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EXTERNAL GRIFFITH CRACK IN TRANSVERSELY ISOTROPIC MATERIAL*

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The problem of external Griffith crack for transversely isotropic material is investigated under the assumptions of plane strain. The corresponding mixed boundary value problem is solved by reducing it to the solution of dual integral equations. The dual integral equations are solved by the method of Lowengrub. The variations of normal stress are calculated numerically for three different transversely isotropic materials. It is interesting to note that the expressions for stress intensity factors are independent of elastic constants of materials, while the effect of these can be noticed on the distribution of displacements and stresses. The corresponding thermoelastic problem is also solved.

1. INTRODUCTION

Recently, Konishi (1972) has solved the problem of two coplanar Griffith cracks in an infinite transversely isotropic medium. Konishi and Atsumi (1973) have considered the problem of two coplanar Griffith cracks in a transversely isotropic strip. The problem of external Griffith crack in an infinite isotropic medium has been solved by Lowengrub (1966). In this paper we consider the problem of external Griffith crack in a transversely isotropic medium.

Section 2 deals with the formulation of the problem and basic equations of isothermal elasticity. The mixed boundary value problem is reduced to dual integral equations and solved in section 3. The quantities of physical interest are derived in this section. Section 4 deals with the numerical calculation of normal stress. The corresponding thermoelastic problem is considered in sections 5 and 6.

2. FORMULATION OF THE PROBLEM

Under the usual assumptions of plane strain, the crack is supposed to occupy the region $|x| > 1$, $y = 0$ of the two dimensional transversely isotropic medium. The crack is opened by the application of internal pressure to its faces. We assume

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that the pressure on the upper face of the crack is equal to that of lower face. It is clear that there is symmetry about x-axis but not necessarily about y-axis. The boundary conditions on $y = 0$ may be written as

$$\sigma_{xy}(x, 0) = 0, \qquad -\infty < x < \infty \qquad \text{...(2.1)}$$

$$\sigma_{yy}(x, 0) = -Ep(x), \quad |x| > 1 \qquad \text{...(2.2)}$$

$$v(x, 0) = 0, \qquad |x| < 1 \qquad \text{...(2.3)}$$

where E is the Young's modulus of the direction of x-axis and $p(x)$ is some known function of x.

Following Konishi (1972) and Lowengrub (1966), the appropriate expressions for displacement and stress components can be written as

$$v(x, y) = \frac{2}{\pi} \int_0^\infty \left\{\left(-\frac{C_{22}\beta^2 + C_{12}\xi^2}{C_{22}\alpha^2 + C_{12}\xi^2}\right) e^{-\alpha y} + e^{-\beta y}\right\} \times [\psi_s(\xi) \cos \xi x + \psi_a(\xi) \sin \xi x] \, d\xi \qquad \text{...(2.4)}$$

$$\sigma_{yy}(x, y) = -\frac{2}{\pi} \frac{C_{66}}{C_{12} + C_{66}} \int_0^\infty \left(-\frac{1}{\alpha} e^{-\alpha y} + \frac{1}{\beta} e^{-\beta y}\right) (C_{22}\beta^2 + C_{12}\xi^2) \times [\psi_s(\xi) \cos \xi x + \psi_a(\xi) \sin \xi x] \, d\xi \qquad \text{...(2.5)}$$

$$\sigma_{xy}(x, y) = -\frac{2}{\pi} \frac{C_{66}}{C_{12} + C_{66}} \int_0^\infty \xi^{-1}(C_{22}\beta^2 + C_{12}\xi^2)(e^{-\alpha y} + e^{-\beta y}) \times [\psi_s(\xi) \sin \xi x - \psi_a(\xi) \cos \xi x] \, d\xi \qquad \text{...(2.6)}$$

where $C_{11} \ldots C_{66}$ are the elastic constants and α and β are given by (2.6) of Konishi (1972).

3. Solution of Dual Integral Equations

We see that the boundary condition (2.1) is automatically satisfied. If we write

$$p(x) = p_s(x) + p_a(x)$$

and $\quad p_s(x) = \frac{1}{2}[p(x) + p(-x)], \; p_a(x) = \frac{1}{2}[p(x) - p(-x)]$

we see that the boundary conditions (2.2) and (2.3) lead to the following dual integral equations :

$$\left.\begin{aligned} &\int_0^\infty \xi^{-1} f_s(\xi) \cos \xi x \, d\xi = 0, \qquad x < 1 \\ &\int_0^\infty f_s(\xi) \cos \xi x \, d\xi = g_s(x), \qquad x > 1 \end{aligned}\right\} \qquad \text{...(3.1)}$$

and

$$\left.\begin{aligned} &\int_0^\infty \xi^{-1} f_a(\xi) \sin \xi x \, d\xi = 0, \qquad x < 1 \\ &\int_0^\infty f_a(\xi) \sin \xi x \, d\xi = g_a(x), \qquad x > 1 \end{aligned}\right\} \qquad \ldots(3.2)$$

where

$$\left[\frac{C_{22}(\alpha^2 - \beta^2)}{C^{22}\alpha^2 + C_{12}\xi^2}\right] \psi_s(\xi) = \xi^{-1} f_s(\xi) \qquad \ldots(3.3)$$

$$\left[\frac{C_{22}(\alpha^2 - \beta^2)}{C_{22}\alpha^2 + C_{12}\xi^2}\right] \psi_a(\xi) = \xi^{-1} f_a(\xi) \qquad \ldots(3.4)$$

$$g_s(x) = \frac{\pi}{2} \frac{C_{12} + C_{66}}{C_{66}} \frac{E}{K} p_s(x) \qquad \ldots(3.5)$$

$$g_a(x) = \frac{\pi}{2} \frac{C_{12} + C_{66}}{C_{66}} \frac{E}{K} p_a(x) \qquad \ldots(3.6)$$

with

$$K = \frac{(C_{12}\xi^2 + C_{22}\beta^2)(C_{12}\xi^2 + C_{22}\alpha^2)}{\alpha\beta(\alpha + \beta)\, \xi\, C_{22}}. \qquad \ldots(3.7)$$

The solutions of eqns. (3.1) and (3.2) are well known (Lowengrub 1966) and are given by

$$f_s(\xi) = \frac{2}{\pi} \int_1^\infty g_s(x) \cos \xi x \, dx + \frac{2}{\pi} \left[\frac{J_0(\xi) - 1}{\xi} \int_1^\infty g_s(x)\, dx - \xi \int_0^1 J_1(\xi t) \int_0^\infty \frac{u g_s(u)}{(u^2 - t^2)^{1/2}} \, du \, dt\right] \qquad \ldots(3.8)$$

$$f_a(\xi) = \frac{2}{\pi} \int_1^\infty g_a(x) \sin \xi x \, dx + \frac{2}{\pi} \int_0^1 t^2 J_1(\xi t) \int_1^\infty \frac{g_a(u)}{(u^2 - t^2)^{3/2}} \, du \, dt. \qquad \ldots(3.9)$$

The normal stress in $0 < x < 1$ is given by the following relation

$$\begin{aligned} \sigma_{yy}(x, 0) &= -\frac{2}{\pi} \frac{K C_{66}}{C_{12} + C_{66}} \int_0^\infty [f_s(\xi) \cos \xi x + f_a(\xi) \sin \xi x] \, d\xi \\ &= -\frac{2}{\pi} \frac{K C_{66}}{C_{12} + C_{66}} \sigma(x), \quad 0 < x < 1 \end{aligned} \qquad \ldots(3.10)$$

where

$$\sigma(x) = \frac{1}{(1 - x^2)^{1/2}} \left[\int_1^\infty g_s(u)\, du + \int_1^\infty \frac{u(u^2 - 1)^{1/2}}{u^2 - x^2} g_s(u)\, du \right.$$

$$\left. + x(1 - x^2)^{1/2} \int_1^\infty \frac{g_a(u)\, du}{(u^2 - x^2)(u^2 - 1)^{1/2}} \right], \; 0 < x < 1. \qquad ...(3.11)$$

Let us now examine cases of particular loadings:

Case (a) — If we take

$$p_s(x) = \frac{2P}{E} \delta(d - x), \; p_a(x) = 0, \; d > 1 \qquad ...(3.12)$$

we get the symmetrical loading shown in Fig. 1. In this case it is not difficult to see that

$$\sigma(x) = \frac{C_{12} + C_{66}}{C_{66}} \frac{P\pi}{K} (1 - x^2)^{-1/2} \left[1 + \frac{d(d^2 - 1)^{1/2}}{d^2 - x^2} \right]. \qquad ...(3.13)$$

The stress intensity factor is defined by

$$N = \underset{x \to 1^-}{\mathrm{Lt}} \, [\sqrt{2(1 - x)}\, \sigma_{yy}(x, 0)]. \qquad ...(3.14)$$

Substituting for $\sigma_{yy}(x, 0)$ in (3.14), we get

$$N = - 2P \left[1 + \frac{d}{(d^2 - 1)^{1/2}} \right]. \qquad ...(3.15)$$

Case (b) — If we take

$$p_s(x) = \frac{P}{E} (d - x), \; p_a(x) = \frac{P}{E} (d - x), \; d > 1 \qquad ...(3.16)$$

we get the loading shown in Fig. 2.

In this case the expression for $\sigma(x)$ takes the following form

$$\sigma(x) = \frac{\pi}{2} \frac{C_{12} + C_{66}}{C_{66}} \frac{P}{K} \left[(1 - x^2)^{-1/2} \left\{ 1 + \frac{d(d^2 - 1)^{1/2}}{d^2 - x^2} \right\} \right.$$

$$\left. + \frac{x(1 - x^2)^{1/2}}{(d^2 - x^2)(d^2 - 1)^{1/2}} \right]. \qquad ...(3.17)$$

The stress intensity factor in this case is given by

$$N = - P \left[1 + \frac{d}{(d^2 - 1)^{1/2}} \right]. \qquad ...(3.18)$$

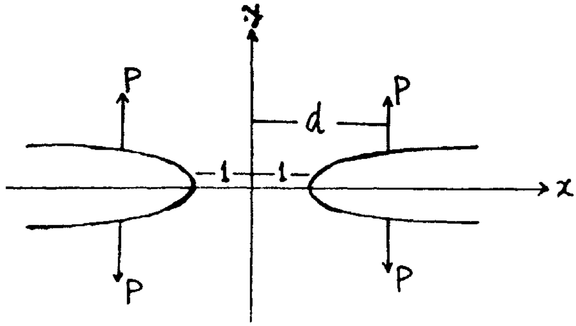

FIG. 1. Loading case (a).

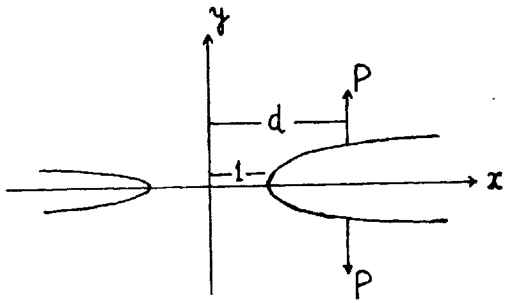

FIG. 2. Loading case (b).

4. NUMERICAL CALCULATIONS AND DISCUSSIONS

The variations of $\frac{1}{\pi P}\,\sigma(x)$ with x ($d = 1.2, 1.6$) for magnesium (Mg), cadmium (Cd) and zinc (Zn) are shown in Figs. 3, 4 and 5 respectively for different loadings. The continuous curves are for the loadings in Case (a) while dotted curves correspond to Case (b).

It is seen from (3.13) that the isotropic state appears for $E'/E \rightarrow 1.0$ and $\frac{C_{12} + C_{66}}{KC_{66}} \rightarrow 2(1 - \nu^2)$, where E' is the Young's modulus for y-axis direction.

5. THE THERMOELASTIC PROBLEM

When a simple uniform steady heat flow is disturbed by the presence of a cavity or inclusion a local intensification of temperature gradient develops with associated thermal stress. Sih (1962) has given a solution of the problem concerning a Griffith crack in an infinite material by using complex variables.

In this note, we obtain the solution for an external Griffith crack in transversely isotropic medium. We shall assume that the heat flow is perpendicular to the crack surface, which is perfectly insulated. The external Griffith crack is supposed to occupy the region $| x | > 1$, $y = 0$. The boundary conditions on $y = 0$ may be written as

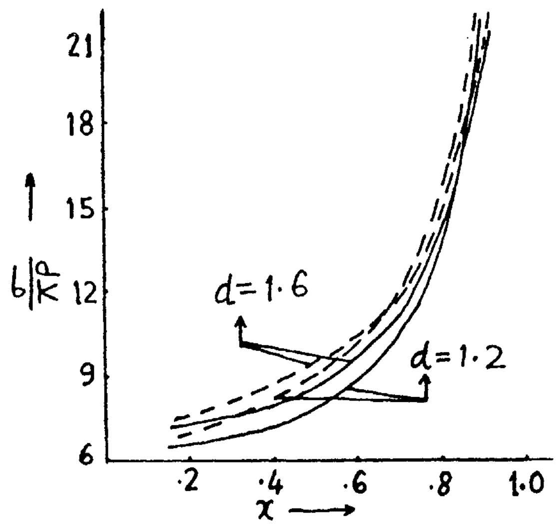

FIG. 3. Variation of $\sigma/\pi P$ with x for Mg.

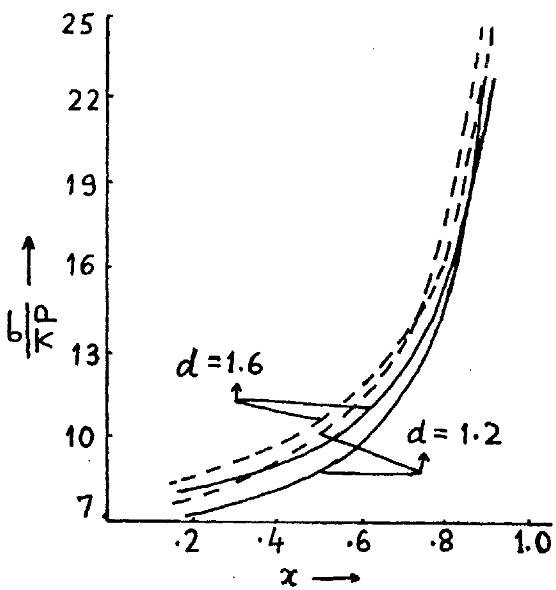

FIG. 4. Variation of $\sigma/\pi P$ with x for Cd.

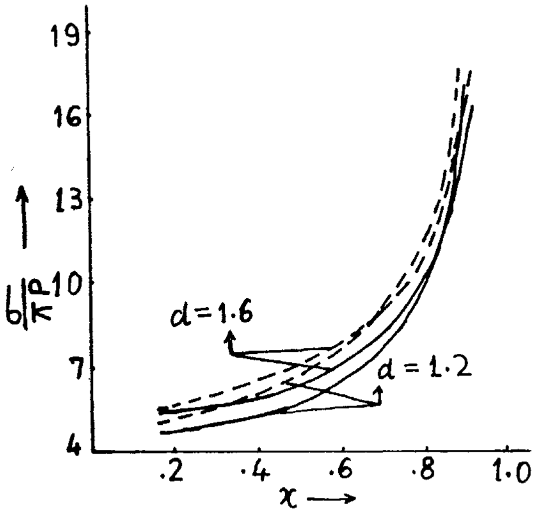

FIG. 5. Variation of $\sigma/\pi P$ with x for Zn.

$$\frac{\partial}{\partial y} T(x, y)\Big]_{y=0} = T(x), \mid x \mid > 1 \qquad ...(5.1)$$

$$T(x, 0) = 0, \mid x \mid < 1 \qquad ...(5.2)$$

$$\sigma_{yy}(x, 0) = 0, \mid x \mid > 1 \qquad ...(5.3)$$

$$v(x, 0) = 0, \mid x \mid < 1 \qquad ...(5.4)$$

$$\sigma_{xy}(x, 0) = 0, \text{ for all } x. \qquad ...(5.5)$$

For symmetrical deformation the displacement vector U may be taken to have components $(u, v, 0)$ and stress tensor the components $(\sigma_{xx}, \sigma_{xy}, \sigma_{yy})$. For a transversely isotropic medium, the stress-strain relations in terms of displacements can be written as

$$\left.\begin{aligned} \sigma_{xx} &= C_{11}u_{,x} + C_{12}v_{,y} - b_1 T \\ \sigma_{yy} &= C_{12}u_{,x} + C_{22}v_{,y} - b_1 T \\ \sigma_{xy} &= C_{66}(u_{,y} + v_{,x}) \end{aligned}\right\} \qquad ...(5.6)$$

where C_{11} to C_{66} are the elastic constants, T the deviation of absolute temperature and

$$b_1 = (C_{11} + C_{12})\,\alpha_1. \qquad ...(5.7)$$

In the above equation α_1 is the coefficient of linear expansion along the axis of elastic symmetry. The equations of equilibrium in the absence of body forces take the form

$$\sigma_{xx,x} + \sigma_{xy,y} = 0, \quad \sigma_{yy,y} + \sigma_{xy,x} = 0 \qquad ...(5.8)$$

where $\sigma_{xx,x}$ stands for $\frac{\partial}{\partial x}\sigma_{xx}$. Substitute (5.6) in (5.8) to get

$$\left.\begin{aligned} C_{11}u_{,xx} + (C_{12} + C_{66})\, v_{,xy} + C_{66}u_{,yy} - b_1 T_{,x} = 0 \\ C_{22}v_{,yy} + (C_{12} + C_{66})\, u_{,xy} + C_{66}v_{,xx} - b_1 T_{,y} = 0. \end{aligned}\right\} \quad \ldots(5.9)$$

In the steady state condition the temperature function $T(x, y)$ in transversely isotropic medium free from thermal sources satisfies the equation

$$T_{,xx} + k^2 T_{,yy} = 0 \quad \ldots(5.10)$$

where $k^2 = \beta_2/\beta_1$; β_1 and β_2 are the thermal conductivity coefficients of x- and y-axis directions. Keeping in mind that the disturbance is localized, the appropriate solution of (5.10) may be taken as

$$T(x, y) = \frac{2k}{\pi} \int_0^\infty \xi^{-1} A(\xi)\, e^{-\xi(ix+(y/k))}\, d\xi$$

which takes the following form in the absence of symmetry about y-axis.

$$T(x, y) = \frac{2k}{\pi} \int_0^\infty \xi^{-1} e^{-\xi y/k} [A_s(\xi) \cos \xi x + A_a(\xi) \sin \xi x]\, d\xi. \quad \ldots(5.11)$$

The boundary conditions (5.1) and (5.2) are satisfied provided

$$\left.\begin{aligned} \int_0^\infty \xi^{-1} [A_s(\xi) \cos \xi x + A_a(\xi) \sin \xi x]\, d\xi = 0, \; x < 1 \\ \int_0^\infty [A_s(\xi) \cos \xi x + A_a(\xi) \sin \xi x]\, d\xi = \frac{\pi}{2} T(x), \; x > 1. \end{aligned}\right\} \quad \ldots(5.12)$$

If we write $T(x) = T_s(x) + T_a(x)$, we get the following set of dual integral equations

$$\left.\begin{aligned} \int_0^\infty \xi^{-1} A_s(\xi) \cos \xi x\, d\xi = 0, \; x < 1 \\ \int_0^\infty A_s(\xi) \cos \xi x\, d\xi = \frac{\pi}{2} T_s(x), \; x > 1 \end{aligned}\right\} \quad \ldots(5.13)$$

and

$$\left.\begin{aligned} \int_0^\infty \xi^{-1} A_a(\xi) \sin \xi x\, d\xi = 0, \; x < 1 \\ \int_0^\infty A_a(\xi) \sin \xi x\, d\xi = \frac{\pi}{2} T_a(x), \; x > 1. \end{aligned}\right\} \quad \ldots(5.14)$$

The solutions of these integral equations are well known (viz. Lowengrub 1966) and need not be written here.

6. Solution of Thermoelastic Problem

Let us seek the solution of (5.9) in the form

$$\left.\begin{aligned} u &= \psi_{,x} + \phi_{,x} \\ v &= \mu\psi_{,y} + \lambda\phi_{,y} \end{aligned}\right\} \quad \text{...(6.1)}$$

where μ and λ are constants to be determined later and ψ is given by

$$\psi = \frac{2}{\pi} \int_0^\infty A(\xi)\, B(\xi)\, e^{-\xi(ix+(y/k))}. \quad \text{...(6.2)}$$

Substituting u and v in (5.9), we find that eqns. (5.9) are satisfied provided

$$\left.\begin{aligned} C_{11}\psi_{,xx} + [C_{66} + \mu(C_{12} + C_{66})]\, \psi_{,yy} &= b_1 T \\ (\mu C_{66} + C_{12} + C_{66})\, \psi_{,xx} + \mu C_{22}\psi_{,yy} &= b_1 T \end{aligned}\right\} \quad \text{...(6.3)}$$

and

$$\left.\begin{aligned} C_{11}\phi_{,xx} + [C_{66} + \lambda(C_{12} + C_{66})]\, \phi_{,yy} &= 0 \\ (\lambda C_{66} + C_{12} + C_{66})\, \phi_{,xx} + \lambda C_{22}\phi_{,yy} &= 0. \end{aligned}\right\} \quad \text{...(6.4)}$$

Equations (6.3) are satisfied if

$$\mu = \frac{(C_{66} - k^2C_{11}) + k^2(C_{12} + C_{66})}{(C_{22} - k^2C_{66}) + (C_{12} + C_{66})} \quad \text{...(6.5)}$$

$$\xi^2 B(\xi) = \frac{b_1k^2\, [(C_{22} - k^2C_{66}) - (C_{12} + C_{66})]}{(C_{22} - k^2C_{66})\,(C_{66} - k^2C_{11}) + k^2(C_{12} + C_{66})^2} = \eta \text{ (say)}. \quad \text{...(6.6)}$$

A non-zero solution of (6.4) is possible if these two equations are identical, i.e.

$$\frac{C_{66} + \lambda(C_{12} + C_{66})}{C_{11}} = \frac{\lambda C_{22}}{\lambda C_{66} + C_{12} + C_{66}} = \nu^2 \quad \text{...(6.7)}$$

where ν^2 is a constant. Eliminating λ from (6.7) we get

$$C_{11}C_{66}\nu^4 + (C_{12}^2 + 2C_{12}C_{66} - C_{11}C_{22})\, \nu^2 + C_{22}C_{66} = 0. \quad \text{...(6.8)}$$

Associated with each root ν_i^2 $(i = 1, 2)$ of (6.8) is a potential function which satisfies

$$\left(\frac{\partial^2}{\partial x^2} + \nu_i^2 \frac{\partial^2}{\partial y^2}\right) \phi_i = 0,\ i = 1, 2. \quad \text{...(6.9)}$$

The general solution of (6.9) is taken in the form

$$\phi_j = \frac{2}{\pi}\int_0^\infty \xi^{-2} C_j(\xi)\, e^{-\xi(ix+(y/\nu_j))}\, d\xi \qquad \text{...(6.10)}$$

where $C_j(\xi)$ are unknown functions to be determined from boundary conditions.

The displacement components u and v may be expressed as

$$\left.\begin{aligned} u &= (\psi + \phi_1 + \phi_2)_{,x} \\ v &= (\mu\psi + \lambda_1\phi_1 + \lambda_2\phi_2)_{,y} \end{aligned}\right\} \qquad \text{...(6.11)}$$

where λ_1 and λ_2 are given by

$$\begin{pmatrix}\lambda_1\\ \lambda_2\end{pmatrix} = -\frac{\alpha}{2} \pm \left\{\left(\frac{\alpha}{2}\right)^2 - 1\right\}^{1/2}$$

and

$$\alpha = \frac{(C_{12}+C_{66})^2 + C_{66}^2 - C_{11}C_{22}}{C_{66}(C_{12}+C_{66})}. \qquad \text{...(6.12)}$$

The components of stresses are

$$\sigma_{xx} = C_{11}(\psi + \phi_1 + \phi_2)_{,xx} + C_{12}(\mu\psi + \lambda_1\phi_1 + \lambda_2\phi_2)_{,yy} - b_1 T \qquad \text{...(6.13)}$$

$$\sigma_{yy} = C_{12}(\psi + \phi_1 + \phi_2)_{,xx} + C_{22}(\mu\psi + \lambda_1\phi_1 + \lambda_2\phi_2)_{,yy} - b_1 T \qquad \text{...(6.14)}$$

$$\sigma_{xy} = C_{66}\,[(\psi + \phi_1 + \phi_2)_{,xy} + (\mu\psi + \lambda_1\phi_1 + \lambda_2\phi_2)_{,xy}]. \qquad \text{...(6.15)}$$

Let us now satisfy the boundary conditions (5.3) to (5.5). The condition (5.5) is satisfied provided

$$\frac{1+\lambda_1}{\nu_1} C_1(\xi) = -\frac{(1+\mu)\,\eta}{k}\,\xi^{-2} A(\xi) - \frac{\lambda_2+1}{\nu_2} C_2(\xi). \qquad \text{...(6.16)}$$

The appropriate expressions for $v(x, y)$ and $\sigma_{yy}(x, y)$ take the following form

$$\begin{aligned} v(x, y) = -\frac{2}{\pi}\int_0^\infty \Bigg[&\left\{\frac{\mu\eta}{k}\, e^{-\xi y/k} - \frac{\lambda_1\eta(1+\mu)}{(1+\lambda_1)\,k}\, e^{-\xi y/\nu_1}\right\} \xi^{-1}(A_s(\xi)\cos\xi x \\ &+ A_a(\xi)\sin\xi x) + \left\{\frac{\lambda_2}{\nu_2}\, e^{-\xi y/\nu_2} - \frac{\lambda_1(1+\lambda_2)}{\nu_2(1+\lambda_1)}\, e^{-\xi y/\nu_1}\right\} \\ &\times \xi^{-1}(C_{2s}(\xi)\cos\xi x + C_{2a}(\xi)\sin\xi x)\Bigg] d\xi \end{aligned}$$

$$\sigma_{yy}(x, y) = \frac{2}{\pi} \int_0^\infty \Big[\Big\{ \Big(\frac{\mu\eta C_{22}}{k^2} - \eta C_{12} - b_1 \Big) e^{-\xi y/k}$$

$$- \frac{\nu_1(1+\mu)\eta}{k(1+\lambda_1)} \Big(\frac{\lambda_1 C_{22}}{\nu_1^2} - C_{12} \Big) e^{-\xi y/\nu_1} \Big\}$$

$$\times (A_s(\xi) \cos \xi x + A_a(\xi) \sin \xi x)$$

$$+ \Big\{ \Big(\frac{\lambda_2 C_{22}}{\nu_2^2} - C_{12} \Big) e^{-\xi y/\nu_2} - \frac{\nu_1(1+\lambda_2)}{\nu_2(1+\lambda_1)} \Big(\frac{\lambda_1 C_{22}}{\nu_1^2}$$

$$- C_{12} \Big) e^{-\xi y/\nu_1} \Big\} (C_{2s}(\xi) \cos \xi x + C_{2a}(\xi) \sin \xi x) \Big] d\xi.$$

The boundary conditions (5.3) and (5.4) are satisfied if

$$\int_0^\infty \xi^{-1} D_s(\xi) \cos \xi x \, d\xi = - \frac{d_1}{d_2} \int_0^\infty \xi^{-1} A_s(\xi) \cos \xi x \, d\xi = 0, \; 0 < x < 1$$

$$\int_0^\infty D_s(\xi) \cos \xi x \, d\xi = - \frac{d_3}{d_4} \int_0^\infty A_s(\xi) \cos \xi x \, d\xi = - \frac{\pi}{2} \frac{d_3}{d_4} T_s(x), \; 1 < x < \infty \quad \ldots(6.17)$$

$$\int_0^\infty \xi^{-1} D_a(\xi) \sin \xi x \, d\xi = - \frac{d_1}{d_2} \int_0^\infty \xi^{-1} A_a(\xi) \sin \xi x \, d\xi = 0, \; 0 < x < 1$$

$$\int_0^\infty D_a(\xi) \sin \xi x \, d\xi = - \frac{d_3}{d_4} \int_0^\infty A_a(\xi) \sin \xi x \, d\xi = - \frac{\pi}{2} \frac{d_3}{d_4} T_a(\xi), \; 1 < x < \infty \quad \ldots(6.18)$$

where $C_{2s}(\xi) = D_s(\xi)$, $C_{2a}(\xi) = D_a(\xi)$ and

$$d_1 = \frac{\eta}{k} \Big\{ \mu - \frac{\lambda_1(1+\mu)}{1+\lambda_1} \Big\}, \quad d_2 = \frac{\lambda_2 - \lambda_1}{\nu_2(1+\lambda_1)}$$

$$d_3 = \frac{\eta C_{22}}{k} \Big\{ \frac{\mu}{k} - \frac{\lambda_1(1+\mu)}{\nu_1(1+\lambda_1)} \Big\} + \eta C_{12} \Big\{ \frac{\nu_1(1+\mu)}{1+\lambda_1} - 1 \Big\} - b_1$$

$$d_4 = \frac{C_{22}}{\nu_2} \Big\{ \frac{\lambda_2}{\nu_2} - \frac{\lambda_1(1+\lambda_2)}{\nu_1(1+\lambda_1)} \Big\} + C_{12} \Big\{ \frac{\nu_1(1+\lambda_2)}{\nu_2(1+\lambda_1)} - 1 \Big\}.$$

Solutions of integral eqns. (6.17) and (6.18) are well known from Lowengrub (1966), and need not be repeated here. The component of normal stress on $y = 0$ takes the form in $0 < x < 1$

$$\sigma_{yy}(x, 0) = - d_3 \left[\frac{1}{(1 - x^2)^{1/2}} \int_1^\infty T_s(u)\, du + \frac{1}{(1 - x^2)^{1/2}} \int_1^\infty \frac{u(u^2 - 1)^{1/2}}{u^2 - x^2} \right.$$

$$\left. \times\, T_s(u)\, du + x(1 - x^2)^{1/2} \int_1^\infty \frac{T_a(u)\, du}{(u^2 - x^2)(u^2 - 1)^{1/2}} \right] + d_3 T(x, 0). \qquad \ldots(6.19)$$

The stress singularity at the tip of external crack is defined as

$$N = \underset{x \to 1^-}{\mathrm{Lt}} \left[\sqrt{2(1 - x)}\, \sigma_{yy}(x, 0) \right]. \qquad \ldots(6.20)$$

Particular Cases of Temperature

Case (*a*) — Let $T_s(x) = 2T_0 \delta(d - x)$, $T_a(x) = 0$, $d > 1$. Then

$$N = - 2T_0 d_3 \left[1 + \frac{d}{(d^2 - 1)^{1/2}} \right]. \qquad \ldots(6.21)$$

Case (*b*) — Let $T_s(x) = T_0 \delta(d - x)$, $T_a(x) = T_0 \delta(d - x)$, $d > 1$. Then

$$N = - T_0 d_3 \left[1 + \frac{d}{(d^2 - 1)^{1/2}} \right]. \qquad \ldots(6.22)$$

It is interesting to note that the nature of stress singularity at the tip of external crack in this case is exactly similar to that of isothermal case. This fact has been mathematically established by Sih (1962) for two dimensional elastic systems.

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