Indian J. pure appl. Math., 10(1) : 100-111, January 1979

ON DISLOCATIONS AND EXTENDED SOURCES IN AN ELASTIC HALF SPACE

by ARABINDA ROY, *Centre of Advanced Study in Applied Mathematics, Calcutta University*, 92, *A.P.C. Road, Calcutta* 700009

(*Received* 18 *May* 1978)

A general method has been suggested for obtaining the displacement field in an elastic half space due to finite dislocations or stress discontinuity across an inclined fault plane. The displacement field in an infinite medium is first obtained in source coordinate system by application of Laplace and Fourier transform w.r.t. time and space coordinates. Transforming from source coordinate system to the other coordinate system the displacement field is obtained in an elastic half space. Results obtained by the present method are in agreement with those of previous workers. In particular the use of independent coordinate system for specifying the sources is found to be suitable in obtaining the displacement field for a variety of physically important sources.

INTRODUCTION

There exist different methods for finding the displacement field in an elastic half space due to dislocations or multipoles. The standard approach for a point dipole or a couple source is by differentiating the solution for a point force or by reducing the body force to stress or displacement discontinuity across a plane parallel to the surface, which can be done however the dipole is oriented. An extended source may be obtained by integration. Ben-Menahem and Singh (1968) derived explicit expression for the displacement field due to internal dislocation of arbitrary orientation and depth. Singh *et al.* (1973) gave a unified approach to the representation of seismic sources.

In the present paper we solve the same problem from a different angle. Earlier Roy (1965) gave a method for obtaining the displacement field in a half space where the body force terms appear through their transform. It was shown that for any specific point couple or dislocation or finite force the problem reduces to simply calculating the transform of body force. However, since only one coordinate system was used the results of Roy (1965) could not be applied for finite dislocations and sources across a plane inclined to the free surface except via integration process. In this paper we overcome the difficulty by using two coordinate systems, one of which describes the source system and the other is chosen such that it is perpendicular to the free surface. By application of Laplace and Fourier transforms w.r.t. the time and space coordinate and a suitable change over from one coordinate system to the other, the displacement field in a half space is derived in a suitable form particularly useful for finite dislocation across a plane inclined to the free surface.

In a later section we obtain the surface displacement, for finite dislocations and normal stress over a circular area on an inclined fault plane. As in nature the stresses are more likely to be distributed over a surface, not necessarily parallel to the free surface, the results of the present paper are readily applicable for a variety of physically important sources.

Equation of Motion

Let the elastic half space be referred to an orthogonal Cartesian coordinate system x, y, z (or x_1, x_2, x_3 whichever is convenient). The z-axis is taken to be perpendicular to the free surface and x- and y-axis are in the plane of the boundary surface. We introduce another coordinate system $(\zeta_1, \zeta_2, \zeta_3)$, referred to as the source coordinate system in the text, to describe the force system in the medium. The two coordinate systems are related by

$$\mathbf{r} = A\mathbf{r}' \qquad \ldots(1)$$

or

$$\mathbf{r}' = A^{-1}\mathbf{r} \qquad \ldots(1a)$$

where $A = [a_{ij}]$ and $A^{-1} = [a_{ji}]$ are orthogonal matrix; $\mathbf{r}$ and $\mathbf{r}'$ denote column matrices with elements (x, y, z) and $(\zeta_1, \zeta_2, \zeta_3)$ respectively.

The displacement at any point in the medium is denoted by $\mathbf{u}(\mathbf{r}, t)$ with components u, v, w and u_1, u_2, u_3 referred to x, y, z and $(\zeta_1, \zeta_2, \zeta_3)$ coordinate systems respectively. The stress tensor is denoted by τ_{ij} and σ_{ij} in the two coordinate systems.

The equation of motion, at points not containing the source can be written in source coordinate system as

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial}{\partial \zeta_j} \sigma_{ij} \qquad \ldots(2)$$

where

$$\sigma_{ij} = \lambda \frac{\partial u_i}{\partial \zeta_i} \delta_{ij} + \mu \left(\frac{\partial u_i}{\partial \zeta_j} + \frac{\partial u_j}{\partial \zeta_i} \right) \qquad \ldots(3)$$

i and j run over 1, 2 and 3. δ_{ij} is Kronecker delta function; λ and μ are Lame's constants and ρ is the density of the medium.

The equation of motion at points containing the source can be written as

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial}{\partial \zeta_j} \sigma_{ij} + F_i \qquad \ldots(4)$$

where F_1, F_2, F_3 are the components of body forces in the $(\zeta_1, \zeta_2, \zeta_3)$ coordinate system.

Since F_i vanishes outside the source region, (4) can be taken as the general equation of motion at all points both inside and outside the source region, in the medium.

If the disturbance at the focus happens to be due a dislocation across a fault plane, (2) can be taken as the general equation of motion regarding the derivative in the generalized sense (Jones 1966). In this case also (2) can be written in the form (4) and F_i are to be regarded then as the equivalent body forces in the medium, which produce the same radiation as the dislocation itself. The form of equivalent body forces in case of dislocations in the medium is given below.

Let us assume that across a plane ζ_3 = constant = ζ_{30}, the displacement or stress is discontinuous. We have, following Jones (1966)

$$\left.\begin{aligned}
\frac{\partial}{\partial\zeta_3}\{u_i\} &= \frac{\partial u_i}{\partial\zeta_3} + [u_i]\,\delta(\zeta_3-\zeta_{30})\\
\frac{\partial^2}{\partial\zeta_3^2}\{u_i\} &= \frac{\partial^2 u_i}{\partial\zeta_3^2} + \left[\frac{\partial u_i}{\partial\zeta_3}\right]\delta(\zeta_3-\zeta_{30}) + [u_i]\,\delta'(\zeta_3-\zeta_{30})\\
\frac{\partial^2}{\partial\zeta_j^2}\{u_i\} &= \frac{\partial^2 u_i}{\partial\zeta_j^2}\ (j=1,2)
\end{aligned}\right\} \quad \ldots(5)$$

where

$$[u_i] = u_i^+(\zeta_1,\zeta_2) - u_i^-(\zeta_1,\zeta_2) \quad \ldots(6)$$

denotes the jump in the displacement between the two faces of the surface $\zeta_3 = \zeta_{30}$; $\frac{\partial}{\partial\zeta_j}\{u_i\}$ denotes the generalized derivative of u_i with respect to ζ_j; $\delta(x)$ is Dirac's delta function.

Using (5) we can write easily the equivalent body force for faulting across $\zeta_3 = \zeta_{30}$ as,

$$\left.\begin{aligned}
F_1 &= -[\sigma_{13}]\,\delta(\zeta_3-\zeta_{30}) - \lambda\left[\frac{\partial u_3}{\partial\zeta_1}\right]\delta(\zeta_3-\zeta_{30}) - \mu[u_1]\,\delta'(\zeta_3-\zeta_{30})\\
F_2 &= -[\sigma_{23}]\,\delta(\zeta_3-\zeta_{30}) - \lambda\left[\frac{\partial u_3}{\partial\zeta_2}\right]\delta(\zeta_3-\zeta_{30}) - \mu[u_2]\,\delta'(\zeta_3-\zeta_{30})\\
F_3 &= -[\sigma_{33}]\,\delta(\zeta_3-\zeta_{30}) - \mu\left(\left[\frac{\partial u_1}{\partial\zeta_1}\right] + \left[\frac{\partial u_2}{\partial\zeta_2}\right]\right)\delta(\zeta_3-\zeta_{30})\\
&\quad - (\lambda+2\mu)\,[u_3]\,\delta'(\zeta_3-\zeta_{30})
\end{aligned}\right\} \quad \ldots(7)$$

where $[\sigma_{i3}]$ and $[u_i]$ denote the jump in the stress and displacement across $\zeta_3 = \zeta_{30}$.

Since the normal to a fault plane can always be taken as the ζ_3-axis, (7) can be used as the equivalent body force for any dislocation or stress discontinuity across a plane.

In (7) the body force equivalents are written in the source coordinate system. An alternative derivation for the body force equivalents has been given by Burridge and Knopoff (1964). It is easily seen that our result is identical with that of Burridge and Knopoff (1964). Thus we note that (4) can be regarded as the equation of motion in the medium where F_i may be regarded as body forces, volume or surface or the equivalent body forces in case of dislocation. In the following section we give a general method of solution of (4) in a half space.

Displacement in an Infinite Medium

We assume for the present that with the system of axes chosen as before, the medium extends to infinity. We make the following transform

$$\bar{\mathbf{u}} = \int_0^\infty \mathbf{u}\, e^{-pt} dt,$$

$$\mathbf{u}^{kvs} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{u} \exp\left(-i(k\zeta_1 + v\zeta_2 + s\xi_3)\right) d\zeta_1 d\zeta_2 d\zeta_3, \qquad \ldots(8)$$

with inverse transform defined by

$$\mathbf{u} = \frac{1}{2\pi i}\int_{Br} \bar{\mathbf{u}}\, e^{pt} dp,$$

$$\mathbf{u}^{kvs} = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{u}^{kvs} \exp\left(i(k\zeta_1 + v\zeta_2 + s\zeta_3)\right) dk\, dv\, ds, \qquad \ldots(9)$$

where Br denotes the Bromwich contour.

Transforming the equation of motion (4) by means of (8) we get

(a) $$(\beta^2 s^2 + \alpha^2 k^2 + \beta^2 v^2 + p^2)\, \bar{u}_1^{kvs} + (\alpha^2 - \beta^2)\, kv \bar{u}_2^{kvs} + (\alpha^2 - \beta^2)\, ks\, \bar{u}_3^{kvs} = \frac{\bar{F}_1^{kvs}}{\rho},$$

(b) $$(\alpha^2 - \beta^2)\, kv\, \bar{u}_1^{kvs} + (\beta^2 s^2 + \alpha^2 v^2 + \beta^2 k^2 + p^2)\, \bar{u}_2^{kvs} + (\alpha^2 - \beta^2)\, vs\, \bar{u}_3^{kvs} = \frac{\bar{F}_2^{kvs}}{\rho},$$

(equation continued on p. 104)

(c) $$(\alpha^2 - \beta^2)\, ks\, \bar{u}_1^{kvs} + (\alpha^2 - \beta^2)\, vs\, \bar{u}_2^{kvs} + (\alpha^2 s^2 + \beta^2 k^2 + \beta^2 v^2 + p^2)\, \bar{u}_3^{kvs} = \frac{\bar{F}_3^{kvs}}{\rho} \qquad \ldots(10)$$

where

$$\alpha^2 = \frac{\lambda + 2\mu}{\rho}, \quad \beta^2 = \frac{\mu}{\rho}. \qquad \ldots(11)$$

Solving from (10) for $\bar{u}_j^{kvs}$ etc. and applying the inverse transform (9) we get

$$u_j = \frac{1}{(2\pi)^4 i} \int_{Br} e^{pt}\, dp \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{P_j \exp\left(i(k\zeta_1 + v\zeta_2 + s\zeta_3)\right) dk\, dv\, ds}{\alpha^2 \beta^4 (s^2 + s_\alpha^2)(s^2 + s_\beta^2)^2} \qquad \ldots(12)$$

where

$$\rho P_1 = \bar{F}_1^{kvs} [\{\alpha^2(k^2 + v^2 + s^2) + p^2\} - (\alpha^2 - \beta^2) k^2] - \bar{F}_2^{kvs} (\alpha^2 - \beta^2) kv - \bar{F}_3^{kvs} (\alpha^2 - \beta^2) ks,$$

$$\rho P_2 = -\bar{F}_1^{kvs} (\alpha^2 - \beta^2) kv + \bar{F}_2^{kvs} [\{\alpha^2(k^2 + v^2 + s^2) + p^2\} - (\alpha^2 - \beta^2) v^2] - \bar{F}_3^{kvs} (\alpha^2 - \beta^2) vs,$$

$$\rho P_3 = -\bar{F}_1^{kvs} (\alpha^2 - \beta^2) ks - \bar{F}_2^{kvs} (\alpha^2 - \beta^2) vs + \bar{F}_3^{kvs} [\{\alpha^2(k^2 + v^2 + s^2) + p^2\} - (\alpha^2 - \beta^2) s^2],$$

$$s_\alpha = \left(k^2 + v^2 + \frac{p^2}{\alpha^2}\right)^{1/2}, \quad s_\beta = \left(k^2 + v^2 + \frac{p^2}{\beta^2}\right)^{1/2}. \qquad \ldots(13)$$

With a view to apply the results in a half space with the free surface boundary condition, in (12) we change over from source coordinate system $(\zeta_1, \zeta_2, \zeta_3)$ to (x, y, z) system through the relation (1a) and the variables of integration (k, v, s) are changed to (ξ, η, ζ), also through the relation

$$\Omega = A\Omega' \qquad \ldots(14)$$

or

$$\Omega' = A^{-1}\Omega \qquad \ldots(14a)$$

where Ω', Ω denote column matrix with elements (k, v, s) and (ξ, η, ζ) respectively.

Then, on changing over to new variable (ξ, η, ζ), (12) takes the form

$$u_j = \frac{1}{(2\pi)^4 i} \int_{Br} e^{pt} dp \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{P_j' \exp (i(\xi x + \eta y + \zeta z) \, d\xi \, d\eta \, d\zeta}{\alpha^2 \beta^4 (\zeta^2 + \zeta_\alpha^2)(\zeta^2 + \zeta_\beta^2)^2} \quad \text{...(15)}$$

where

$$\zeta_\alpha = \left(\xi^2 + \eta^2 + \frac{p^2}{\alpha^2} \right)^{1/2},$$

$$\zeta_\beta = \left(\xi^2 + \eta^2 + \frac{p^2}{\beta^2} \right)^{1/2}, \quad \text{...(16)}$$

with the Riemann sheet chosen by the condition $\mathrm{Re}\,(\zeta_\alpha) > 0$ and $\mathrm{Re}\,(\zeta_\beta) > 0$ and P_j' are obtained from P_j after making use of transformation rule (14*a*).

We now evaluate (15). Let us assume that the body forces are distributed over a region extending from $z = 0$ to $z = h_1$. We note that the only poles in the ζ-plane are at $\zeta = \pm i\zeta_\alpha$ and $\zeta = \pm i\zeta_\beta$. Then on evaluating (15) at the respective poles, (i.e. at $\zeta = - i\zeta_\alpha$, $\zeta = - i\zeta_\beta$ for $z < 0$) the displacement in an infinite medium, after some simplification, can be written in x, y, z system as, for $z < 0$,

$$\mathbf{u}_{inc} = \int_{Br} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\exp (pt + i(\xi x + \eta y))}{16\pi^3 p^2 i \rho}$$

$$\times \left[- (\xi \mathbf{e}_x + \eta \mathbf{e}_y - i\zeta_\alpha \mathbf{e}_z) \frac{A}{\zeta_\alpha} \exp (\zeta_\alpha z) \right.$$

$$+ B \exp (\zeta_\beta z) \left(\xi \mathbf{e}_x + \eta \mathbf{e}_y - i \frac{(\xi^2 + \eta^2)}{\zeta_\beta} \mathbf{e}_z \right)$$

$$\left. + \frac{p^2}{\beta^2} C (\eta \mathbf{e}_x - \xi \mathbf{e}_y) \exp (\zeta_\beta z) \right] d\xi \, d\eta \, dp \quad \text{...(17)}$$

where $\mathbf{u}_{inc}$ denotes the incident field i.e. the displacement in an infinite medium; $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ are unit vectors in the direction of x-, y- and z-axis.

$$A = D_\alpha \xi + E_\alpha \eta - i\zeta_\alpha G_\alpha,$$

$$B = (D_\beta \xi + E_\beta \eta) \frac{\zeta_\beta}{(\xi^2 + \eta^2)} - iG_\beta,$$

$$C = \frac{D_\beta \eta - E_\beta \xi}{(\xi^2 + \eta^2) \zeta_\beta},$$

$$D = \bar{F}_1^{kvs} a_{11} + \bar{F}_2^{kvs} a_{12} + \bar{F}_3^{kvs} a_{13},$$

$$E = \bar{F}_1^{kvs} a_{21} + \bar{F}_2^{kvs} a_{22} + \bar{F}_3^{kvs} a_{23},$$

$$F = \bar{F}_1^{kvs} a_{31} + \bar{F}_2^{kvs} a_{32} + \bar{F}_3^{kvs} a_{33}. \quad \text{...(18)}$$

D_α and D_β are obtained from D after changing over from variable (k, v, s) to (ξ, η, ζ) through transformation rule (14*a*) and setting $\zeta = -i\zeta_\alpha$ and $\zeta = -i\zeta_\beta$ respectively. Similar meaning is attached to E_α, E_β, G_α, G_β.

In particular, when the source coordinate system is the same as (x, y, z) coordinate system, then

$$D = \bar{X}^{\xi\eta\zeta},\ E = \bar{Y}^{\xi\eta\zeta},\ G = \bar{Z}^{\xi\eta\zeta} \qquad \ldots(19)$$

where (X, Y, Z) are the components of forces along x, y, z axes and

$$(\bar{X}^{\xi\eta\zeta}, \bar{Y}^{\xi\eta\zeta}, \bar{Z}^{\xi\eta\zeta}) = \int_0^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty (X, Y, Z) \exp(-pt - i(\xi x + \eta y + \zeta z))\, dx\, dy\, dz\, dt. \qquad \ldots(20)$$

For $z > h_1$, the incident displacement is given by, on evaluating (15) at the poles $\zeta = i\zeta_\alpha$ and $\zeta = i\zeta_\beta$,

$$\begin{aligned} \mathbf{u}_{inc} = \int_{Br} \int_{-\infty}^\infty \int_{-\infty}^\infty & \frac{\exp(pt + i(\xi x + \eta y))}{16 p^2 \pi^3 i \rho} \\ & \times \Big[-(\xi \mathbf{e}_x + \eta \mathbf{e}_y + i\zeta_\alpha \mathbf{e}_z) \frac{A'}{\zeta_\alpha} \exp(-\zeta_\alpha z) \\ & + B' \exp(-\zeta_\beta z) \left(\xi \mathbf{e}_x + \eta \mathbf{e}_y + i \frac{(\xi^2 + \eta^2)}{\zeta_\beta} \mathbf{e}_z \right) \\ & + \frac{p^2}{\beta^2} C' \exp(-\zeta_\beta z)(\eta \mathbf{e}_x - \xi \mathbf{e}_y) \Big] d\xi\, d\eta\, dp \end{aligned} \qquad \ldots(21)$$

where

$$\begin{aligned} A' &= D'_\alpha \xi + E'_\alpha \eta + i\zeta_\alpha G'_\alpha, \\ B' &= (D'_\beta \xi + E'_\beta \eta) \frac{\zeta_\beta}{(\xi^2 + \eta^2)} + iG'_\beta, \\ C' &= (D'_\beta \eta - E'_\beta \xi) \frac{1}{(\xi^2 + \eta^2)\zeta_\beta}, \end{aligned} \qquad \ldots(22)$$

D'_α and D'_β are obtained from D on changing first (k, v, s) to (ξ, η, ζ) by using (14*a*) and setting $\zeta = i\zeta_\alpha$ and $\zeta = i\zeta_\beta$. Similarly E'_α, E'_β, G'_α, G'_β etc., are obtained from E, G.

Displacement in an Elastic Half Space

We now consider the case when the disturbance is inside an elastic half space with the bounding surface $z = -h$ being free, i.e.

$$\tau_{zx} = \tau_{zy} = \tau_{zz} = 0 \quad \text{at } z = -h. \qquad ...(23)$$

In this case the displacement at any point in the medium can be written as

$$\mathbf{u} = \mathbf{u}_{inc} + \mathbf{u}_{ref} \qquad ...(24)$$

where the incident field $\mathbf{u}_{inc}$ is given by (17), for $z < 0$ and $\mathbf{u}_{ref}$ is the field, due to the presence of free surface and satisfies the equation of motion (2), without the force term.

The general expression for $\mathbf{u}_{ref}$ can be written as

$$\mathbf{u}_{ref} = \int_{Br} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\exp(pt + i(\xi x + \eta y))}{16\pi^3 p^2 i \rho}$$

$$\times \Big[A_1(\xi \mathbf{e}_x + \eta \mathbf{e}_y + i\zeta_\alpha \mathbf{e}_z) \exp(-\zeta_\alpha z)$$

$$- B_1 \exp(-\zeta_\beta z) \left(\xi \mathbf{e}_x + \eta \mathbf{e}_y + \frac{i(\xi^2 + \eta^2)}{\zeta_\beta} \mathbf{e}_z \right)$$

$$+ C_1 \exp(-\zeta_\beta z)(\eta \mathbf{e}_x - \xi \mathbf{e}_y) \Big] d\xi\, d\eta\, dp. \qquad ...(25)$$

The constants A_1, B_1, C_1 are determined from the condition that (25) satisfies (23). On evaluating A_1, B_1, and C_1 and on substitution, $\mathbf{u}_{ref}$ can be written as

$$\mathbf{u}_{ref} = \int_{Br} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\exp(pt + i(\xi x + \eta y))}{16\pi^3 p^2 i \rho}$$

$$\times \Big[\exp(-\zeta_\alpha(z + 2h))(\xi \mathbf{e}_x + \eta \mathbf{e}_y + i\zeta_\alpha \mathbf{e}_z))$$

$$\times \left\{ \frac{A}{\zeta_\alpha} \frac{F^*(\xi, \eta)}{F(\xi, \eta)} - \frac{4B(\xi^2 + \eta^2)}{F(\xi, \eta)} \left(2\xi^2 + 2\eta^2 + \frac{p^2}{\beta^2} \right) \right\}$$

$$- \{(\xi \mathbf{e}_x + \eta \mathbf{e}_y)\zeta_\beta + i(\xi^2 + \eta^2)\mathbf{e}_z\} \exp(-\zeta_\beta(z + 2h))$$

$$\times \left\{ \frac{4A}{F(\xi, \eta)} \left(2\xi^2 + 2\eta^2 + \frac{p^2}{\beta^2} \right) - \frac{B}{\zeta_\beta} \frac{F^*(\xi, \eta)}{F(\xi, \eta)} \right\}$$

$$+ \frac{p^2}{\beta^2} C \exp(-\zeta_\beta(z + 2h))(\eta \mathbf{e}_x - \xi \mathbf{e}_y) \Big] d\xi\, d\eta\, dp \qquad ...(26)$$

where

$$F^*(\xi, \eta) = \left(2\xi^2 + 2\eta^2 + \frac{p^2}{\beta^2} \right)^2 + 4(\xi^2 + \eta^2)\zeta_\alpha \zeta_\beta,$$

$$F(\xi, \eta) = \left(2\xi^2 + 2\eta^2 + \frac{p^2}{\beta^2} \right)^2 - 4(\xi^2 + \eta^2)\zeta_\alpha \zeta_\beta. \qquad ...(27)$$

In particular the surface displacement can be written from (24) and (26) by setting $z = -h$ and are given by

$$\mathbf{u}(-h) = \int_{Br} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\exp(pt + i(\xi x + \eta y))}{8\pi^3 \mu i F(\xi, \eta)}$$

$$\times \Big[-A \exp(-\zeta_\alpha h) \Big\{ 2\zeta_\beta(\xi \mathbf{e}_x + \eta \mathbf{e}_y) - i\Big(2\xi^2 + 2\eta^2 + \frac{p^2}{\beta^2}\Big) \mathbf{e}_z \Big\}$$

$$+ \Big\{ \Big(2\xi^2 + 2\eta^2 + \frac{p^2}{\beta^2}\Big)(\xi \mathbf{e}_x + \eta \mathbf{e}_y) - 2i\zeta_\alpha(\xi^2 + \eta^2)\, \mathbf{e}_z \Big\}$$

$$\times B \exp(-\zeta_\beta h) \Big] \, d\xi \, d\eta \, dp + \int_{Br} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{C \exp(-\zeta_\beta h)}{8\pi^3 \mu i}$$

$$\times (\eta \mathbf{e}_x - \xi \mathbf{e}_y) \exp(pt + i(\xi x + \eta y)) \, d\xi \, d\eta \, dp \qquad \ldots(28)$$

where A, B and C are given by (18).

Applications

As an application we consider the case of finite dislocation across a fault plane. Let the faulting take place along the plane $\zeta_3 = 0$. Let ζ_1-axis be taken along the slip direction and ζ_2-axis be perpendicular to the slip direction. The x-axis is taken along the strike of the fault. Then if λ be the slip angle and δ be the dip of the fault plane, the two coordinate systems are related by (Fig. 1)

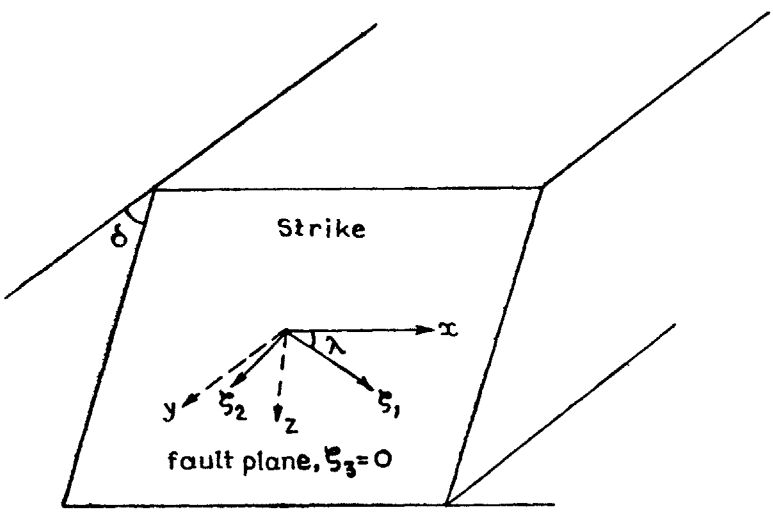

FIG. 1. Two co-ordinate systems.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{pmatrix} = \begin{pmatrix} \cos\lambda & -\sin\lambda & 0 \\ \sin\lambda\cos\delta & \cos\lambda\cos\delta & -\sin\delta \\ \sin\lambda\sin\delta & \cos\lambda\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \xi_3 \end{pmatrix}. \quad \ldots(29)$$

Let the dislocation be specified by

$$[u_1] = \delta(\zeta_1)\,[H(\zeta_2) - H(\zeta_2 - b)]\,H(t) \quad \ldots(30)$$

where b is the extent of dislocation along ζ_2-axis.

Equivalent body forces following (6) are

$$F_1 = -\mu\delta(\zeta_1)\,[H(\zeta_2) - H(\zeta_2 - b)]\,\delta'(\zeta_3)\,H(t),$$

$$F_2 = -0,$$

$$F_3 = -\mu\delta'(\zeta_1)\,[H(\zeta_2) - H(\zeta_2 - b)]\,\delta(\zeta_3)\,H(t),$$

$$\bar{F}_1^{kvs} = -i\mu s\left(\frac{1}{p}\right)\left\{\frac{1 - e^{ibv}}{iv}\right\},$$

$$\bar{F}_2^{kvs} = 0,$$

$$\bar{F}_3^{kvs} = -i\mu k\left(\frac{1}{p}\right)\left\{\frac{1 - e^{ivb}}{iv}\right\}. \quad \ldots(31)$$

Changing over from the variables (k, v, s) to the variables (ξ, η, ζ) through

$$\begin{pmatrix} k \\ v \\ s \end{pmatrix} = A^{-1} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = \begin{pmatrix} \cos\lambda & \sin\lambda\cos\delta & \sin\lambda\sin\delta \\ -\sin\lambda & \cos\lambda\cos\delta & \cos\lambda\sin\delta \\ 0 & -\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} \quad \ldots(32)$$

and using eqn. (18) we get

$$A = -\mu[-2i\xi\eta\cos\lambda\sin\delta + 2\xi\zeta_\alpha\cos\lambda\cos\delta + 2\eta\zeta_\beta\sin\lambda\cos 2\delta$$
$$- i(\eta^2 + \zeta_\alpha^2)\sin\lambda\sin 2\delta]\left(\frac{1}{p}\right)\left\{\frac{1 - \exp(-iv_\alpha b)}{iv_\alpha}\right\},$$

$$B = -\mu\Big[-2i\xi\eta\zeta_\beta\cos\lambda\sin\delta + \xi\left(2\xi^2 + 2\eta^2 + \frac{p^2}{\beta^2}\right)\cos\lambda\cos\delta$$
$$+ \eta\left(2\xi^2 + 2\eta^2 + \frac{p^2}{\beta^2}\right)\sin\lambda\cos 2\delta - i\zeta_\beta(2\eta^2 + \xi^2)\sin\lambda\sin 2\delta\Big]$$
$$\times \frac{1}{(\xi^2 + \eta^2)} \times \left(\frac{1}{p}\right)\left\{\frac{1 - \exp(-iv_\beta b)}{iv_\beta}\right\},$$

$$C = -\mu[-i(\eta^2 - \xi^2)\cos\lambda\sin\delta + \eta\zeta_\beta\cos\lambda\cos\delta + i\xi\eta\sin\lambda\sin 2\delta$$
$$- \xi\zeta_\beta\sin\lambda\cos 2\delta]\left(\frac{1}{p}\right)\cdot\frac{1}{(\xi^2 + \eta^2)\,\zeta_\beta}\left\{\frac{1 - \exp(-iv_\beta b)}{iv_\beta}\right\} \quad \ldots(33)$$

where

$$\nu_\alpha = \xi \cos\lambda + \eta \sin\lambda \cos\delta - i\zeta_\alpha \sin\lambda \sin\delta,$$

$$\nu_\beta = -\xi \sin\lambda + \eta \cos\lambda \cos\delta - i\zeta_\beta \cos\lambda \sin\delta,$$

$$\zeta_\alpha = \left(\xi^2 + \eta^2 + \frac{p^2}{\alpha^2}\right)^{1/2}, \quad \zeta_\beta = \left(\xi^2 + \eta^2 + \frac{p^2}{\beta^2}\right)^{1/2}. \qquad \ldots(34)$$

Substituting the above values of A, B, and C in (28) we get the surface displacement in an elastic half space. In case of point dislocation, factor inside the curly bracket is set equal to 1. For a vertical strikeslip fault (i.e. $\lambda = 0$, $\delta = \pi/2$) or a vertical dip slip fault (i.e. $\lambda = \pi/2$, $\delta = \pi/2$) the surface displacement as obtained from above, on transforming to the cylindrical coordinate system and using the standard formula for Bessel function, can be shown to be in agreement with those of Ben-Menahem and Singh (1968).

For tensile dislocation across the fault plane $\zeta_3 = 0$, we have

$$[u_3] = \delta(\zeta_1)\,[H(\zeta_2) - H(\zeta_2 - b)]\,H(t). \qquad \ldots(35)$$

In this case, A, B, and C have the following values :

$$A = \mu\left[i\left(2\xi^2 + 2\eta^2 + \frac{p^2}{\beta^2}\right) + 2\zeta_\alpha\eta \sin 2\delta - i2(\eta^2 + \zeta_\alpha^2)\sin^2\delta\right]\left(\frac{1}{p}\right)\left\{\frac{1 - \exp(-i\nu_\alpha b)}{i\nu_\alpha}\right\},$$

$$B = \mu\left[2i(\xi^2 + \eta^2)\,\zeta_\beta + \sin 2\delta\left(2\xi^2 + 2\eta^2 + \frac{p^2}{\beta^2}\right)\eta - i2\sin^2\delta\,(\xi^2 + 2\eta^2)\right]\frac{1}{p(\xi^2 + \eta^2)}\left\{\frac{1 - \exp(-i\nu_\beta b)}{i\nu_\beta}\right\},$$

$$C = \mu(2i\xi\eta \sin^2\delta - \xi\zeta_\beta \sin 2\delta)\,\frac{1}{(\xi^2 + \eta^2)\,\zeta_\beta}\left(\frac{1}{p}\right)\left\{\frac{1 - \exp(-i\nu_\beta b)}{i\nu_\beta}\right\}. \qquad \ldots(36)$$

We now consider discontinuity in stress component normal to the fault plane in the form

$$[\sigma_{33}] = H(a - \sqrt{\zeta_1^2 + \zeta_2^2})\,H(t).$$

In this case we have

$$A = (\eta \sin\delta + i\zeta_\alpha \cos\delta)\,\frac{J_1(k_\alpha a)}{k_\alpha}\left(\frac{1}{p}\right),$$

$$B = [\eta \sin\delta + i(\xi^2 + \eta^2)\cos\delta]\,\frac{1}{(\xi^2 + \eta^2)\,p}\,\frac{J_1(k_\beta a)}{k_\beta a},$$

$$C = \frac{\xi \sin\delta}{(\xi^2 + \eta^2)\,p}\,\frac{J_1(k_\beta a)}{k_\beta},$$

where

$$k_\alpha = [\xi^2 + \cos^2\delta\,\eta^2 - \zeta_\alpha^2 \sin^2\delta - i\eta\zeta_\alpha \sin 2\delta]^{1/2},$$

$$k_\beta = [\xi^2 + \cos^2\delta\,\eta^2 - \zeta_\beta^2 \sin^2\delta - i\eta\zeta_\beta \sin 2\delta]^{1/2}. \quad \ldots(37)$$

Conclusion

To show the applicability of our method we have considered two typical cases of finite dislocation and a stress distribution over a finite area on an inclined fault plane. We note that our methods are valid for any arbitrary type of body forces, volume or surface, both in case of stationary or moving ones. Nature of body forces that are of importance in Earthquake source mechanism study has been discussed by Roy (1965) and recently by Singh *et al.* (1973). In our expression for surface displacement in a half space the body force terms appear through their transform. As such for any particular type of body force, surface displacement can be immediately written simply by calculating the corresponding transform of the body forces.

Throughout our analysis we have used Cartesian coordinate as there is distinct advantage in describing the force system in Cartesian coordinate, especially in case of faulting along an inclined plane. However, if the force system is given in any other coordinate system, e.g. in cylindrical system, the corresponding results can be obtained by resolving the force system along the coordinate axis and changing over from Cartesian to cylindrical coordinate system.

The use of Cartesian coordinate system gives us the flexibility to apply our results to more complicated source models. One can use Cagniard-De-Hoop transformation directly for obtaining the theoretical seismograms.

Our method of analysis is perfectly general. As such it can be easily applied to find the displacement field for arbitrary dislocations or source both point and finite.

References

Ben-Menahem, A., and Singh, S. J. (1968). Multi-polar elastic fields in a layered half space. *Bull. Seis. Soc. Am.*, **58**, 1519-72.

Burridge, R., and Knopoff, L. (1964). Body force equivalents for seismic dislocations. *Bull. Seis. Soc. Am.*, **54**, 1875-88.

Jones, D. S. (1966). Generalized Functions. McGraw-Hill Book Co., Inc., New York.

Roy, A. (1965). On the general solution of elastic wave equations in a homogeneous medium with applications to the study of earthquake source mechanisms. *Bull. Seis. Soc. Am.*, **55**, 283-301.

Singh, S. J., Ben-Menahem, A., and Vered, M. (1973). A unified approach to the representation of seismic sources. *Proc. R. Soc.*, A **331**, 525-51.