Indian J. pure appl. Math., 27(9) : 931-941, September 1996

FREE CONVECTION IN BOUNDARY LAYER FLOWS OF POWER-LAW FLUIDS PAST A VERTICAL FLAT PLATE WITH SUCTION/INJECTION

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(*Received* 21 *November* 1994; *after final revision* 19 *February* 1996; *accepted* 11 *March* 1996)

Free convective laminar boundary layer flow of power-law fluids both pseudoplastic and dilatant have been considered and the effect of suction/injection has been studied on the velocity and temperature fields. The results have been correlated for velocity, temperature distribution, skin-friction and heat transfer parameters. It is observed that suction/injection has significant effects on velocity and temperature fields. Suction decreases skin-friction and increases the heat transfer parameters while it is reverse in the case of blowing. The average Nusselt number for a non-isothermal plate is less than that of isothermal plate. The temperature in boundary layer increases (decreases) due to injection (suction). With the increase in value of generalized Prandtl number the effects of suction/injection become very significant.

NOMENCLATURE

A = a dimensional constant

a = a constant

B = a dimensional constant

b = a constant

C_p = specific heat of power-law fluid

f = non-dimensional stream function

f'_{max} = non-dimensional maximum velocity

f_w = suction/injection parameter

g = acceleration due to gravity

K = consistency factor for power-law fluids

k = thermal conductivity of fluids

L = a characteristic length

m = temperature variation index at wall

N_{gx} = local Grashof number

N_{gl} = Grashof number with reference to a characteristic length L

N_{Pr} = generalized Prandtl number

N_{ul} = average Nusselt number

N_{ux} = local Nusselt number

T = temperature of power-law fluid in boundary layer

T_∞ = ambient temperature of fluid in boundary layer

T_w = wall temperature of power-law fluid

u = velocity component along the plate

v = velocity component perpendicular to plate

x = coordinate along the plate

y = coordinate perpendicular to the plate.

Greek letter

α = thermal diffusivity

η = non-dimensional coordinate perpendicular to the wall

θ = non-dimensional temperature

ρ = density

τ_{xy} = shear stress

ψ = dimensional stream function.

INTRODUCTION

Free convective phenomenon has been the object of extensive research. The importance of this phenomenon is due to enhanced concern in science and technology about buoyancy induced motions in the atmosphere, in bodies of water and quasisolid bodies such as earth. Free convective flow has been extensively studied[1, 2] for Newtonian and some class of non-Newtonian fluids.

Free convective heat transfer from an isothermal vertical plate to power-law fluids has been analysed by Acrivos[3], Tien[4], Shulman *et al.*[5] and Shenoy and Ulbrecht[6]. A similarity analysis for the case of non-isothermal vertical plate natural convection of non-Newtonian fluids of Ostwald-de-Waele type has already been reported[7, 8]. For other models of non-Newtonian fluids, a non-uniformly heated vertical flat plate problem has been tackled by Mathur[9] and Mishra[10]. Jena and Mathur[11, 12] have obtained similar solutions for the case of non-uniformly heated vertical flat plate for thermomicropolar fluids with or without suction/injection.

In the present work, we have studied free convective flow of power-law fluids past a non-uniformly heated vertical flat plate with suction/injection. Earlier works[7,8] have obtained the set of nonlinear coupled ordinary differential equations for velocity and temperature fields but no numerical solution have been obtained for the set of physically meaningful values of power-law fluid parameters. We have attempted to obtain numerical solutions of the problem for a set of parameters whose values have been chosen from experiments involving realistic power-law fluids. We have solved this problem with and without suction/injection and compared theorectical and experimental results wherever possible.

FORMULATION OF THE PROBLEM

The equations governing the two-dimensional plane flow of a power-law fluid in the laminar boundary region close to a vertical flat plate are :

Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.0 \qquad \dots (1)$$

Momentum

$$\rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = K \frac{\partial}{\partial y} \left[\left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \right] + \rho g \beta \, (T - T_{\infty}) \qquad \dots (2)$$

Energy

$$\rho C_p \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2} . \qquad \dots (3)$$

The boundary conditions are

$$\left. \begin{aligned} & y = 0; \quad u = 0.0, \quad v = v_w(x), \quad T = T_w(x) \\ & y \to \infty; \quad u \to 0.0, \quad T \to T_{\infty}. \end{aligned} \right\} \qquad \dots (4)$$

Here $v_w(x)$ and $T_w(x)$ are respectively the non-uniform suction/injection velocity and temperature of the plate. $v_w < 0.0$ means suction and $v_w > 0.0$ implies injection at the plate. In eqn. (3), we have neglected the viscous dissipation term since the velocities encountered in free convection are rather small.

METHOD OF SOLUTION

We try to search for self-similar solutions of the eqns. (1)-(4). For this purpose, we introduce the following quantities :

$$\left.\begin{aligned} &u = \psi_y, \quad v = -\psi_x, \\ &\psi = Ax^a f(\eta), \quad \eta = Byx^b, \\ &\theta = \frac{T - T_\infty}{T_w(x) - T_\infty}, \quad T_w - T_\infty = Nx^m, \quad N = \frac{T_w(0) - T_\infty}{L^m}. \end{aligned}\right\} \quad \text{... (5)}$$

Substituting the expressions (5) in eqns. (1)-(4), we obtain,

$$A^2B^2 x^{2a+2b-1} [(a+b)(f')^2 - aff'']$$
$$= \frac{K}{\rho} A^n B^{2n+1} \frac{d}{d\eta} [\,|f''|^{n-1} f''\,] x^{n(a+2b)+b} + g\beta N x^m \theta \quad \text{... (6)}$$

$$A\,B\,N\,x^{a+b+m-1} [m\theta f' - af\theta'] = \alpha N B^2 x^{m+2b} \theta''. \quad \text{... (7)}$$

Since eqns. (6) and (7) must hold for all values of x if similarity is to exist, we have

$$2a + 2b - 1 = n(a + 2b) + b = m, \quad a + b + m - 1 = m + 2b. \quad \text{... (8)}$$

Solving eqn. (8), we get

$$a = \frac{2}{3}, \; b = m = -\frac{1}{3}.$$

Thus, we have

$$\left.\begin{aligned} &\psi = Ax^{2/3} f(\eta), \quad \eta = Byx^{-1/3}, \quad T_w - T_\infty = Nx^{-1/3} \\ &u = ABx^{1/3} f', \quad v = -Ax^{-1/3}\left(\frac{2}{3}f - \frac{1}{3}\eta f'\right) \\ &T - T_\infty = Nx^{-1/3}\theta(\eta). \end{aligned}\right\} \quad \text{... (9)}$$

With this, we note that $v_w(x) = -\frac{2}{3} Ax^{-1/3} f_w$ where $f_w = f(0)$. The equations determining f and θ are

$$(f')^2 - 2ff'' = 3n\,|f''|^{n-1} f''' + 3\theta \quad \text{... (10)}$$

$$\theta'' = -\frac{N_{Pr}}{3}(\theta f' + 2f\theta') \quad \text{... (11)}$$

$$\left.\begin{aligned} &\eta = 0.0; \; f = f_w, \; f' = 0.0; \; \theta = 1 \\ &\eta \to \infty; \; f' \to 0.0; \; \theta \to 0.0. \end{aligned}\right\} \quad \text{... (12)}$$

where dash denotes the differentiation with respect to η. The eqns. (10)-(12) have been obtained after properly choosing the dimensional constants A, B and α in the following form :

$$A = \left(\frac{K}{\rho}\right)^{\frac{1}{n+1}} (gN\beta)^{\frac{2n-1}{2(n+1)}}$$

$$B = \left(\frac{K}{\rho}\right)^{\frac{-1}{n+1}} (gN\beta)^{\frac{2-n}{2(n+1)}}$$

$$\alpha = \frac{k}{\rho C_p} .$$

The dimensionless parameter N_{Pr} is defined as

$$N_{Pr} = \frac{\rho C_p}{k}\left(\frac{K}{\rho}\right)^{\frac{2}{n+1}} (gN\,\beta)^{\frac{3(n-1)}{2(n+1)}} .$$

For $n = 1$, N_{Pr} becomes the usual Prandtl number. Newtonian case has to be obtained differently as has been solved by Sparrow and Gregg[13]. The set of eqns. (10)-(12) has been solved numerically. This system of equation is highly nonlinear. We have applied quasilinearization technique to solve this system. This method converts the nonlinear two-point boundary value problem into an iterative scheme of solution which involves the step-by-step integration of linearized differential equations, with two point boundary conditions. This method is discussed in detail by Bellman and Kalaba[14]. This technique has been used successfully by Redbill[15], Libby and Chen[16] and Inouye and Tate[17] for the solution of the Falkner-Skan type equations.

Applying quasilinearization technique to eqns. (10) and (11), we have

$$f'''_{i+1} = A_i f''_{i+1} + B_i f'_{i+1} + C_i f_{i+1} + D_i \theta_{i+1} + E_i \qquad \ldots (13)$$

and

$$\theta''_{i+1} = F_i \theta'_{i+1} + G_i \theta_{i+1} + H_i f'_{i+1} + I_i f_{i+1} + J_i \qquad \ldots (14)$$

where

$$A_i = -\frac{2}{3n} f_i |f_i''|^{1-n} - (1-n)\,\alpha_1 |f_i''|^{-n} \left[\frac{2}{3n} f_i f_i'' - \frac{1}{3n} f_i'^2 + \frac{1}{n}\theta_i\right]$$

$$B_i = \frac{2}{3n} f_i' |f_i''|^{1-n}, \quad C_i = -\frac{2}{3n} f_i'' |f_i''|^{1-n}, \quad D_i = \frac{1}{n} |f_i''|^{1-n}$$

$$E_i = -A_i f_i'' - B_i f_i' - C_i f_i - D_i \theta_i + |f_i''|^{1-n}\left[-\frac{2}{3n} f_i f_i'' + \frac{1}{3n} f_i'^2 - \frac{1}{n}\theta_i\right]$$

$$F_i = -\frac{2}{3} N_{Pr} f_i, \quad G_i = -\frac{1}{3} N_{Pr} f_i', \quad H_i = -\frac{1}{3} N_{Pr}\,\theta_i, \quad I_i = -\frac{2}{3} N_{Pr}\,\theta_i'$$

$$J_i = -F_i\theta_i' - G_i\theta_i - H_i f_i' - I_i f_i - \frac{N_{Pr}}{3}(\theta_i f_i' + 2f_i\theta_i')$$

$$\alpha_i = -1 \text{ if } f_i'' < 0 \text{ and } \alpha_1 = 1 \text{ if } f_i'' > 0.0$$

The subscripts i and $i + 1$ denote the values of the variables at two successive iterations. The iterative process is terminated when $|f_i - f_{i+1}| < 10^{-4}$ and $|\theta_i - \theta_{i+1}| < 10^{-4}$. Equations (13) and (14) are integrated by using fourth order Runge-Kutta method and the boundary conditions are satisfied with the application of principle of superposition. It is worthwhile to mention here that as the values of N_{Pr} increases the thickness of thermal boundary layer decreases while that of velocity boundary layer thickness increases. Further, for a large value of N_{Pr} (N_{Pr} >> 1.0) eqns. (13) and (14) behave as a set of stiff ordinary differential equation[22], hence the classical Runge-Kutta method diverges if both the momentum and energy equations are integrated simultaneously for a large value of η_∞. To overcome this difficulty care has been taken to decouple the momentum equation from the energy equation after a certain value of η say η_t (since θ and θ' approach zero very fast). Five different step sizes have been chosen to check the convergence of the solution with reference to grid size. It is observed that a step width of 0.1 gives a grid independent solution (Table I). However, it is found that the convergence of the iterative process depends basically on the initial profiles and relaxation parameter. In first attempt, it is not possible to obtain a converged solution for a desired value of η_∞ for N_{Pr} >> 1, which satisfies the asymptotic conditions. Therefore, first a converged solution is obtained

TABLE I

Effects of grid size on local Nusselt number parameter $[-\theta'(0)]$

($f_w = 0$, $N_{Pr} = 10$, $n = 0.891$, $\eta_t = 6.0$, $\eta_\infty = 24.0$)

Step size	0.3	0.2	0.1	0.05	0.025
$-\theta'(0)$	0.53810	0.53966	0.53904	0.53905	0.53892

by introducing less number of grid points (say N_0) and with crude initial profiles. Afterwards the obtained profiles are used as initial solution for a desired value of η_∞ which satisfies the asymptotic boundary conditions. As the number of grids increases, the values of the variables at the grid point N_0 are used to initialise the variables at the grid points greater than N_0. It may be pointed out here that the values of η_∞ and η_t should be properly chosen to satisfy the asymptotic conditions, which is very crucial for obtaining an accurate solution. The value of the relaxation parameter used in this calculations lies between 0.5 and 1.0, depending upon the value of the Prandtl number (N_{Pr}).

We have obtained the numerical solutions for the functions f and θ for the different values of n, N_{Pr} and f_w. With the knowledge of f and θ, we compute the skin-friction coefficient C_f and the rate of heat transfer coefficient N_{ux}. The expressions for C_f and N_{ux} are :

$$C_f = \frac{\tau_{yx} \mid y = 0}{\rho U_c^2} = \frac{[f''(0)]^n}{N_{gx}^{\frac{1}{2(n+1)}} f_{\max}'^2}$$

$$N_{ux} = \frac{h(x)\, x}{k} = -\theta'(0)\, N_{gx}^{\frac{1}{2(n+1)}}$$

where

$$U_c = (g N \beta)^{1/2} f_{\max}', \qquad N_{gx} = \left(\frac{\rho}{K}\right)^2 x^{n+2} (g N \beta\, x^{-1/3})^{2-n}.$$

Result and Discussions

Figure 1 shows the velocity profiles for the various values of power-law index n, suction/injection parameter f_w and the Prandtl number type non-Newtonian parameter N_{Pr}. Suction decreases the velocity of the Newtonian as well as power-law fluids, while injection increases the velocity in the boundary layer region. The physical explanation for such a behaviour is that while stronger blowing is provided, the heated fluid is pushed farther from the wall where the buoyancy forces can act to accelerate the flow with less influence of the viscosity. This effect acts to increase the shear by increasing the maximum velocity within the boundary layer. For suction the same principle operates but acting in the reverse direction, viz., the wall shear decreases due to suction. This type of behaviour is confirmed by Eichhorn[19] for Newtonian fluids. It is observed from Fig. 1, that the maximum velocity $f_{\max}$ increases with an increase in the value of flow behaviour index n. Further, the velocity boundary layer thickness for a pseudoplastic fluid is very thick compared to Newtonian as well as dilatant fluids (Fig. 1).

Temperature in the boundary layer region for the different values of n, f_w and N_{Pr} increases with decreasing value of flow behaviour index n with or without suction/injection (Fig. 2). However with suction (injection) the temperature in boundary layer decreases (increases). The thermal boundary layer thickness increases (decreases) with injection (suction) which causes a decrease (increase) in rate of heat transfer (Table II). The explanation for such a behaviour is that the fluid at ambient conditions is brought closer to the surface and reduces the thermal boundary layer thickness. Such a behaviour is also observed for Newtonian fluids[1, 19, 20]. With an increase in generalized Prandtl number the effects of suction/injection become more pronounced. For a given value of N_{gx} the local Nusselt number increases with a decreasing in value of n (Fig. 3). The local Nusselt number obtained by present method well agrees with the matched asymptotic results[21]. Unfortunately no

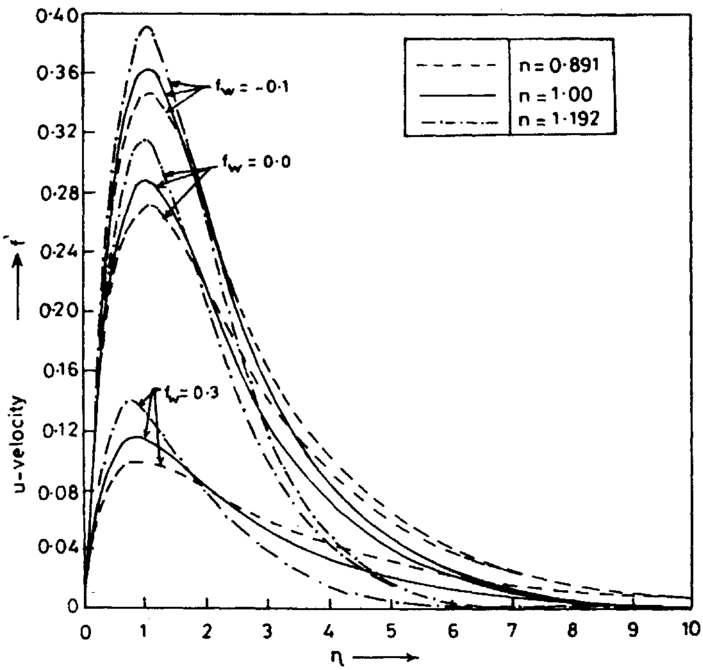

FIG. 1. Variation of velocity profiles with f_w and n for N_{Pr} = 10.

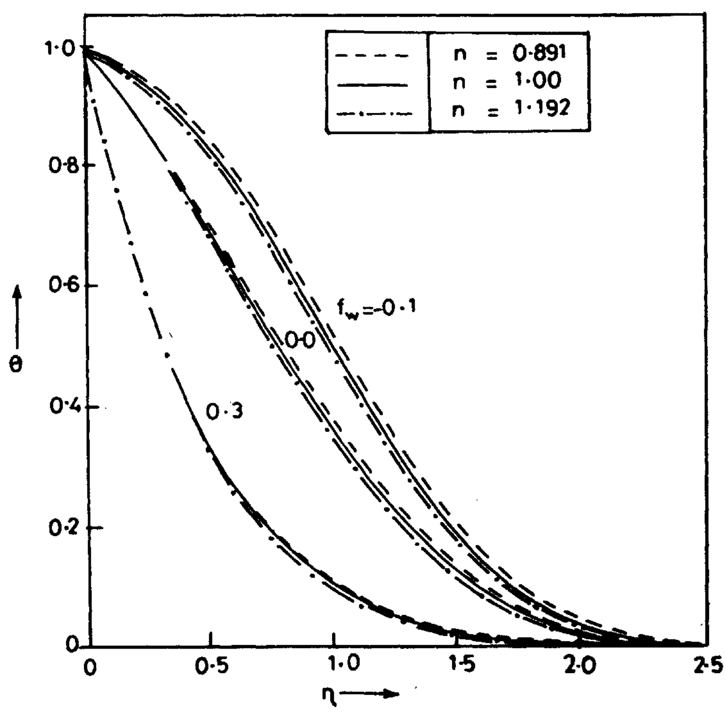

FIG. 2. Variation of temperature profiles with f_w and n for N_{Pr} = 10.

TABLE II

Variation of local Nusselt number $N_{ux}/N_{gx}^{1/2(n+1)}$ *with* f_w *and* n

	N_{Pr} = 10			N_{Pr} = 100		
f_w n	0.891	1.0	1.192	0.891	1.0	1.192
0.5	3.361	3.368	3.377	-	-	-
0.3	2.094	2.104	2.120	20.002	20.003	20.004
0.1	0.982	0.995	1.015	6.705	6.716	6.738
0.0	0.539	0.551	0.568	1.006	1.050	1.103
	(0.535)	(0.552)	(0.562)	(1.007)	(1.046)	(1.100)
− 0.01	0.462	0.473	0.4490	0.341	0.373	0.422
− 0.02	0.195	0.204	0.217	-	-	-

Results within brackets are from Sahu[21].

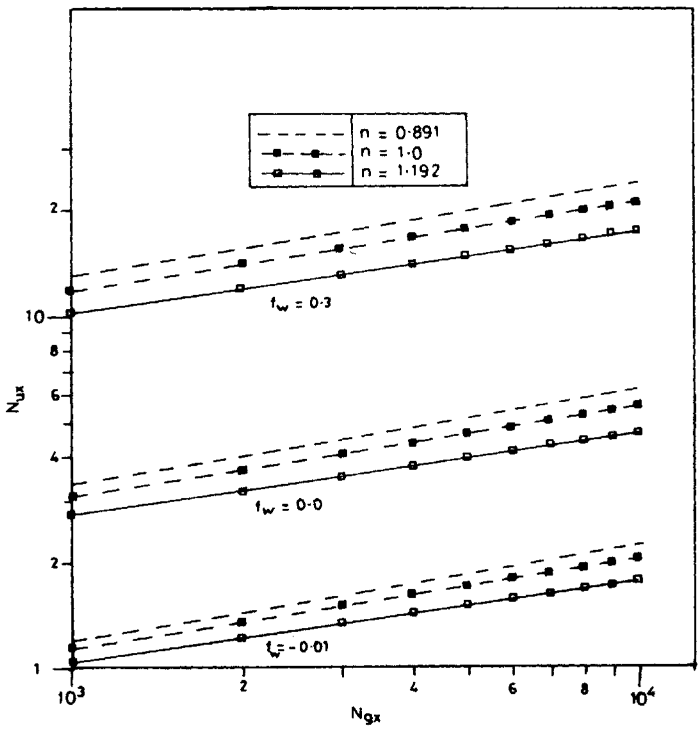

FIG. 3. Variation of local Nusselt number N_{ux} with Grashof number N_{gx} for different values of f_w and n for N_{Pr} = 10.

experimental data are available for a non-isothermal plate. However, it is observed that the average Nusselt number for an isothermal plate is higher than that of a non-isothermal plate (Fig. 4). For an isothermal plate the comparison of average Nusselt number between experimental and predicted[3] values are made by redefining the N_{Pr} and N_{gl} according to the reference[18]. It may be noted that the agreement is very good (f_w = 0.0) (Fig. 4). Based on this information the same conclusion may be drawn for a non-isothermal plate. This study may be useful in problems related to pervaporation and pertraction.

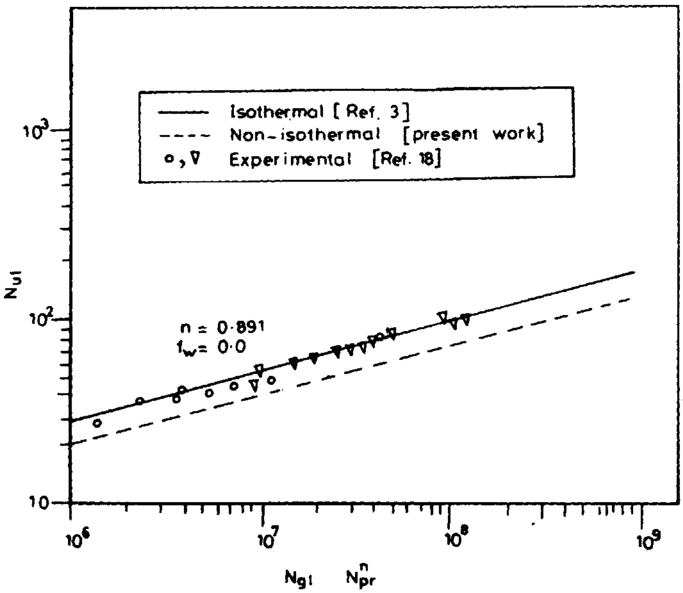

FIG. 4. Comparison of average Nusselt number N_{gl} for isothermal and non-isothermal plate (0, ∇ experimental data with plate height 0.323 ft and 0.650 ft respectively, for an isothermal case).

ACKNOWLEDGEMENT

We thank Dr K. S. Rao for his help in computer programming of the numerical part of this work.

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