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LAMINAR NATURAL CONVECTION HEAT TRANSFER FROM A CURVED SURFACE TO A NON-NEWTONIAN PSEUDOPLASTIC LIQUID

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The problem deals with the natural convection heat transfer into a pseudoplastic power law liquid, initially cold and at rest, from an isothermal curved surface which is kept at constant temperature. By expressing the conservation equations in an explicit transient form and using a finite difference scheme, the solutions at each time-step have been found and then stepped forward in time until we reach at the steady state solutions. The scheme has been carried over an IBM 1130 electronic computer. The velocity and temperature fields, thus found, are tabulated. The velocity and temperature dependence on the distance from the immersed surface at a particular height above the surface are shown graphically.

NOMENCLATURE

C = specific heat capacity, Btu/lbm. deg F

g = acceleration due to gravity, ft/sec^2

K = thermal conductivity, Btu/sec. ft. deg F

k = consistency, lbm/sec^{2-n}. ft

n = flow index

P_r = Prandtl number, $\rho c/K \left(\frac{k}{\rho}\right)^{1/(2-n)} L^{2(n-1)/(n-2)}$

T = temperature, deg F

θ = dimensionless temperature, $(T = T_\infty)/(T_w - T_\infty)$

u, v = velocity components in the x and y directions, ft/sec

x = distance along the wall, ft

y = distance normal to the wall, ft

β = coefficient of thermal expansion, 1/deg F

ρ = density, lbm/ft^3

η = angle shown in Fig. 1.

Subscripts

w = wall, ∞ = infinity, m = mass.

1. Introduction

Laminar natural convection heat transfer between an isothermal object kept at constant temperature and a power law liquid was studied by Acrivos (1960) analytically. An asymptotic solution for large Prandtl number was obtained using similarity transformation. This approach does not seem to be applicable for other types of boundary conditions such as uniform flux or arbitrary plate temperature. Tien (1967) while studying natural convection heat transfer from a vertical plate at constant temperature to a power law liquid with large Prandtl number, gave a more generalized analytical method, viz. integral method, to extend the problem to the case where the heat flux of the plate was uniform and the case where the plate temperature was maintained arbitrarily. Other studies germane to this investigation include those of Reilly *et al.* (1965) and a few others.

Now-a-days a rapidly increasing interest is being given on finite difference method to solve a set of non-linear equations with appropriate boundary conditions numerically. In this study, we present a finite difference scheme to study the laminar natural convection heat transfer from an isothermal curved surface kept at constant temperature to a non-Newtonian pseudoplastic liquid. This problem for a vertical plate with a uniform wall heat flux was studied by Dale and Emery (1972) using same technique.

2. Governing Equations

An isothermal curved surface at constant temperature is immersed in an infinite expanse of a pseudoplastic power law liquid which is initially cold and at rest (Fig. 1). The liquid is assumed to have temperature dependent buoyancy effect. Under the

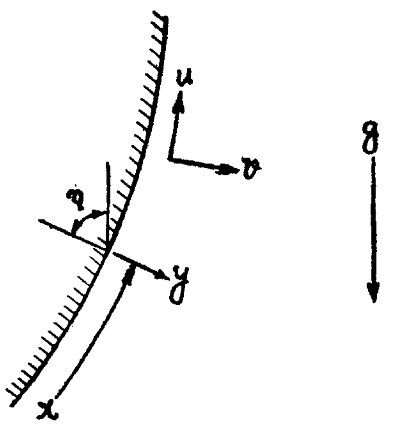

Fig. 1. The position directions of the coordinates x and y, the velocity components u and v, and the angle η.

assumptions of steady state and two dimensional boundary layer flow, the conservation equations can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots(1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty)\sin\eta + \frac{1}{\rho}\frac{\partial}{\partial y}\left(k\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) \qquad \ldots(2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho c}\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) \qquad \ldots(3)$$

with the boundary conditions

$$\left.\begin{aligned} u = v = 0,\ T = T_w \text{ at } y = 0 \\ u = 0,\ T = T_\infty \text{ at } y = \infty \end{aligned}\right\} \qquad \ldots(4)$$

where T_w = wall temperature and T_∞ = temperature at infinity.

The density and specific heat have been assumed to be constant except in the buoyancy term.

By expressing the conservation equations in an explicit transient form, the equations are made parabolic and the solutions can be stepped forward in time until a steady state is reached. To do so we have to ensure the consistency, convergency and stability of the difference scheme used. The system of equations we are dealing with is a properly posed initial value problem. The theorem due to Lax and Richtmyer (1956) states 'Given a properly posed initial value problem and a finite difference approximation to it that satisfies the consistency conditions, stability is the necessary and sufficient condition for convergence'.

We first introduce the following dimensionless variables and make the conservation equations dimensionless

$$x^* = \frac{x}{L},\ y^* = \frac{y}{L},\ u^* = \frac{u}{U},\ v^* = \frac{v}{U}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \text{ and } t^* = \frac{Ut}{L}$$

where L is a suitable length scale and U is a typical velocity, which are connected by the relations

$$\frac{g}{m} = \frac{U^n}{L^{n+1}},\ m = \frac{k}{\rho}$$

Dropping stars we obtain the transformed set of transient conservation equations as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad ...(1')$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = M\theta + \frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) \qquad ...(2')$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} \qquad ...(3')$$

with $M = \beta(T_w - T_\infty)\sin\eta$.

Hence, the approximate set of finite difference equations are

$$\frac{u'_{i,j} - u'_{i-1,j}}{\triangle x} + \frac{v'_{i,j} - v'_{i,j-1}}{\triangle y} = 0 \qquad ...(4')$$

$$\frac{u'_{i,j} - u_{i,j}}{\triangle t} + u_{i,j}\frac{u_{i,j} - u_{i-1,j}}{\triangle x} + v_{i,j}\frac{u_{i,j+1} - u_{i,j}}{\triangle y} = M\theta'_{i,j}$$

$$+ \left\{\left|\frac{u_{i,j+1} - u_{i,j}}{\triangle y}\right|^{n-1}\left(\frac{u_{i,j+1} - u_{i,j}}{\triangle y}\right) - \left|\frac{u_{i,j} - u_{i,j-1}}{\triangle y}\right|^{n-1}\left(\frac{u_{i,j} - u_{i,j-1}}{\triangle y}\right)\right\} \Big/ \triangle y \qquad ...(5')$$

$$\frac{\theta'_{i,j} - \theta_{i,j}}{\triangle t} + u_{i,j}\frac{\theta_{i,j} - \theta_{i-1,j}}{\triangle x} + v_{i,j}\frac{\theta_{i,j+1} - \theta_{i,j}}{\triangle y}$$

$$= \frac{1}{P_r}\left(\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\triangle y)^2}\right) \qquad ...(6')$$

where the primed variables indicate the values of the variables at a new time and (i, j) represent grid points.

3. Consistency of the Finite Difference Scheme

The term consistency applied to a finite difference procedure, means that the procedure may in fact approximate the solution of the partial differential equation under study and not the solution of any other partial differential equation. The consistency is measured in terms of the difference between a differential equation and a difference equation.

Here

$$\frac{\partial u}{\partial t} = \frac{u'_{i,j} - u_{i,j}}{\triangle t} + O(\triangle t)$$

$$\frac{\partial u}{\partial x} = \frac{u_{i,j} - u_{i-1,j}}{\triangle x} + O(\triangle x)$$

$$\frac{\partial u}{\partial y} = \frac{u_{i,j+1} - u_{i,j}}{\triangle y} + O(\triangle y)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{(\triangle y)^2} + O(\triangle y)^2$$

For consistency of (5′) we estimate

$$\Bigg[\frac{u'_{i,j} - u_{i,j}}{\triangle t} + u_{i,j}\frac{u_{i,j} - u_{i-1,j}}{\triangle x} + \dots\dots$$
$$- \left\{\left|\frac{u_{i,j+1} - u_{i,j}}{\triangle y}\right|^{n-1}\left(\frac{u_{i,j+1} - u_{i,j}}{\triangle y}\right) - \left|\frac{u_{i,j} - u_{i,j-1}}{\triangle y}\right|^{n-1}\left(\frac{u_{i,j} - u_{i,j-1}}{\triangle y}\right)\right\}\Big/\triangle y\Bigg]$$
$$- \left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \dots\dots - \frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right)\right]_{i,j}$$
$$= O(\triangle t) + u_{i,j}\,O(\triangle x) + v_{i,j}\,O(\triangle y) + O(\triangle y)^{n+1} \qquad \dots(7')$$

For (6′) we have

$$\left[\frac{\theta'_{i,j} - \theta_{i,j}}{\triangle t} + u_{i,j}\frac{\theta_{i,j} - \theta_{i-1,j}}{\triangle x} + \dots\dots - \frac{1}{P_r}\,\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\triangle y)^2}\right]$$
$$- \left[\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + \dots\dots - \frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2}\right]_{i,j}$$
$$= O(\triangle t) + u_{i,j}\,O(\triangle x) + v_{i,j}\,O(\triangle y) + \frac{1}{P_r}\,O(\triangle y)^2. \qquad \dots(8')$$

R.H.S. of (7′) and (8′) represent truncation error. As $\triangle t \to 0$ with $\triangle x$, $\triangle y \to 0$ the truncation error $\to 0$. Hence our explicit scheme is consistent.

4. Stability Condition of the Scheme

We present here a treatment for determining the stability conditions of the finite difference scheme based on O'Brien *et al.* (1951). As an explicit procedure is used, we intent to investigate the largest time-step consistent with stability. As t does not occur in the equation of continuity we consider only the momentum and energy conservation equations. The general terms of the Fourier expansions at $t = 0$ for u and θ are both $e^{iax}e^{iby}$, $(i = \sqrt{-1})$. After a time t these terms become

$$u : \psi(t)\, e^{ipx}e^{iqy}$$

$$\theta : \phi(t)\, e^{ipx}e^{iqy}.$$

Substituting in (5′) and (6′) and regarding the coefficients u and v as constants over any one time-step and denoting the values of ψ and ϕ after the time-step by ψ' and ϕ' we obtain after a little simplification

$$\frac{\psi' - \psi}{\Delta t} + u\,\frac{\psi(1 - e^{-ip\Delta x}}{\Delta x} + v\,\frac{\psi(e^{iq\Delta y} - 1)}{\Delta y}$$

$$= M\phi' + \psi\left\{\left|\frac{u_{i,j+1} - u_{i,j}}{\Delta y}\right|^{n-1}\frac{(e^{iq\Delta y} - 1)}{\Delta y} - \left|\frac{u_{i,j} - u_{i,j-1}}{\Delta y}\right|^{n-1}\frac{(1 - e^{-iq\Delta y})}{\Delta y}\right\}\Big/\Delta y \qquad \ldots(9')$$

when $\left|\frac{u_{i,j+1} - u_{i,j}}{\Delta y}\right|^{n-1}$ and $\left|\frac{u_{i,j} - u_{i,j-1}}{\Delta y}\right|^{n-1}$

are considered to be constant over any one time-step for the values of n, $(0 < n < 1)$, very close to unity. This also indicates that the liquid is a pseudoplastic power law liquid. Also we have

$$\frac{\phi' - \phi}{\Delta t} + u\,\frac{\phi(1 - e^{-ip\Delta x})}{\Delta x} + v\,\frac{\phi(e^{iq\Delta y} - 1)}{\Delta y}$$

$$= \frac{2\phi}{P_r}\,\frac{(\cos q\,\Delta\,y - 1)}{(\Delta\,y)^2}. \qquad \ldots(10')$$

Now let

$$A = 1 - \frac{u\,\Delta\,t}{\Delta\,x}(1 - e^{-ip\Delta x}) - \frac{v\,\Delta\,t}{\Delta\,y}(e^{iq\Delta y} - 1)$$

$$+ \frac{\Delta\,t}{(\Delta\,y)^2}\left\{\left|\frac{u_{i,j+1} - u_{i,j}}{\Delta y}\right|^{n-1}(e^{iq\Delta y} - 1) - \left|\frac{u_{i,j} - u_{i,j-1}}{\Delta y}\right|^{n-1}(1 - e^{-iq\Delta y})\right\}$$

$$B = 1 - \frac{u\,\Delta\,t}{\Delta\,x}(1 - e^{-ip\Delta x}) - \frac{v\,\Delta\,t}{\Delta\,y}(e^{iq\Delta y} - 1)$$

$$+ \frac{2\,\Delta\,t}{P_r(\Delta\,y)^2}(\cos q\,\Delta\,y - 1).$$

Hence eqns. (9′) and (10′) become

$$\psi' = A\psi + MB\,\Delta\,t\phi$$

$$\phi' = B\phi$$

i.e. $$\begin{bmatrix}\psi' \\ \phi'\end{bmatrix} = \begin{bmatrix}A & MB\,\Delta\,t \\ 0 & B\end{bmatrix}\begin{bmatrix}\psi \\ \phi\end{bmatrix}, \quad \therefore \quad \zeta' = D\zeta$$

where $\zeta' = \begin{bmatrix}\psi' \\ \phi'\end{bmatrix}$, $D = \begin{bmatrix}A & MB\Delta t \\ 0 & B\end{bmatrix}$ and $\zeta = \begin{bmatrix}\psi \\ \phi\end{bmatrix}$.

For stability the moduli of each of the eigen values λ_1 and λ_2 of the amplification matrix D should be less or equal to unity. Here $\lambda_1 = A$, $\lambda_2 = B$. Hence the stability conditions are

$$|\,A\,| \leqslant 1 \text{ and } |\,B\,| \leqslant 1 \text{ for all } p \text{ and } q.$$

As the liquid in contact with the immersed surface becomes heated and varied, it goes up in the x-direction. Naturally liquid from positive y-direction flows towards the surface. Hence we may assume u to be everywhere non-negative and v non-positive. Let us substitute

$$\alpha = \frac{u \triangle t}{\triangle x}, \ \gamma = \frac{| v | \triangle t}{\triangle y}, \ \delta = \frac{\triangle t}{(\triangle y)^2}$$

$$C_1 = \left| \frac{u_{i,j+1} - u_{i,j}}{\triangle y} \right|^{n-1} \text{ and } C_2 = \left| \frac{u_{i,j} - u_{i,j-1}}{\triangle y} \right|^{n-1}$$

$$\therefore \quad A = 1 - \alpha(1 - e^{-ip\triangle x}) + \gamma(e^{iq\triangle y} - 1) + \delta\,[C_1(e^{iq\triangle y} - 1) - C_2(1 - e^{-iq\triangle y})]$$

$$= 1 - \alpha - \gamma - C_1\delta - C_2\delta + \alpha e^{-ip\triangle x} + \gamma e^{iq\triangle y} + C_1\delta e^{iq\triangle y} + C_2\delta e^{-iq\triangle y}$$

α, γ, δ are all real and non-negative. Representing A on an Argand diagram, the maximum value of $| A |$ occurs when $p \triangle x = r\pi$, $q \triangle y = s\pi$ where r and s are $+ve$ integers. Here $\triangle t$ sufficiently large $| A |$ is maximum when r and s are odd integers, in which case we have

$$A = (1 - \alpha - \gamma - C_1\delta - C_2\delta) - \alpha - \gamma - C_1\delta - C_2\delta.$$

Now to satisfy $| \lambda_1 | \leqslant 1$ the most negative allowable value is $A = -1$. Hence the stability condition can be written as

$$-1 \leqslant 1 - 2(\alpha + \gamma + C_1\delta + C_2\delta)$$

or $$\alpha + \gamma + C_1\delta + C_2\delta \leqslant 1$$

i.e. $$\frac{u \triangle t}{\triangle x} + \frac{| v | \triangle t}{\triangle y} + \frac{\triangle t}{(\triangle y)^2}\left\{\left|\frac{u_{i,j+1} - u_{i,j}}{\triangle y}\right|^{n-1} + \left|\frac{u_{i,j} - u_{i,j-1}}{\triangle y}\right|^{n-1}\right\} \leqslant 1. \quad \text{...(11')}$$

Similarly the condition $| \lambda_2 | \leqslant 1$ gives

$$\frac{u \triangle t}{\triangle x} + \frac{| v | \triangle t}{\triangle y} + \frac{1}{P_r}\frac{2 \triangle t}{(\triangle y)^2} \leqslant 1. \quad \text{...(12')}$$

Hence combining (11') and (12') we obtain the condition of stability as

$$\frac{u \triangle t}{\triangle x} + \frac{| v | \triangle t}{\triangle y} + \frac{\triangle t}{(\triangle y)^2} \times \left\{\left|\frac{u_{i,j+1} - u_{i,j}}{\triangle y}\right|^{n-1} + \left|\frac{u_{i,j} - u_{i,j-1}}{\triangle y}\right|^{n-1} + \frac{2}{P_r}\right\} \leqslant 1. \quad \text{...(13')}$$

5. Method of Solution

The choice of mesh lengths, time-step and parameters becomes convenient when the finite difference scheme is considered in terms of variables with proper dimensions and hence the difference equations (4′) - (6′) and the stability condition (13′) are written as

$$\frac{u'_{i,j} - u'_{i-1,j}}{\Delta x} + \frac{v'_{i,j} - v'_{i,j-1}}{\Delta y} = 0 \qquad \ldots(5)$$

$$\frac{u'_{i,j} - u_{i,j}}{\Delta t} + u_{i,j}\frac{u_{i,j} - u_{i-1,j}}{\Delta x} + v_{i,j}\frac{u_{i,j+1} - u_{i,j}}{\Delta y} = g\beta \sin\eta(T_w - T_\infty)\theta'_{i,j}$$
$$+ \frac{k}{\rho \Delta y}\left\{\left|\frac{u_{i,j+1} - u_{i,j}}{\Delta y}\right|^{n-1}\left(\frac{u_{i,j+1} - u_{i,j}}{\Delta y}\right)\right.$$
$$\left. - \left|\frac{u_{i,j} - u_{i,j-1}}{\Delta y}\right|^{n-1}\left(\frac{u_{i,j} - u_{i,j-1}}{\Delta y}\right)\right\} \qquad \ldots(6)$$

$$\frac{\theta'_{i,j} - \theta_{i,j}}{\Delta t} + u_{i,j}\frac{\theta_{i,j} - \theta_{i-1,j}}{\Delta x} + v_{i,j}\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta y}$$
$$= \frac{K}{\rho c}\left(\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta y)^2}\right) \qquad \ldots(7)$$

and

$$\frac{u \Delta t}{\Delta x} + \frac{|v| \Delta t}{\Delta y} + \frac{\Delta t}{(\Delta y)^2}\left\{\frac{2K}{\rho c} + \frac{k}{\rho}\left(\left|\frac{u_{i,j+1} - u_{i,j}}{\Delta y}\right|^{n-1}\right.\right.$$
$$\left.\left. + \left|\frac{u_{i,j} - u_{i,j-1}}{\Delta y}\right|^{n-1}\right)\right\} \leqslant 1. \qquad \ldots(8)$$

The numerical calculations were performed by taking 12 × 15 meshes over the region, $x = 1$ ft and $y = 0.44$ ft. The mesh lengths were not all equal and were defined by

Δx : 0.032, 0.032, 0.032, 0.032, 0.040, 0.056, 0.064, 0.064, 0.096, 0.160, 0.194 and 0.198 ft

Δy : 0.008, 0.008, 0.008, 0.008, 0.01, 0.014, 0.016, 0.016, 0.024, 0.048, 0.056, 0.056, 0.056, 0.056 and 0.056 ft

The time-steps were uniform and given by $\Delta t = 0.15$ sec., while the parameters assumed the following values :

$n = 0.888$, $g = 32.0$ ft/sec², $\beta = 0.00017$/°F,

$T_w - T_\infty = 7.0$°F, $\sin\eta = 0.9659$,

$k = 0.006316$ lb mass/sec^{2-n} ft, $\rho = 62.5$ lb mass/ft³,

$c = 1.0$ Btu/lb mass°F, $K = 0.0001$ Btu/sec. ft² °F/ft.

An examination of the print out indicates that there has been hardly any change, not more than 0.0001 in 10 time-steps, in the variables u, v and θ when $t = 61.5$ secs. It has also been found that the temperature and velocity increase with the passage of time and reach maximum values and then decrease slightly to attain steady state values. Here the steady state solutions are found after 61.5 secs.

6. Discussion

The values of the parameters supplied in the calculation refer to an aqueous solution of 0.05% carboxymethylcellulose (CMC) at 90°F. The velocity and temperature fields are given in Tables I and II, while in Figs. 2 and 3 are shown the degree of dependence of the velocity and temperature profiles on the distance from the surface. The running time taken by the computer is about 115 minutes. The rate of heat transfer hence poses no problem after the determination of the velocity and temperature fields.

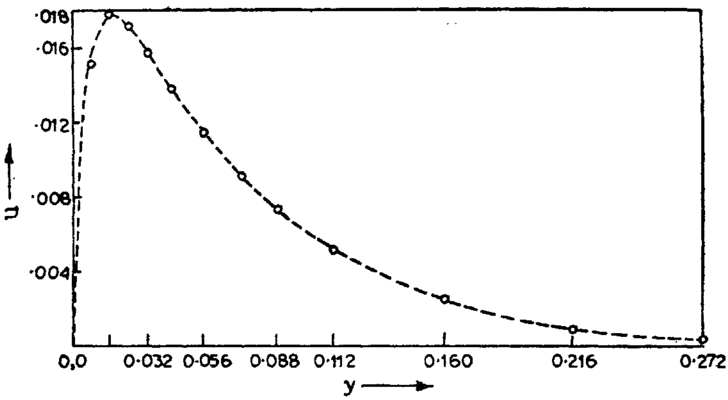

Fig. 2. The x-component of velocity (u) at $x = 0.802$ ft.

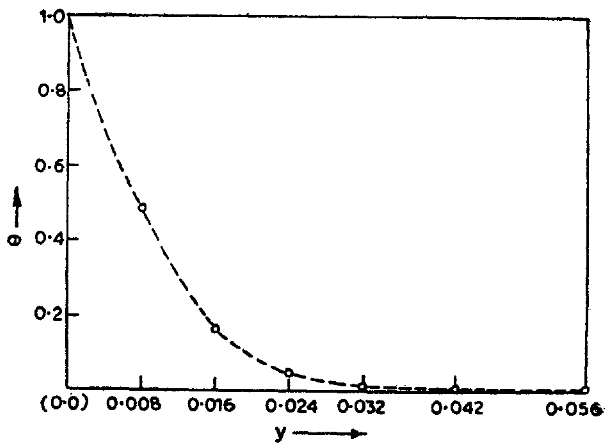

Fig. 3. The temperature profile (θ) at $x = 0.802$ ft.

TABLE I

Values of u at t = 61.5 secs.

x \ y	0.0	0.008	0.016	0.024	0.032	0.042	0.056	0.072	0.088	0.112	0.160	0.216	0.272	0.328	0.384	0.440
0.032	0.0000	0.0023	0.0021	0.0018	0.0015	0.0012	0.0009	0.0006	0.0005	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.064	0.0000	0.0038	0.0037	0.0032	0.0027	0.0022	0.0017	0.0012	0.0010	0.0006	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
0.096	0.0000	0.0050	0.0049	0.0043	0.0037	0.0031	0.0024	0.0018	0.0014	0.0010	0.0004	0.0002	0.0000	0.0000	0.0000	0.0000
0 128	0.0000	0.0060	0.0060	0.0053	0.0046	0.0039	0.0031	0.0024	0.0018	0.0013	0.0006	0.0002	0.0001	0.0000	0.0000	0.0000
0.168	0.0000	0.0070	0.0072	0.0064	0.0057	0.0048	0.0038	0.0030	0.0023	0.0017	0.0008	0.0003	0.0001	0.0000	0.0000	0.0000
0.224	0.0000	0.0082	0.0086	0.0078	0.0069	0.0059	0.0048	0.0037	0.0030	0.0021	0.0010	0.0004	0.0001	0.0000	0.0000	0.0000
0.288	0.0000	0.0093	0.0100	0.0092	0.0082	0.0071	0.0057	0.0045	0.0036	0.0026	0.0012	0.0005	0.0002	0.0000	0.0000	0.0000
0.352	0.0000	0.0103	0.0112	0.0105	0.0094	0.0081	0.0066	0.0053	0.0042	0.0030	0.0015	0.0006	0.0002	0.0000	0.0000	0.0000
0.448	0.0000	0.0116	0.0129	0.0121	0.0110	0.0096	0.0079	0.0063	0.0050	0.0036	0.0017	0.0007	0.0002	0.0001	0.0000	0.0000
0.608	0.0000	0.0133	0.0153	0.0146	0.0133	0.0116	0.0096	0.0077	0.0062	0.0044	0.0021	0.0008	0.0003	0.0001	0.0000	0.0000
0.802	0.0000	0.0151	0.0177	0.0171	0.0157	0.0138	0.0114	0.0091	0.0073	0.0052	0.0024	0.0009	0.0003	0.0001	0.0000	0.0000
1.000	0.0000	0.0166	0.0198	0.0193	0.0178	0.0156	0.0129	0.0103	0.0082	0.0058	0.0027	0.0010	0·0003	0.0001	0.0000	0.0000

x, *y* and *u* carry aforesaid units.

TABLE II

Values of θ at $t = 61.5$ secs.

x \ y	0.0	0.008	0.016	0.024	0.032	0.042	0.056	0.072	0.088	0.112	0.160	0.216	0.272	0.328	0.384	0·440
0.032	1.0000	0.1337	0.0137	0.0012	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.064	1.0000	0.1967	0.0260	0.0028	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.096	1.0000	0.2386	0.0372	0.0046	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0..128	1.0000	0.2700	0.0474	0.0066	0.0008	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.168	1.0000	0.3003	0.0590	0.0091	0.0013	0.0001	0.0000	0.0000	0.0000	0.0000	0 0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.224	1.0000	0.3327	0.0733	0.0127	0.0020	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.288	1.0000	0.3618	0.0878	0.0168	0.0029	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.352	1.0000	0.3856	0.1010	0.0208	0.0039	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.448	1.0000	0.4138	0.1184	0.0268	0.0055	0.0008	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.608	1.0000	0.4489	0.1432	0.0364	0.0083	0.0013	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.802	1.0000	0.4812	0.1689	0.0476	0.0119	0.0020	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.000	1.0000	0.5068	0.1917	0.0585	0.0155	0.0028	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

θ is dimensionless and x, y carry aforesaid units.

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