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BEHAVIOUR OF VISCOELASTIC LUBRICANTS IN REFERENCE TO HUMAN JOINTS

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Analysis of "boosted lubrication" in between two approaching solids one of which is porous has been presented in reference to normally loaded living human joints. Viscoelastic fluid has been considered to represent the synovial fluid in the fluid film region in between the approaching surfaces. The pore size of the porous material is small so that only the suspending medium of the lubricant enters into the porous matrix and thus the concentration of the suspended particles increases in the fluid film region. The problem has been analyzed in the two regions separately and it has been observed that load capacity increases when the gap decreases and the concentration of the suspended particles increases in the fluid film region in accordance to results of Dowson *et al.* (1970).

INTRODUCTION

The load bearing human joint is a self acting dynamically loaded bearing which employs a porous pad and elastic bearing material (articular cartilage) and a highly non-Newtonian lubricant (synovial fluid). Dowson (1966-67) showed that a squeeze film action is capable of providing considerable protection to the cartilage surfaces, once a fluid film is generated. Furthermore, Fein (1966-67) discussed the possibility of increasing concentration of hyaluronic acid in synovial fluid during the squeeze film action due to the filtration into the porous cartilage. Walker *et al.* (1968) introduced the concept of "boosted lubrication" as an important feature of the joint behaviour. A mathematical analysis of the concept of boosted lubrication of human joints has been presented by Dowson *et al.* (1970). The predictions of their analysis were in close agreement with the experimental findings of Edwards (1966-67). The increased concentration of hyaluronic acid increases the viscosity of synovial fluid in accordance to Negami's (1964) findings.

Dowson *et al.* (1970) also suggested that best squeeze film models may be given by parallel surfaces. Mc-Cutchen (1959) and Maroudas (1966-67) have discussed the non-Newtonian nature of synovial fluid. Thus, the fluid under consideration must include the salient properties of synovial fluid (Mow 1968). The viscoelastic fluids exhibit non-Newtonian flow properties observed in the flow of suspended particles in viscous fluid as considered by Oldroyed (1958) and Mow and Lau (1974). Tanner (1965) showed that the movement of the suspending medium of the synovial fluid into and out of the cartilage contribute considerably to its damped elasticity or visco-elasticity. Two dimensional squeeze film lubrication with viscoelastic fluid as the lubricant in between the two approaching surfaces, one of which is covered with the porous material has been considered in this paper. The pore size of the porous material has been considered to be so small that only the suspending fluid (i.e. viscous fluid) enters into the porous matrix. Although the model used here is very much simpler than the real mechanism of living human joint, but it includes the effect of porosity of the cartilage.

The problem has been analyzed in the two regions i.e. flow of viscous fluid into the porous matrix and the squeeze film lubrication in between two approaching surfaces with viscoelastic lubricant (Fig. 1). Exact solution not being possible, the perturbation method has been used to obtain approximate solution.

GOVERNING EQUATION AND SOLUTION

Consider the two dimensional flow problem in the two regions with (u, v), and $(\bar{u}, \bar{v})$ as the velocity components and p and $\bar{p}$ the pressures in the fluid film and porous regions respectively.

Following Darcy's law, we have

$$\left.\begin{aligned} \bar{u} &= \frac{-k}{\eta_0}\frac{\partial \bar{p}}{\partial x} \\ \bar{v} &= \frac{-k}{\eta_0}\frac{\partial \bar{p}}{\partial y}. \end{aligned}\right\} \qquad \ldots(1)$$

Thus, the pressure $\bar{p}$ satisfies the Laplace equation

$$\frac{\partial^2 \bar{p}}{\partial x^2} + \frac{\partial^2 \bar{p}}{\partial y^2} = 0. \qquad \ldots(2)$$

Assuming that the pressure gradient normal to film is negligible and following Mow (1974), the governing equation in the fluid film region is given by

$$\frac{dp}{dx} = \frac{\partial}{\partial y}(\tau_{xy}) \qquad \ldots(3)$$

where

$$\tau_{xy} = \eta\left[\frac{\partial u}{\partial y} F\left(\frac{\partial u}{\partial y}\right)\right] \qquad \ldots(4)$$

$$F\left(\frac{\partial u}{\partial y}\right) = \frac{c^2 + \alpha\beta\Lambda^2\left(\frac{\partial u}{\partial y}\right)^2}{c^2 + \beta\Lambda^2\left(\frac{\partial u}{\partial y}\right)^2} \qquad \ldots(5)$$

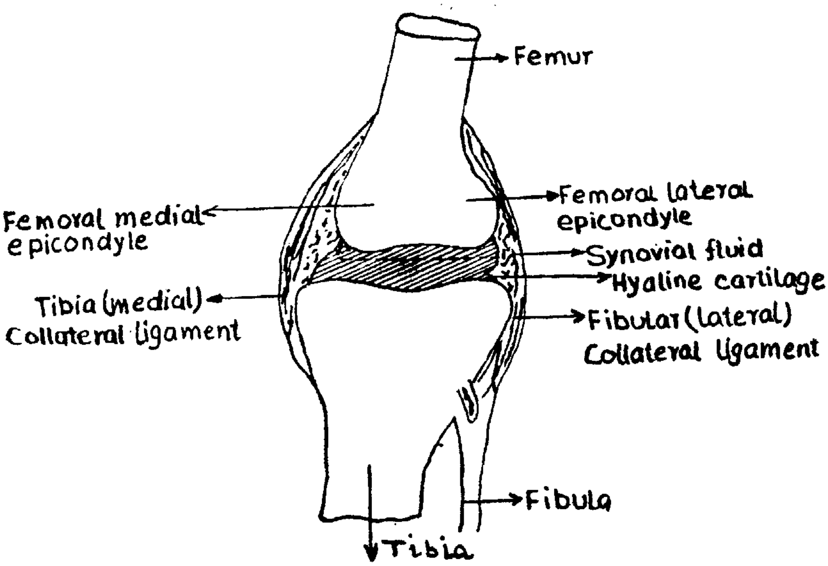

FIG. 1(a). Model for knee joint.

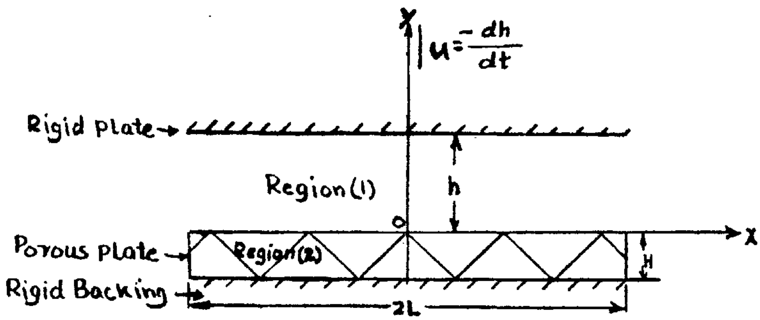

FIG. 1(b). Geometrical representation of knee joint.

$$\beta\Lambda^2\left(\frac{\partial u}{\partial y}\right)^2 \leqslant c^2 \qquad \ldots(6)$$

$$\Lambda = \frac{\lambda_1 V}{\xi},\ \alpha = \frac{\lambda_2}{\lambda_1},\ \beta = \frac{\mu}{\lambda_1},\ \mu_0 = \frac{\mu V^2}{\xi^2 c^2} \qquad \ldots(7)$$

V, ξ being constants with dimensions of velocity and length respectively, and μ_0, λ_1, λ_2 being constants with the dimensions of time, $\eta = \eta_0(1 + 2.5\phi)$,

where $\phi = \dfrac{\text{Volume occupied by particles}}{\text{Total volume of suspension}}$,

η is the viscosity of suspension and η_0 the viscosity of fluid entering porous region.

BOUNDARY CONDITIONS

$$\left.\begin{aligned} \bar{p} &= 0 \quad \text{at } x = \pm L \\ \frac{\partial \bar{p}}{\partial y} &= 0 \text{ at } y = -H \end{aligned}\right\} \qquad \ldots(8)$$

$$\left.\begin{aligned} u &= 0 \quad \text{at } y = 0, h \\ v &= \frac{-k}{\eta_0} \frac{\partial \bar{p}}{\partial y} \text{ at } y = 0 \text{ and } p = 0 \text{ at } x = \pm L \\ v &= \frac{dh}{dt} \text{ at } y = h. \end{aligned}\right\} \qquad \ldots(9)$$

Solution of eqn. (2) under boundary conditions (8) may be written in the form

$$\bar{p} = \sum_{n=1}^{\infty} An \{e^{n\pi y/L} + e^{(-2n\pi H/L - n\pi y/L)}\} \sin \frac{n\pi x}{L}. \qquad \ldots(10)$$

Combining eqns. (3) to (7) we have

$$\frac{dp}{dx} = \eta \frac{\partial^2 u}{\partial y^2} \left[1 - 3(\lambda_1 - \lambda_2) \mu_0 \left(\frac{\partial u}{\partial y}\right)^2\right]. \qquad \ldots(11)$$

Solution of eqn. (11) may be written as

$$\frac{dp}{dx} y + c(x, t) = \eta \frac{\partial u}{\partial y} \left[1 - \mu_0(\lambda_1 - \lambda_2) \left(\frac{\partial u}{\partial y}\right)^2\right]. \qquad \ldots(12)$$

In order to proceed further we represent all the variables in the form of sequences of functions in terms of small parameter

$$\epsilon = \frac{(\lambda_1 - \lambda_2) \mu_0}{T^2} \text{ as given below :}$$

$$f = f_0 + \epsilon f_1 + \epsilon^2 f_2 + \ldots \qquad \ldots(13)$$

T being some characteristic time. Introducing such sequences for all the variables and collecting the terms of zeroeth and first order in ϵ we obtain the following system of differential equations

$$\frac{dp_0}{dx} y + c_0 = \eta \frac{\partial u_0}{\partial y} \qquad \ldots(14)$$

$$\frac{1}{\eta} \frac{dp_1}{dx} y + c_1 = \left[\frac{\partial u_1}{\partial y} - T^2 \left(\frac{\partial u_0}{\partial y}\right)^3\right]. \qquad \ldots(15)$$

Transformed boundary conditions are

$$\left.\begin{aligned} & p_0 = p_1 = \ldots = 0 \text{ at } x = \pm L \\ & u_0 = u_1 = u_2 = \ldots = 0 \text{ at } y = 0,\ y = h \\ & v_0 = \frac{-k}{\eta_0} \left.\frac{\partial \bar{p}}{\partial y}\right|_{y=0} \text{ and } v_1 = v_2 = \ldots = 0 \text{ at } y = 0,\ y = h \\ & v_0 = \frac{dh}{dt} \text{ at } y = h. \end{aligned}\right\} \qquad \ldots(16)$$

Equation of continuity also gives

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0 \text{ and } \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0. \qquad \ldots(17)$$

Solution of the problem of eqns. (14) to (17) may be put in the form

$$u = u_0 + \epsilon u_1 \qquad \ldots(18)$$

$$p = p_0 + \epsilon p_1 \qquad \ldots(19)$$

where

$$u_0 = \frac{1}{2\eta} \frac{dp_0}{dx} (y - h) y \qquad \ldots(20)$$

$$u_1 = \frac{1}{2\eta} \frac{dp_1}{dx} (y^2 - hy) - \frac{216T^2}{h^9} \left[\frac{k}{\eta_0} \sum A_n (1 - e^{-2n\pi H/L}) \cos \frac{n\pi x}{L} - x \frac{dh}{dt} \right]^3 \left[\frac{(2y - h)^4}{8} - \frac{h^4}{8} \right] \qquad \ldots(21)$$

where

$$A_n = \frac{\eta_0 L \left(\frac{dh}{dt}\right) [2(-1)^n - 2 - n^2\pi^2]\, e^{2n\pi H/L}}{k(n\pi)^2 [(1 + nM)\, e^{2n\pi H/L} + (nM - 1)]} \qquad \ldots(22)$$

$$p_0 = \frac{-12\eta_0 L^2}{h^3} \left(\frac{dh}{dt}\right) \times \left[\frac{(2(-1)^n - 2 - n^2\pi^2)\,(e^{2n\pi H/L} - 1) \sin \frac{n\pi x}{L}}{(n\pi)^3 \{(nM + 1)\, e^{2n\pi H/L} + (nM - 1)\}} \right] + \tfrac{1}{2}\left(1 - \frac{x^2}{L^2}\right) \qquad \ldots(23)$$

$$p_1 = \frac{259.2\eta_0 T^2 L^4}{h^7} \left(\frac{dh}{dt}\right)^3 \times \left[\frac{3\, \Sigma \{2(-1)^n - 2 - n^2\pi^2\}\, (e^{2n\pi H/L} - 1)}{(n\pi)^5 \{(nM + 1)\, e^{2n\pi H/L} + (nM - 1)\}} \right] \times$$

(*equation continued on p.* 48)

$$\times \left[\left(\frac{n^2\pi^2x^2}{L^2} - 2\right)\sin\frac{n\pi x}{L} + \frac{2n\pi x}{L}\left(\cos\frac{n\pi x}{L} + (-1)^{n+1}\right)\right]$$

$$+ \tfrac{1}{4}\left(1 - \frac{x^4}{L^4}\right) \qquad \ldots(24)$$

where

$$M = \frac{\pi h^3}{12kL(1 + 2.5\phi)}. \qquad \ldots(25)$$

The normalized load-carrying capacity is given by

$$\bar{W} = \frac{\bar{W}_0 + \epsilon\bar{W}_1}{\bar{W}_0} \qquad \ldots(26)$$

where

$$\bar{W}_0 = 2\int_0^1 \bar{p}_0 d\bar{x} = -2\left[\tfrac{1}{3} + \tfrac{1}{2}\frac{(4 + \pi^2)(e^{\lambda} - 1)}{(M + 1)e^{\lambda} + (M - 1)}\right.$$

$$+ \frac{(4 + 3^2\pi^2)(e^{3\lambda} - 1)}{3^4\{(3M + 1)e^{3\lambda} + (3M - 1)\}}$$

$$\left. + \frac{(5^2\pi^2 + 4)(e^{5\lambda} - 1)}{5^4\{(5M + 1)e^{5\lambda} + (5M - 1)\}} + \ldots\right] \qquad \ldots(27)$$

$$\bar{W}_1 = 2\int_0^1 \bar{p}_1 d\bar{x} = \tfrac{2}{5} - \frac{12}{\pi^6}\left[\frac{(\pi^4 - 16)(e^{\lambda} - 1)}{\{(M + 1)e^{\lambda} + (M - 1)\}}\right.$$

$$- \frac{2^4\pi^4(e^{2\lambda} - 1)}{2^6\{(2M + 1)e^{2\lambda} + (2M - 1)\}} + \frac{(3^4\pi^4 - 16)(e^{3\lambda} - 1)}{3^6\{(3M + 1)e^{3\lambda} + (3M - 1)\}}$$

$$\left. - \frac{4^4\pi^4(e^{4\lambda} - 1)}{4^6\{(4M + 1)e^{4\lambda} + (4M - 1)\}} + \frac{(5^4\pi^4 - 16)(e^{5\lambda} - 1)}{5^6\{(5M + 1)e^{5\lambda} + (5M - 1)\}}\right] \qquad \ldots(28)$$

where $\frac{2\pi H}{L} = \lambda.$

RESULTS AND DISCUSSION

On the basis of the analysis presented above, following conclusions have been made.

(1) It is apparent from Fig. 2 that normalized load-carrying capacity $\bar{W}$ increases when the parameter $M \left(= \frac{\pi h^3}{12kL(1 + 2.5\phi)}\right)$ decreases. It is also obseved that M

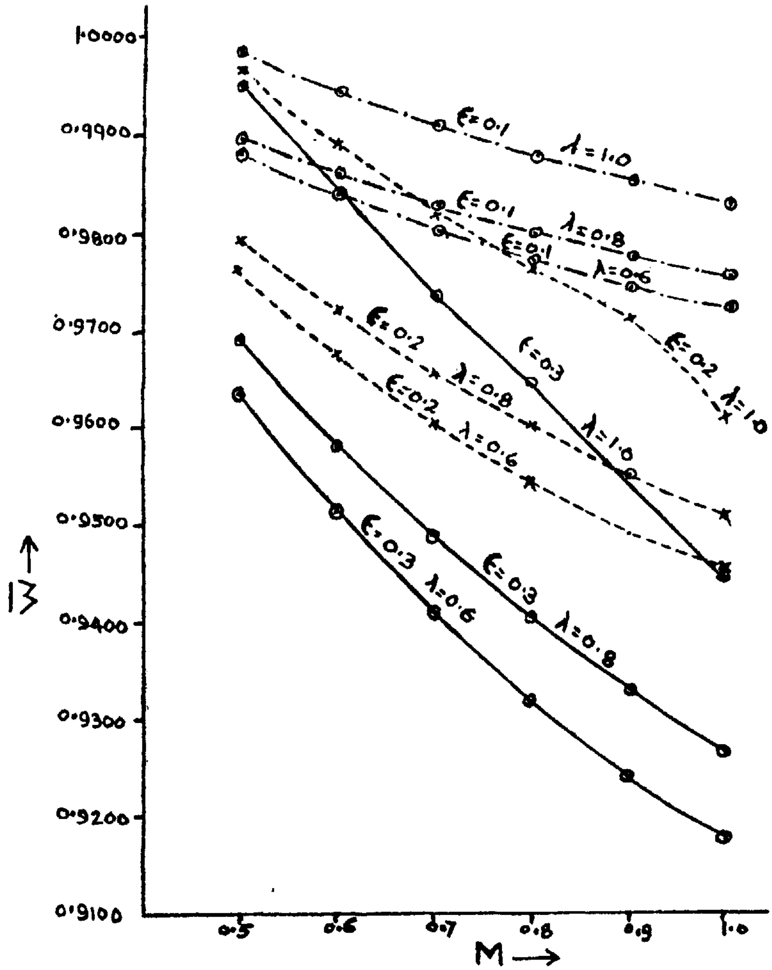

FIG. 2. Variation of load-carrying capacity $\bar{W}$ with bearing characteristic M for different values of λ and ϵ.

decreases when h, the gap between the approaching surfaces, decreases, or k the porosity/ϕ the concentration of suspension increases. The load-carrying capacity increases when the concentration of the suspending particles increases or the gap decreases.

(2) It is also clear from Fig. 2 that for a particular set of values of the parameters ϵ and M, the load-carrying capacity $\bar{W}$ increases when λ $(= 2\pi H/L)$ increases. The parameter λ increases when H, the thickness of the porous material increases. Hence load-carrying capacity increases when the porous pad thickness increases.

(3) Figure 2 also shows that load-carrying capacity $\bar{W}$ decreases when ϵ, the viscoelastic parameter increases.

The results (1) and (2) are in agreement with the proposals made by Dowson *et al.* (1970). As has been recently said that the synovial fluid is viscoelastic and non-linear, the results presented above are in good agreement with the experimental observations made by Dowson *et al.* (1970) and Negami (1964).

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