Indian J. pure appl. Math., **10**(1) : 124-127, January 1979

ON A FUNCTION DEFINED BY GENERALIZED RODRIGUES' FORMULA

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(*Received* 23 *May* 1978 ; *after revision* 19 *August* 1978)

Patil and Thakare (1977) have obtained a bilateral generating relation for the general function $P_n^\alpha(x; r, s, p, k, \lambda)$. In this note we present a more general bilateral generating relation by specializing known results due to Srivastava and Lavoie (1975). We then extend this bilateral generating relation in the form of a mixed trilateral generating relation.

§1. Employing the differential operator $\theta = x^k(\lambda + xD)$, $D \equiv d/dx$, Patil and Thakare (1975) have defined a general function $P_n^\alpha(x; r, s, p, k, \lambda)$ as

$$P_n^\alpha(x; r, s, p, k, \lambda) \equiv P_n^\alpha(x) = x^{-\alpha} \exp(px^r)\, \theta^n(x^{\alpha+sn} \exp(-px^r)). \qquad \text{...(1.1)}$$

Recently, Patil and Thakare (1977) established the following bilateral generating relation :

$$\sum_{n=0}^{\infty} P_n^{\alpha-sn}(x)\, \phi_n(y)\, t^n = (1 - tk\, x^{s+k})^{-(\alpha+\lambda)/k}$$
$$\times \exp[px^r\{1 - (1 - tk\, x^{s+k})^{-r/k}\}]$$
$$\times F(x(1 - tk\, x^{s+k})^{-1/k}, yt(1 - tk\, x^{s+k})^{s/k}) \qquad \text{...(1.2)}$$

where

$$F(x, t) = \sum_{n=0}^{\infty} \frac{1}{n!}\, \mu_n\, P_n^{\alpha-sn}(x)\, t^n. \qquad \text{...(1.3)}$$

In fact, the bilateral generating relation (1.2) is contained in a theorem due to Srivastava and Singhal (1972).

The object of the present note is to give a generalization of (1.2) by specializing certain known results due to Srivastava and Lavoie (1975) and then to extend this bilateral generating relation in the form of a mixed trilateral generating relation.

§2. Patil and Thakare (1977) have obtained the following generating relation for $P_n^{\alpha}(x; r, s, p, k, \lambda)$:

$$\sum_{n=0}^{\infty} P_{m+n}^{\alpha-sn}(x)\{t^n/n!\} = (1 - tk\, x^{s+k})^{-(\alpha+\lambda)/k}$$
$$\times \exp[px^r\{1 - (1 - tk\, x^{s+k})^{-r/k}\}]$$
$$\times P_m^{\alpha}(x(1 - tk\, x^{s+k})^{-1/k}). \quad \text{...(2.1)}$$

In view of the relations (105) through (108) in the paper of Srivastava and Lavoie (1975), we obtain the following bilateral generating relation :

$$\sum_{n=0}^{\infty} P_{n+m}^{\alpha-sn}(x)\, \sigma_n^q(y)\{t^n/n!\} = (1 - tk\, x^{s+k})^{-(\alpha+\lambda)/k}$$
$$\times \exp[px^r\{1 - (1 - tk\, x^{s+k})^{-r/k}\}]$$
$$\times F_q[x(1 - tk\, x^{s+k})^{-1/k}, y\{t(1 - tk\, x^{s+k})^{s/k}\}^q] \quad \text{...(2.2)}$$

where $\sigma_n^q(y)$ is a polynomial of degree $[n/q]$ in y defined by

$$\sigma_n^q(y) = \sum_{j=0}^{[n/q]} \binom{n}{qj} a_j\, y^j \quad \text{...(2.3)}$$

and

$$F_q[x, t] = \sum_{n=0}^{\infty} \frac{a_n}{(qn)!} P_{qn+m}^{\alpha-sqn}(x)\, t^n \quad \text{...(2.4)}$$

where q is an arbitrary positive integer and the $a_n \neq 0$ are arbitrary constants.

Taking $m = 0$ and $q = 1$ in (2.2), we obtain (1.2).

Taking $m = 0$ and noting the relationship

$$P_n^{\alpha-sn}(x; r, s, p, k, 0) = n!\, x^{n(s+k)}\, G_n^{(\alpha)}(x, r, p, k) \quad \text{...(2.5)}$$

where $G_n^{(\alpha)}(x, r, p, k)$ are the polynomials defined by Srivastava and Singhal (1971), we obtain from (2.2) a known bilateral generating relation for $G_n^{(\alpha)}(x, r, p, k)$ (Srivastava and Lavoie 1975, p. 314, Corollary 13).

We now give, in the following theorem, an extension of the bilateral generating relation (2.2) in the form of a mixed trilateral generating relation.

Theorem — For the polynomials generated by (2.1), let

$$F_q[x, y, t] = \sum_{n=0}^{\infty} \frac{a_n}{(qn)!} P_{qn+m}^{\alpha-sqn}(x)\, g_n(y)\, t^n \qquad \ldots(2.6)$$

then

$$\sum_{n=0}^{\infty} P_{n+m}^{\alpha-sn}(x)\, \sigma_n^q(y, z)\, \{t^n/n!\} = (1 - tk\, x^{s+k})^{-(\alpha+\lambda)/k}$$

$$\times \exp\left[px^r\{1 - (1 - tk\, x^{s+k})^{-r/k}\}\right]$$

$$\times F_q[x(1 - tk\, x^{s+k})^{-1/k}, y, z\{t(1 - tk\, x^{s+k})^{s/k}\}^q] \qquad \ldots(2.7)$$

where

$$\sigma_n^q(y, z) = \sum_{j=0}^{[n/q]} a_j \binom{n}{qj} g_j(y)\, z^j. \qquad \ldots(2.8)$$

Proof of the Theorem : Consider the sum

$$\sum_{n=0}^{\infty} P_{n+m}^{\alpha-sn}(x)\, \sigma_n^q(y, z)\, \{t^n/n!\}$$

$$= \sum_{n=0}^{\infty} P_{n+m}^{\alpha-sn}(x)\, \{t^n/n!\} \sum_{j=0}^{[n/q]} a_j \binom{n}{qj} g_j(y)\, z^j$$

$$= \sum_{j=0}^{\infty} a_j g_j(y)\, z^j \frac{t^{qj}}{(qj)!} \sum_{n=0}^{\infty} P_{n+qj+m}^{\alpha-sqj-sn}(x)\, \{t^n/n!\}.$$

Now the theorem would follow if we interpret the above expression by (2.1) and (2.6).

Taking $m = 0$ and $q = 1$ in (2.7), we obtain a novel extension of the bilateral generating relation (1.2).

ACKNOWLEDGEMENT

The authors are grateful to the referee for his valuable suggestions.

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