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ON THE ANALOGY IN SLIP FLOWS

B. S. BHATT* AND N. C. SACHETI**

Department of Mathematics, Indian Institute of Technology, Kanpur 208016

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The purpose of this article is to propose an analogy in slip flows for viscous incompressible fluid flow (i) through and around smooth solid boundaries and (ii) through and around naturally permeable boundaries allowing small permeabilities. It has been shown that the results in either cases could be directly taken over to the other one by the proper choice of few parameters.

INTRODUCTION

The existence of slip phenomenon at the boundaries and the interface has been observed in the flows of rarefied gases, hypersonic flows of chemically reacting binary mixture, etc. (Street 1963, Davis 1970).

Under certain approximated situations the researchers studied the slip phenomenon in the case of viscous incompressible fluid also. In this case, the use of slip boundary condition in preference to the no-slip condition was made due to the fact that walls allowed, the fluid particles to slip. It was obviously better and more realistic approach to obtain the flow behaviours.

Later on with the important paper of Beavers and Joseph (1967) giving the slip boundary condition at a naturally permeable plane boundary, the interest of the workers shifted towards the coupled flow problems in porous media dividing the whole flow field into two regions namely, (i) free fluid region (where Navier-Stokes equations hold) and (ii) the porous region (where Darcy's law holds) with continuity of normal velocity and pressure at the interface, slip condition for the tangential component of velocity in the free fluid region at the outer surface of the porous material and no-slip at the impermeable wall.

For small permeability, the boundary condition proposed by Beavers and Joseph (1967) was simplified by Saffman (1971) as

$$\frac{du}{dy} = \frac{\alpha}{\sqrt{k}}\, u, \qquad \ldots(1)$$

**Present address* : Department of Mathematics, University of West Indies, St. Augustine, Trinidad, West Indies.

***Present address* : Department of Mathematics, Monash University, Clayton, Victoria 3168, Australia.

where α is a constant depending only upon the properties of the porous material and not on its structure, k is the permeability parameter of the porous material. Of course, here, for the sake of simplicity, only single velocity component system for plane boundary has been considered.

As is well known in the case of solid smooth boundary (Lance and Rogers 1962), the corresponding slip condition is

$$\frac{du}{dy} = L'u \qquad \text{...(2)}$$

where

$$L' = \frac{f_1}{(2 - f_1)\,L} \qquad \text{...(3)}$$

L is the mean free path and f_1 is the Maxwell's relaxation coefficient.

Analysis

Now we establish the analogy. From (1) and (2) it may be easily noted that if L' is replaced by $\alpha/\sqrt{k}$ [in both the cases, the flow, however, is being governed by NS equations], all the results that hold good in the case of viscous incompressible fluid through and around solid boundaries may be directly taken over to the case of viscous incompressible fluid flow through and around corresponding naturally permeable boundaries. Of course in the later case, the usual simplification as necessitated due to small permeability approximation has to be effected. Similarly if we want to derive the result of first case from the second case, $\alpha/\sqrt{k}$ should be replaced by L' in the results before where the simplification of small permeability is applied.

It will be worthwhile to make following remarks :

(i) The results will also hold good for the curved surfaces,

(ii) the results still hold good when the bounding walls are in motion,

(iii) in case of unsteady flows, the above results will be true,

(iv) (i) – (iii) will hold good even if we take MHD effects into account.

The results of first case may also be derived from the results obtained from Beavers and Joseph (1967) slip condition by replacing $\alpha/\sqrt{k}$ by L' and neglecting the terms of the type (k/μ) [as in this simplification the Beavers and Joseph condition reduces to (2)].

As for example, the velocity distribution for Poiseuille flow over a naturally permeable bed is given as (Beavers and Joseph 1967)

$$u = u_B\left(1 + \frac{\alpha}{\sqrt{k}}\,y\right) + \frac{1}{2\mu}\,(y^2 + 2\alpha y\sqrt{k})\,\frac{dP}{dx} \qquad \text{...(4)}$$

where

$$u_B = -\frac{k}{2\mu}\left(\frac{\sigma^2 + 2\alpha\sigma}{1 + \alpha\sigma}\right)\frac{dP}{dx} \qquad \ldots(5)$$

with

$$\sigma = \frac{h}{\sqrt{k}} \qquad \ldots(6)$$

which on replacing $\alpha/\sqrt{k}$ by L' and neglecting the terms of $O(k/\mu)$ gives

$$u = -\frac{1}{2\mu}\left(\frac{h^2}{1 + L'h}\right)\frac{dP}{dx}(1 + L'y) + \frac{1}{2\mu}\frac{dP}{dx}y^2. \qquad \ldots(7)$$

It may easily be verified that (7) is the plane Poiseuille flow when there is a slip at the lower surface.

Similarly the drag expression as obtained for a porous sphere of radius a (in the case of a flow past a porous sphere) by Jones (1973),

$$D_r = \frac{6\pi\mu Ua(2 + a\beta)}{3 + a\beta + \frac{3k}{a^2} + \frac{3k\beta}{2a}} \qquad \ldots(8)$$

with

$$\beta = \frac{\alpha}{\sqrt{k}} \qquad \ldots(9)$$

on replacing β by L' and neglecting the terms of $O(k/\mu)$ gives the drag experienced by a solid smooth sphere in the case of a flow past it, as

$$D_r = \frac{6\pi\mu Ua(2 + aL')}{3 + aL'}. \qquad \ldots(10)$$

Conversely, replacing L' by $\alpha/\sqrt{k}$ in (10) and simplifying the result up to $O(\sqrt{k})$, we get

$$D_r = 6\pi\mu Ua\left(1 - \frac{\sqrt{k}}{a\alpha}\right) \qquad \ldots(11)$$

which is the drag experienced by a porous sphere when the permeability is small [in full agreement with Jones (1973), Verma and Bhatt (1975)].

The validity of the analogy in remaining situations will be discussed in detail in the forthcoming papers.

Concluding Remarks

In the present investigation an observation looking to the governing equations and boundary conditions in two types of flows as mentioned i.e. the flow through and around smooth solid boundaries and corresponding flow through and around naturally permeable boundaries, has been pointed out with the result we get similarity in these flows. Though the analogy is true in the restricted sense yet it provides further strength to the study of slip flows over smooth solid surfaces. Further, it will help us to define slip condition for temperature in the case of porous surfaces and we will obtain exactly the same condition as proposed by Rudraiah and Nagaraj (1977).

To sum up the results it may be emphasised that the analogy helps us in knowing the results of flows over naturally permeable walls up to $O(\sqrt{k})$ without solving the flow equations in porous media and simultaneously the flow picture of slip flows over smooth surfaces.

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