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EXTENDED JACOBI POLYNOMIALS AND GROUP REPRESENTATIONS—II

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In this paper, the author has obtained a generating function for the matrix elements $P^l_{mn}(x/b)$, of the representation $T_l(g)$, which subsumes the generating function of extended Jacobi polynomials and thereby of Jacobi, Hermite and Bessel polynomials. Integral representations of matrix elements $P^l_{mn}(x/b)$ and thereby of extended Jacobi polynomials have also been obtained of which Schlaeflies integral is a particular case.

§1. The author (Chiney 1978) has derived the Rodrigue's formula (1.1) for the matrix elements, $P^l_{mn}(x/b)$, of the representation $T_l(g)$, in the space H_l, where $g \in SU(2) \subset SL(2, C)$ and H_l denote the space of all polynomials of degree $2l$ (l being an integer or semi-integer)

$$P^l_{mn}\left(\frac{x}{b}\right) = \sqrt{\frac{(l+m)!}{(l-n)!\,(l+n)!\,(l-m)!}}\;\frac{(-1)^{l-m}(b-x)^{(n-m)/2}}{(2b)^l(b+x)^{(n+m)/2}} \times \frac{d^{l-m}}{dx^{l-m}}[(b-x)^{l-n}(b+x)^{l+n}]. \quad \ldots(1.1)$$

In this paper we derive a generating function for the matrix elements $P^l_{mn}(x/b)$ following Fujiwara (1966) which is similar to one found in Szegö (1959). And due to the relationship of these matrix elements with extended Jacobi polynomials [established by the author (Chiney 1978)], we get the generating function for the extended Jacobi polynomials. The generating functions of Jacobi polynomial, Hermite polynomial and Bessel polynomial can be obtained from it.

§2. *Results required* — Fujiwara (1966) defined the polynomials by generalized Rodrigue's formula (2.1) on the interval (a, b). We call these polynomials as extended Jocobi polynomials and denote them by $F_n(\alpha, \beta; x)$.

$$F_n(\alpha, \beta; x) = \frac{1}{K_n W(x)} \frac{d^n}{dx^n}[W(x)\,\{X(x)\}^n] \quad \ldots(2.1)$$

where $K_n = (-1)^n\, n!$

$X(x) = c(x-a)(b-x); \qquad c > 0$

and the weight function $W(x)$ given by

$$W(x) = \frac{(x-a)^{\alpha}(b-x)^{\beta}}{(b-a)^{\alpha+\beta+1} B(\alpha+1, \beta+1)}; \alpha > -1, \beta > -1.$$

Fujiwara (1966) has proved that when $a = -1$, $b = 1$, $\lambda = 1$ we get

$$F_n(\alpha, \beta; x) = P_n^{(\beta,\alpha)}(x) \qquad \ldots(2.2)$$

when $\alpha = \beta$, $b = -a = \sqrt{\alpha}\,(\alpha > 0)$ and in view of $\lambda \to 2/\sqrt{\alpha}$ we get

$$\lim_{\sqrt{\alpha} \to \infty} F_n(\alpha, \beta; x) = \frac{H_n(x)}{n!}, \text{ when } b = -a = 1 = \lambda. \qquad \ldots(2.3)$$

Thakare (1972) has shown that

$$\lim_{\epsilon \to \infty} \frac{\Gamma(n+1)}{\epsilon^n} F_n\left(r - \epsilon - 1, \epsilon - 1; 1 + \frac{2x\epsilon}{s}\right) = Y_n(x; r, s) \qquad \ldots(2.4)$$

where $Y_n(x; r, s)$ denote the Bessel polynomial considered by Krall and Frink (1949).

The author (Chiney 1978) has expressed the matrix elements $P^l_{mn}(z/b)$ in terms of extended Jacobi polynomial as

$$P^l_{mn}(z/b) = \left[\frac{(l-m)!\,(l+m)!}{(l-n)!\,(l+n)!}\right]^{1/2} \frac{c^m (b+z)^{(m+n)/2}}{\lambda^l (b-z)^{(n-m)/2}} F_k(\beta, \alpha; z) \quad \ldots(2.5)$$

where

$$l = k + \frac{\alpha+\beta}{2}; \; m = \frac{\alpha+\beta}{2}; \; n = \frac{\beta-\alpha}{2}; \; \lambda = 2bc. \qquad \ldots(2.6)$$

§3. The Rodrigue's formula (1.1) can be written as

$$P^{(2k+\alpha+\beta)/2}_{(\alpha+\beta)/2,\,(\beta-\alpha)/2}(x/b) = A(-c)^k D^k [(b-x)^{k+\alpha}(b+x)^{k+\beta}] \qquad \ldots(3.1)$$

where $$A = \left[\frac{(k+\alpha+\beta)!}{k!\,(k+\alpha)!\,(k+\beta)!}\right]^{1/2} \frac{c^{(\alpha+\beta)/2}(b-x)^{-\alpha/2}}{\lambda^{(2k+\alpha+\beta)/2}(b+x)^{\beta/2}};$$

$$l = k + \frac{\alpha+\beta}{2}; \; m = \frac{\alpha+\beta}{2}; \; n = \frac{\beta-\alpha}{2}; \; \lambda = 2bc;$$

$$D \equiv \frac{d}{dx}; \; \alpha > -1, \beta > -1. \qquad \ldots(3.2)$$

By using Cauchy's theorem, we obtain an integral representation of matrix element $P^l_{mn}(x/b)$

$$P^{(2k+\alpha+\beta)/2}_{(\alpha+\beta)/2,\,(\beta-\alpha)/2}(x/b) = \frac{A(-c)^k k!}{2\pi i} \oint \frac{(b-t)^{k+\alpha}(b+t)^{k+\beta}}{(t-x)^{k+1}}\, dt \qquad \text{...(3.3)}$$

where the integration is extended in the positive sense along a suitable curve around x in the complex t-plane.

The generating function is defined, in general, by

$$G(y, \xi) = \sum_{k=0}^{\infty} \xi^k p_k(y).$$

Accordingly, we have

$$G\left(\frac{x}{b}, \xi\right) = \sum_{k=0}^{\infty} \xi^k p_k(x/b)$$

where $\quad p_k(x/b) = P^{(2k+\alpha+\beta)/2}_{(\alpha+\beta)/2,\,(\beta-\alpha)/2}(x/b)$

therefore,

$$G\left(\frac{x}{b}, \xi\right) = \frac{Ak!}{2\pi i} \oint \frac{(b-t)^{\alpha}(b+t)^{\beta}}{(t-x)} \sum_{k=0}^{\infty} \left[\frac{-c(b^2-t^2)}{t-x}\right]^k dt.$$

If the contour of integration is so chosen that

$$\left|\frac{-c\xi(b^2-t^2)}{(t-x)}\right| < 1 \qquad \text{...(3.4)}$$

so that the sum in the above integrand, being an infinite geometric series, is equal to $\dfrac{t-x}{(t-x)+c\xi(b^2-t^2)}$, hence

$$G\left(\frac{x}{b}, \xi\right) = \frac{Ak!}{2\pi i} \oint \frac{(b-t)^{\alpha}(b+t)^{\beta}}{(t-x)+c\xi(b^2-t^2)}\, dt. \qquad \text{...(3.5)}$$

The quadratic function of t in the denominator of the integrand (3.5) is factored as

$$t - x + c\xi(b^2 - t^2) = -c\xi(t-t_0)(t-t_1)$$

and hence

$$\frac{1}{t-x+c\xi(b^2-t^2)} = \frac{1}{c\xi(t_1-t_0)}\left[\frac{1}{t-t_0} - \frac{1}{t-t_1}\right]$$

where $t_0(x, \xi)$ and $t_1(x, \xi)$ are the two zeros of the quadratic function. If t_0 lies inside and t_1 outside the contour defined by (3.4), then by using the residue theorem for (3.5) we have

$$G\left(\frac{x}{b}, \xi\right) = \frac{1}{c\xi(t_1 - t_0)}\left[\frac{k!\,(k+\alpha+\beta)!}{(k+\alpha)!\,(k+\beta)!}\right]^{1/2} \frac{c^{(\alpha+\beta)/2}}{\lambda^{(2k+\alpha+\beta)/2}}$$

$$\times (b - x)^{\alpha/2} (b + x)^{\beta/2} \left(\frac{b - t_0}{b - x}\right)^{\alpha} \left(\frac{b + t_0}{b + x}\right)^{\beta}. \qquad \ldots(3.6)$$

Now we shall investigate the condition (3.4) by setting

$t = x + \xi z$ and $z = re^{i\theta}$

$$\frac{-c\xi(b^2 - t^2)}{t - x} = cz\xi^2 + 2cx\xi + \frac{c(x^2 - b^2)}{z}$$

$$= \cos\theta\left[cr\xi^2 + \frac{c}{r}(x^2 - b^2)\right] + 2cx\xi$$

$$+ i\sin\theta\left[cr\xi^2 - \frac{c}{r}(x^2 - b^2)\right]$$

and

$$\left|\frac{-c\xi(b^2 - t^2)}{t - x}\right|^2 = [Y(r, \theta)]^2 + \lambda^2\xi^2\sin^2\theta \qquad \ldots(3.7)$$

where

$$\lambda = 2bc > 0$$

and

$$Y(r, \theta) = cr\xi^2 + 2c\xi x\cos\theta + \frac{c(x^2 - b^2)}{r}.$$

Here $Y(r, \theta)$ is monotonically increasing function of r, r varying from $-\infty$ to ∞. So that under the condition

$$\lambda\,|\,\xi\,| < 1 \qquad \ldots(3.8)$$

and for a fixed value of θ, four consecutive values of r :

$$r_1(\theta) > d_1(\theta) \geqslant d_0(\theta) > r_0(\theta) > 0 \qquad \ldots(3.9)$$

are defined by

$$Y(r_1(\theta), \theta) = -\,Y(r_0(\theta), \theta) = 1$$

$$Y(d_1(\theta), \theta) = -\,Y(d_0(\theta), \theta) = \lambda\,|\,\xi\cos\theta\,|\,.$$

In (3.9) the equality occurs for $\theta = \pm\,\pi/2$.

For, $\quad d_1(\theta) \geqslant r \geqslant d_0(\theta)$

we have $\quad [Y(r, \theta)]^2 \leqslant (\lambda\xi\cos\theta)^2$

and hence from (3.7) and (3.8) we get

$$\left|\frac{-c\xi(b^2-t^2)}{t-x}\right|^2 \leqslant \lambda^2\xi^2 < 1$$

as required by (3.4).

Consider the equation $t - x + c\xi(b^2 - t^2) = 0$ and putting $t = x + \xi z$, we get

$$cz^2\xi^2 + z(2cx\xi - 1) - c(b^2 - x^2) = 0$$

The two roots of this equation are

$$z_0(x, \xi) = \frac{1 - 2cx\xi - R}{2c\xi^2} \text{ and } z_1(x, \xi) = \frac{1 - 2cx\xi + R}{2c\xi^2} \qquad \text{...(3.10)}$$

where

$$\begin{aligned} R &= (1 - 4cx\xi + 4b^2c^2\xi^2)^{1/2} \\ &= (1 - 4cx\xi + \lambda^2\xi^2)^{1/2}. \end{aligned} \qquad \text{...(3.11)}$$

Hence

$$t_0 = x + \xi z_0 = \frac{1 + \lambda\xi - R}{2c\xi} - b = \frac{1 - \lambda\xi - R}{2c\xi} + b \qquad \text{...(3.12)}$$

where $\lambda = 2bc$. Moreover from (3.11) and (3.12), we have

$$(1 + \lambda\xi + R)(1 + \lambda\xi - R) = 4c\xi(b + x) \qquad \text{...(3.13)}$$

$$(1 - \lambda\xi + R)(1 - \lambda\xi - R) = -4c\xi(b - x). \qquad \text{...(3.14)}$$

Hence (3.12), (3.13) and (3.14) allows us to write

$$\frac{b + t_0}{b + x} = \frac{2}{1 + \lambda\xi + R}; \frac{b - t_0}{b - x} = \frac{2}{1 - \lambda\xi + R}$$

and $\qquad c\xi(t_1 - t_0) = R.$

Hence we can write eqn. (3.6) as

$$G\left(\frac{x}{b}, \xi\right) = \left[\frac{k!\,(k + \alpha + \beta)!}{(k + \alpha)!\,(k + \beta)!}\right]^{1/2} \frac{c^{(\alpha+\beta)/2}}{\lambda^{(2k+\alpha+\beta)/2}} (b - x)^{\alpha/2}(b + x)^{\beta/2} \times \frac{2^{\alpha+\beta}}{R}(1 - \lambda\xi + R)^{-\alpha}(1 + \lambda\xi + R)^{-\beta}$$

$$\sum_{k=0}^{\infty} \xi^k P^l_{mn}(x/b) = \frac{c^m}{\lambda^l}\left[\frac{(l - m)!\,(l + m)!}{(l - n)!\,(l + n)!}\right]^{1/2} (b-x)^{(m-n)/2}(b + x)^{(m+n)/2} \times \frac{2^{\alpha+\beta}}{R}(1 - \lambda\xi + R)^{-\alpha}(1 + \lambda\xi + R)^{-\beta}. \qquad \text{...(3.15)}$$

This is a generating function of the matrix element $P^l_{mn}(x/b)$ with l, m and n given by (3.2). If we substitute for the matrix element in terms of extended Jacobi polynomials as given by (2.5). We get the generating function for extended Jacobi polynomials which is obtained by Fujiwara (1966) as

$$\sum_{k=0}^{\infty} \xi^k F_k(\beta, \alpha; x) = \frac{2^{\alpha+\beta}}{R(x, \xi)} (1 + \lambda\xi + R)^{-\beta} (1 - \lambda\xi + R)^{-\alpha} \quad \ldots(3.16)$$

with the condition given in (3.8).

We shall now explicitly exhibit that the substitutions considered in section 2 lead to the generating functions of Jacobi, Hermite and Bessel polynomials.

(A) *Jacobi Polynomials*

If we put $b = 1$, $\lambda = 1$ and therefore $c = \frac{1}{2}$ since $\lambda = 2bc$ in (3.16) we get by using the result (2.2)

$$\sum_{k=0}^{\infty} \xi^k P_k^{(\alpha,\beta)}(x) = \frac{2^{\alpha+\beta}}{R(x, \xi)} (1 + \xi + R)^{-\beta} (1 - \xi + R)^{-\alpha}$$

or which is the same thing as

$$\sum_{k=0}^{\infty} \xi^k P_k^{(\beta,\alpha)}(x) = \frac{2^{\alpha+\beta}}{R(x, \xi)} (1 + \xi + R)^{-\alpha} (1 - \xi + R)^{-\beta}$$

where $\qquad R = (1 - 2x\xi + \xi^2)^{1/2}$.

(B) *Hermite Polynomials*

If we put $b = \sqrt{\alpha} = \sqrt{\beta} \to \infty$ and in view of $\lambda \to \frac{2}{\sqrt{\alpha}}$ we have $c \to \frac{1}{\alpha}$ because of the relation $\lambda = 2bc$ we have from (3.11)

$$R(x, \xi) = (1 - 4cx\xi + \lambda^2\xi^2)^{1/2}$$

i.e. $\qquad R(x, \xi) \to 1 - 2cx\xi + \frac{1}{2}\lambda^2\xi^2$

i.e. $\qquad R(x, \xi) \to 1 - \frac{2}{\alpha}(\xi x - \xi^2)$.

And eqn. (3.16) gives

$$\sum_{k=0}^{\infty} \xi^k F_k(\alpha, \alpha; x) = \frac{2^{2\alpha}}{R(x, \xi)} (1 + R + \lambda\xi)^{-\alpha} (1 + R - \lambda\xi)^{-\alpha}$$

$$= \frac{2^{\alpha}}{R} (1 - 2cx\xi + R)^{-\alpha}, = \frac{1}{R}\left(\tfrac{1}{2} - cx\xi + \frac{R}{2}\right)^{-\alpha}.$$

Now taking the limit of both the sides as $\alpha \to \infty$

$$\lim_{\alpha\to\infty} \sum_{k=0}^{\infty} \xi^k F_k(\alpha, \alpha; x) = \lim_{\alpha\to\infty} \frac{1}{R} \cdot \lim_{\alpha\to\infty} \left(\tfrac{1}{2} - cx\xi + \frac{R}{2}\right)^{-\alpha}$$

$$= \lim_{\alpha\to\infty} \left(\tfrac{1}{2} + \tfrac{1}{2}\left[1 - \frac{2}{\alpha}(\xi x - \xi^2)\right] - \frac{\xi x}{\alpha}\right)^{-\alpha}$$

$$= \lim_{\alpha\to\infty} \left(1 - \frac{1}{\alpha}(2\xi x - \xi^2)\right)^{-\alpha}$$

$$\sum_{k=0}^{\infty} \frac{\xi^k H_k(x)}{k!} = \exp(2\xi x - \xi^2)$$

which is very well known relation.

(C) *Bessel Polynomials*

When we put $b = 1 = \lambda$, $\alpha = \epsilon - 1$, $\beta = r - \epsilon - 1$, replace x by $1 + \dfrac{2x\epsilon}{s}$ and ξ by $\dfrac{sw}{2\epsilon}$ in (3.11) and (3.16) we have from (3.11)

$$R(x, \xi) = \left[1 - \frac{sw}{\epsilon}\left(1 + \frac{2x\epsilon}{s}\right) + \left(\frac{sw}{2\epsilon}\right)^2\right]^{1/2}$$

and hence

$$R(x, \xi) \to (1 - 2xw)^{1/2} \text{ as } \epsilon \to \infty$$

whereas we have from (3.16)

$$\sum_{k=0}^{\infty} \left(\frac{sw}{2\epsilon}\right)^k F_k\left(r - \epsilon - 1, \epsilon - 1, 1 + \frac{2x\epsilon}{s}\right)$$

$$= \left(2^{r-2} \Big/ \sqrt{\left[1 - \frac{sw}{\epsilon}\left(1 + \frac{2x\epsilon}{s}\right) + \left(\frac{sw}{2\epsilon}\right)^2\right]}\right)$$

$$\times \left(1 + \sqrt{\left[1 - \frac{sw}{\epsilon}\left(1 + \frac{2x\epsilon}{s}\right) + \left(\frac{sw}{2\epsilon}\right)^2\right]}\right)^{2-r}$$

$$\times \left(1 + \frac{(sw/2\epsilon)}{1 + \sqrt{\left[1 - \frac{sw}{\epsilon}\left(1 + \frac{2x\epsilon}{s}\right) + \left(\frac{sw}{2\epsilon}\right)^2\right]}}\right)^{1+s-r}$$

$$\times \left(1 - \frac{(sw/2\epsilon)}{1 + \sqrt{\left[1 - \frac{sw}{\epsilon}\left(1 + \frac{2x\epsilon}{s}\right) + \left(\frac{sw}{2\epsilon}\right)^2\right]}}\right)^{1-s}.$$

Now expand the last two factors by binomial theorem and take the limit of both the sides as $\epsilon \to \infty$, we get

$$\lim_{\epsilon\to\infty} \sum_{k=0}^{\infty} \left(\frac{s}{2}\right)^k \frac{w^k}{\epsilon^k} F_k\left(r - \epsilon - 1,\ \epsilon - 1;\ 1 + \frac{2x\epsilon}{s}\right)$$

$$= (1 - 2xw)^{-1/2} [\tfrac{1}{2} + \tfrac{1}{2}\sqrt{(1 - 2xw)}]^{2-r}$$

$$\times \exp\left(\frac{(sw/2)}{1 + \sqrt{1 - 2xw}}\right) . \exp\left(\frac{(sw/2)}{1 + \sqrt{1 - 2xw}}\right).$$

Therefore by using (2.4)

$$\sum_{k=0}^{\infty} \left(\frac{s}{2}\right)^k Y_k(x; r, s) \frac{w^k}{k!}$$

$$= (1 - 2xw)^{-1/2} (\tfrac{1}{2} + \tfrac{1}{2}\sqrt{1 - 2xw})^{2-r} \exp\left(s(1 - \sqrt{1 - 2xw})/2x\right).$$

This generating function was first obtained by Burchnall (1951). The misprint in Burchnall's paper $(\frac{1}{2} - \frac{1}{2}\sqrt{1 - 2xw})^{2-r}$ where $(\frac{1}{2} + \frac{1}{2}\sqrt{1 - 2xw})^{2-r}$ was intended, was pointed out by Rainville (1953).

§4. *Integral representation of extended Jacobi polynomials* — We rewrite an integral representation of matrix element $P^l_{mn}\left(\frac{x}{b}\right)$ given in (3.3) as

$$P^l_{mn}(x/b) = \frac{Bk!}{2\pi i} c^k (b - x)^{\alpha/2} (b + x)^{\beta/2}$$

$$\oint \frac{(t^2 - b^2)^k}{(t - x)^{k+1}} \left(\frac{b - t}{b - x}\right)^\alpha \left(\frac{b + t}{b + x}\right)^\beta dt$$

where $$B = \left[\frac{(l + m)!}{(l - m)!\,(l + n)!\,(l - n)!}\right]^{1/2} \frac{c^m}{\lambda^l},$$

$$l = k + \frac{\alpha + \beta}{2};\ m = \frac{\alpha + \beta}{2};\ n = \frac{\beta - \alpha}{2};\ \lambda = 2bc.$$

By writing the matrix element $P^l_{mn}(x/b)$ in terms of extended Jacobi polynomial given by (2.5) and simplifying, we have

$$F_k(\beta, \alpha; x) = \frac{c^k}{2\pi i} \oint \frac{(t^2 - b^2)^k}{(t - x)^{k+1}} \left(\frac{b - t}{b - x}\right)^\alpha \left(\frac{b + t}{b + x}\right)^\beta dt \qquad \text{...(4.1)}$$

substituting in this integral $t = x + \sqrt{x^2 - b^2}\, e^{i\phi}$ we get

$$F_k(\beta, \alpha; x) = \frac{(2c)^k}{2\pi} \int_0^{2\pi} (x + \sqrt{x^2 - b^2} \cos \phi)^k$$

$$\times \left[1 - i\sqrt{\frac{b + x}{b - x}} e^{i\phi}\right]^\alpha \left[1 + \sqrt{\frac{x - b}{x + b}} e^{i\phi}\right]^\beta d\phi. \qquad \ldots(4.2)$$

Particular cases — If we set $b = 1 = \lambda$ which forces $c = \frac{1}{2}$ since $\lambda = 2bc$ in (4.1), we get by using (2.2)

$$P_k^{(\alpha,\beta)}(x) = \frac{1}{2\pi i 2^k} \oint \frac{(t^2 - 1)^k}{(t - x)^{k+1}} \left(\frac{1 - t}{1 - x}\right)^\alpha \left(\frac{1 + t}{1 + x}\right)^\beta dt \qquad \ldots(4.3)$$

And, in particular, if we put $\alpha = \beta = 0$ we get Schlaeflis integral representation for Legendre polynomial $P_k(x)$

$$P_k(x) = \frac{1}{2\pi i 2^k} \oint \frac{(t^2 - 1)^k}{(t - x)^{k+1}} dt. \qquad \ldots(4.4)$$

If we put $b = 1 = \lambda$ and $c = \frac{1}{2}$ and $\alpha = \beta = 0$ in (4.2) we get

$$P_k(x) = \frac{1}{2\pi} \int_0^{2\pi} (x + \sqrt{x^2 - 1} \cos \phi)^k d\phi$$

which can be obtained from (4.4) by putting $t = x + \sqrt{x^2 - 1}\, e^{i\phi}$.

Hermite Polynomials — If we put $\alpha = \beta$ and $b = \sqrt{\alpha} \to \infty$, eqn. (4.1) can be written as

$$\lim_{\alpha \to \infty} F_k(\alpha, \alpha; x) = \lim_{\alpha \to \infty} \frac{1}{2\pi i} \frac{1}{\alpha^k} \oint \frac{(t^2 - \alpha)^k}{(t - x)^{k+1}} \left(\frac{\alpha - t^2}{\alpha - x^2}\right)^\alpha dt$$

$$\frac{H_k(x)}{k!} = \frac{1}{2\pi i} \lim_{\alpha \to \infty} \oint \frac{\left(\frac{t^2}{\alpha} - 1\right)^k}{(t - x)^{k+1}} \left(\frac{1 - \frac{t^2}{\alpha}}{1 - \frac{x^2}{\alpha}}\right)^\alpha dt$$

$$= \frac{1}{2\pi i} \oint \frac{(-1)^k e^{-t^2}}{(t - x)^{k+1} e^{-x^2}} dt$$

$$\frac{e^{-x^2} H_k(x)}{k!} = \frac{-1}{2\pi i} \oint \frac{e^{-t^2}}{(x - t)^{k+1}} dt.$$

This is similar to Schlaeflis integral representation for Legendre polynomial and appears to be new integral representation for Hermite polynomials.

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