (2.9) and (2.11) in (3.12) we get

$$A_0 \sin\theta_0 \exp[(ik_0(x_3\sin\theta_0 - c_T t)] - A_1\cos\theta_1 \exp[ik_1(x_3\sin\theta_1 - c_L t)]$$

$$+ A_2 \sin\theta_2 \exp[ik_2(x_3\sin\theta_2 - c_T t)] = 0 \quad \text{... (3.13)}$$

$$- A_0 \cos\theta_0 \exp[ik_0(x_3\sin\theta_0 - c_T t)] + A_1\sin\theta_1 \exp[ik_1(x_3\sin\theta_1 - c_L t)]$$

$$+ A_2 \cos\theta_2 \exp[ik_2(x_3\sin\theta_2 - c_T t)] = 0. \quad \text{... (3.14)}$$

With the help of eqns. (3.4)-(3.6), the algebraic equations for A_1/A_0 and A_2/A_0 from eqns. (3.13) and (3.14) can now be simplified to

$$1 - \frac{A_1\cos\theta_1}{A_0\sin\theta_0} + \frac{A_2}{A_0} = 0 \quad \text{... (3.15)}$$

$$-1 + \frac{A_1\sin\theta_1}{A_0\cos\theta_0} + \frac{A_2}{A_0} = 0. \quad \text{... (3.16)}$$

The solution of this set of equations is

$$\frac{A_1}{A_0} = \frac{2(\bar{k})^{-1}\tan\theta_1}{(1-\tan\theta_0\tan\theta_1)},$$

where $\bar{k} = \dfrac{k_0}{k_1} = \dfrac{c_L}{c_T}$ ... (3.17)

which is the coefficient of reflected *P*-waves and

$$\frac{A_2}{A_0} = \frac{(1-\tan\theta_0\tan\theta_1)}{(1-\tan\theta_0\tan\theta_1)} \quad \text{... (3.18)}$$

is the coefficient of reflected *SV*-waves and $\bar{k}$ is given by eqn. (3.11).

Using (3.11a) eqns. (3.17), (3.18) and (3.11) reduce to

$$\frac{A_2}{A_0} = \frac{(1-\tan\theta_0\tan\theta_1)}{(1-\tan\theta_0\tan\theta_1)} \quad \text{... (3.19)}$$

$$\frac{A_1}{A_0} = \frac{2(\bar{k}_0)^{-1}\tan\theta_1}{(1+\tan\theta_0\tan\theta_1)} \quad \text{... (3.20)}$$

where $\bar{k}_0 = \sqrt{\dfrac{\lambda+2\mu}{\mu}}$ and $\sin\theta_1 = \bar{k}_0 \sin\theta_0$ ... (3.21)

which are reflection coefficients of *SV*-waves and *P*-waves respectively for rigid boundary in an isotropic medium.

4. Numerical Calculations and Discussions

The numerical values of reflection coefficients A_1/A_0 and A_2/A_0 of P and SV-waves respectively for different values of the angle of incidence have been calculated.

The following values have been considered as suggested by Tiersten[11]

$C_{22} = 129.77$ N/m^2, $C_{23} = -7.29$ N/m^2,

$C_{33} = 102.83$ N/m^2,

$C_{42} = 5.7$ N/m^2, $C_{43} = 9.92$ N/m^2,

$C_{44} = 38.61$ N/m^2, $\rho = 2649$ Kg/m^3.

4.1. *Reflection at a Free Boundary in Monoclinic Medium*

In Fig. 2, curve II corresponds to reflection coefficients of P-wave in a monoclinic medium with free boundary. Here all the values of A_1/A_0 are negative. The values of A_1/A_0 for isotropic medium are also negative (curve I). Here we find that the reflection coefficients A_1/A_0 for isotropic case do not exist beyond $\theta_0 = 35°$ which agrees with the curve given by Achenbach[10]. For monoclinic medium

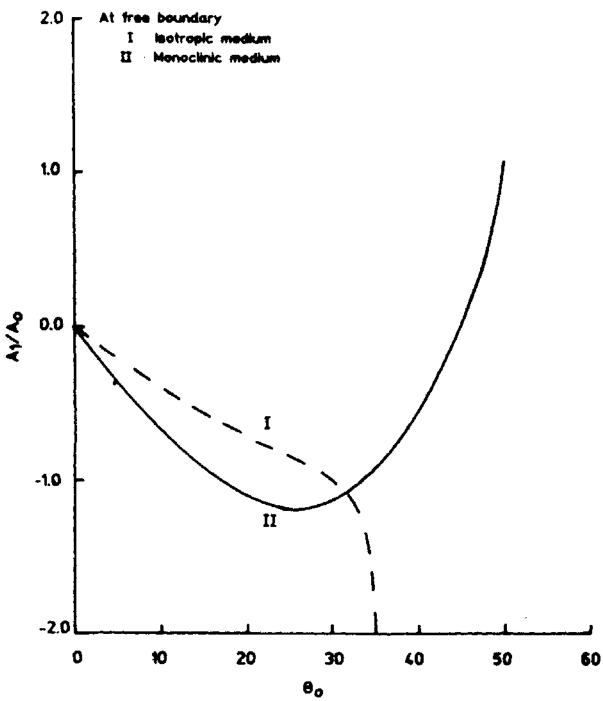

Fig. 2. Variation of A_1/A_0 with respect to θ_0.

the reflection coefficient A_1/A_0 exists upto $\theta_0 = 50°$. A_1/A_0 values of monoclinic medium when compared to isotropic medium, it is found that the values of A_1/A_0 are more in isotropic medium when $\theta_0 = 0°$ to $31°$. At $\theta_0 = 31°$ the curve for isotropic medium intersects the curve of monoclinic medium after which the value of A_1/A_0 for isotropic medium decreases abruptly and does not exist after $\theta_0 = 35°$ where as for monoclinic medium it increases with the increase of θ_0 up to 50° and after 50° it does not exists. $A_1/A_0 = 0$ at $\theta_0 = 0°$ and 44° i.e., at these incidences the *SV*-wave reflected as *SV*-wave whereas for isotropic case it is true for $\theta_0 = 0°$ and coincides with the result of Achenbach[10].

In Fig. 3, the reflection coefficients of *SV*-wave (curve II) for monoclinic medium for different values of θ_0 ranging from 0° to 50° have been plotted, which is permissible range of θ_0 for A_2/A_0 in monoclinic medium, and compared with isotropic case (curve I). The values of reflection coefficient of *SV*-wave for isotropic medium are less than that of monoclinic medium. The value of reflection coefficient of *SV*-wave at $\theta_0 = 0°$ for isotropic medium is same as that of monoclinic medium. The value of reflection coefficient of *SV*-wave for monoclinic medium increases steadily from $\theta_0 = 0°$ to 50°. On comparison of two curves we find the remarkable differences in values of reflection coefficient. The curve plotted for the isotropic is same as given by Achenbach[10].

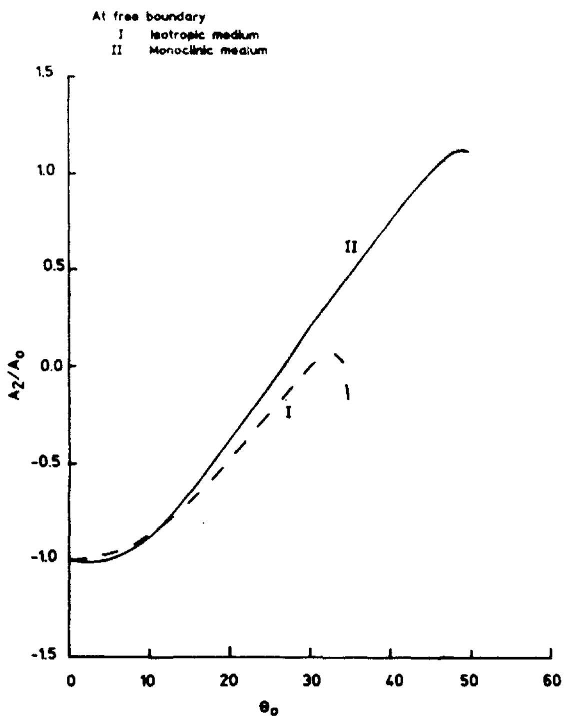

FIG. 3. Variation of A_2/A_0 with respect to θ_0.

4.2. *Reflection at a Rigid Boundary in a Monoclinic Medium*

The rigid boundary plays a very important role in the case of reflection phenomena. A_1/A_0 (reflection coefficient of *P*-wave) in the case of rigid boundary increases from $\theta_0 = 0^\circ$ to 29° for both monoclinic and isotropic medium (Fig. 4). For $\theta_0 = 29^\circ$ to 50°, the value of reflection coefficient in monoclinic medium is increasing slowly. The value of reflection coefficient of *P*-wave for the isotropic medium with rigid boundary coincides with the value of A_1/A_0 for monoclinic medium with rigid boundary at $\theta_0 = 31^\circ$ after that there is an abrupt increase in the value of A_1/A_0 for isotropic medium with rigid boundary and attains the maximum value at $\theta_0 = 35^\circ$ after which it does not exist. If we compare the curves (Fig. 4) with the corresponding curves of free boundary (Fig. 2) we observe that the value of reflection coefficient of *P*-wave for isotropic medium with free boundary decreases uniformly upto $\theta_0 = 31^\circ$ whereas in the case of isotropic medium with rigid boundary the value of A_1/A_0 increases uniformly from $\theta_0 = 0^\circ$ to 31°. From 31° to 35° the value of reflection coefficient of *P*-wave decreases abruptly for the case of isotropic medium with free boundary whereas the value of reflection coefficient of *P*-wave increases abruptly for $\theta_0 = 31^\circ$ to 35° for the case of isotropic medium with rigid boundary.

Fig. 5 shows the reflection coefficient A_2/A_0 for *SV*-waves. The value of A_2/A_0 for monoclinic medium with rigid boundary coincides with value of A_2/A_0 for isotropic medium for $\theta_0 = 0^\circ$ to 24°. From 24° onwards the value of A_2/A_0

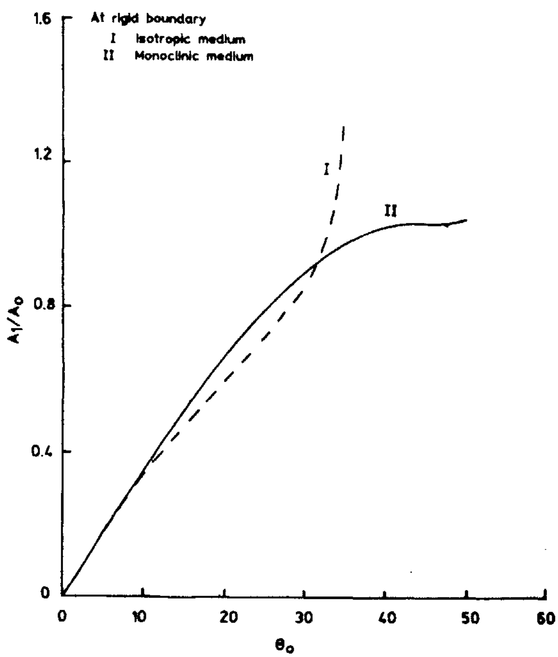

FIG. 4. Variation of A_1/A_0 with respect to θ_0.

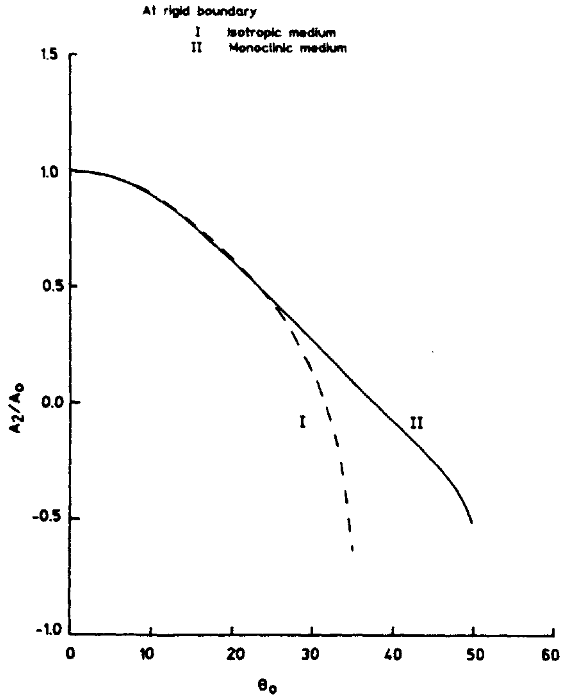

FIG. 5. Variation of A_2/A_0 with respect to θ_0.

decreases rapidly for the isotropic medium with rigid boundary as compared to the monoclinic medium. The value of $A_2/A_0 = 1$ at $\theta_0 = 0°$, for both isotropic and monoclinic medium and which is maximum. The reflection coefficient of reflected *SV*-wave is maximum at $\theta_0 = 0°$ implies that *SV*-wave is totally reflected at this incidence for both monoclinic and isotropic medium (Fig. 5). The value of A_2/A_0 for monoclinic medium with rigid boundary decreases uniformly whereas the value of A_2/A_0 for monoclinic medium with free boundary increases uniformly. In the case of isotropic medium the value of A_2/A_0 increases uniformly with free boundary, and decreases uniformly with rigid boundary. $A_2/A_0 > 0$ for $\theta_0 = 0°$ to 39° for the case of monoclinic medium with rigid boundary (Fig. 5) whereas $A_2/A_0 > 0$ for $\theta_0 = 30°$ to 42° for the case of monoclinic medium with free boundary. Therefore for the ranges mentioned above for the corresponding boundary in monoclinic medium we are getting reflected *SV*-wave. Therefore monoclinic effect is quite significant in this respect.

ACKNOWLEDGEMENT

Authors are grateful to the reviewer for critical review and constructive suggestions for the improvement of the paper.

REFERENCES

1. M. J. P. Musgrave, *Geophys. J. R. astr. Soc.* **3** (1960), 406-18.
2. V. Thapliyal, *Bull. Seism. Soc. Am.* **65** (1974), 1979-91.
3. P. F. Daley and F.Hron, *Bull. Seism. Soc. Am.* **67** (1977), 661-76.
4. C. M. Kieth and S. Crampin, *Geophys. J. R. astr. Soc.* **49** (1977), 181-208.
5. I. Tolstoy, *J. Geophys. Res.* **87** (1982), 6823-27.
6. A. Chattopadhyay and S. Choudhury, *Int. J. Engg. Sci.* **28** (1990), 485-95.
7. A. Chattopadhyay and S. Choudhury, *Int. J. Engg. Sci.* **33** (1995), 195-207.
8. A. N. Norris, *J. Acoust. Soc. Am.* **74** (1983), 1642-43.
9. A. K. Pal and A. Chattopadhyay, *J. Acoust. Soc. Am.* **76** (1984), 924-25.
10. J. D. Achenbach, *Wave Propagation in Elastic Solids*, North-Holland, New York, 1976.
11. H. F. Tiersten, *Linear Piezoelectric Plate Vibrations*, Plenum Press, New York, 1969.