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ORBITAL FRACTIONAL PARENTAGE COEFFICIENTS FOR NUCLEI WITH $A = 3$

S. B. DOMA

Mathematics Department, Faculty of Science, Alexandria University, Alexandria, Egypt

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The bases of the unitary scheme model are used to calculate the two-particle orbital fractional parentage coefficients. Tables of these coefficients are given for nuclei with $A = 3$ and number of quanta of excitations $0 \leqslant N \leqslant 10$.

1. INTRODUCTION

The systematic construction of A-particle states in configuration space, when the particles are in a common harmonic oscillator potential have been attempted by many authors (Kramer and Moshinsky 1966, Pargmann and Moshinsky 1960). The main concept involved in this construction was the realization that the most general symmetry group for the inter-particle motions of this system was the unitary group in $3(A-1)$ dimensions $U_{3(A-1)}$. This realization is achieved in the so-called unitary scheme model (Vanagas 1971), (USM).

The USM bases are classified by the irreducible representations (IR) of the unitary unimodular subgroup of three dimensions SU_3 and its rotational subgroups R_3 and R_2. They are also characterized by the IR of the chain of groups $U_{A-1} \supset O_{A-1} \supset S_A$ where the latter groups are respectively the unitary, orthogonal and symmetric groups.

The A-particle state in a harmonic oscillator potential could be decomposed into a state of the first $A - 2$ particles and a state of the last pair of particles. The expansion coefficients of this decomposition are called two-particle fractional parentage coefficients (FPC). These coefficients are of great importance in the calculation of matrix elements of two-particle operators such as the nucleon-nucleon interactions.

In the present paper the bases of the USM are constructed for nuclei with $A = 3$ and number of quanta of excitations $0 \leqslant N \leqslant 10$.

Furthermore, the two-particle orbital FPC are derived for the considered cases and are tabulated at the end of this paper.

Present address : Faculty of Education, King Abdul Aziz University, Meccah, Saudi Arabia.

2. Construction of the USM Bases

The Hamiltonian operator which describes the inter-particle motions of a nucleus in a common harmonic oscillator potential with respect to the nucleus centre of mass is called the USM Hamiltonian and is defined as

$$H = \frac{1}{A} \sum_{i<j=1}^{A} \left[\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \tfrac{1}{2} m\omega^2 (\vec{r}_i - \vec{r}_j)^2 \right]. \quad \text{...(2.1)}$$

Introducing Jacobi's coordinates (Vanagas 1971) and the corresponding quasi-particle masses and momenta it could be deduced that the Hamiltonian operator H is equivalent to a scalar product of vectors in a $3(A-1)$-dimensional space. This product is invariant with respect to the transformations of the unitary group in $3(A-1)$-dimensions $U_{3(A-1)}$.

The Hamiltonian (2.1) has eigenvalues and corresponding eigenfunctions given by Vanagas (1971)

$$E_N = [N + \tfrac{3}{2}(A-1)]\, \hbar\omega,$$

$$\psi_{\alpha_1 i_1, \ldots, \alpha_N i_N} = C a^{+}_{\alpha_1 i_1} a^{+}_{\alpha_2 i_2} \cdots a^{+}_{\alpha_N i_N} \exp\left\{ -\frac{m\omega}{2\hbar} \sum_{k=1}^{3} \sum_{i=1}^{A-1} \xi^2_{ki} \right\} \quad \text{...(2.2)}$$

where ξ_{ki} are the Jacobi coordinates, a^{+}_{ki} are the creation operators to be associated with these coordinates and with the corresponding momenta, and C is a normalization constant. The functions (2.2) are used as bases for the IR of a symmetric tensor of the rank N. The Young scheme $\{N\}$ is useful for obtaining such IR. The bases (2.2) are usually denoted by

$$| A\Gamma \rangle \equiv | AN \{\rho\} (\nu) [f] \alpha LM \rangle \quad \text{...(2.3)}$$

where

$$\{\rho\} = \begin{cases} \{\rho_1, \rho_2, \rho_3\}, \text{ for } U_n;\ n \geqslant 3 \\ \{\rho_1, \rho_2\};\ \rho_3 = 0, \text{ for } U_2 \\ \{\rho_1\};\ \rho_2 = \rho_3 = 0, \text{ for } U_1 \end{cases}$$

with $\rho_1 \geqslant \rho_2 \geqslant \rho_3 \geqslant 0$, and $\rho = \rho_1 + \rho_2 + \rho_3 = N$.

The bases (2.3) are classified according to the following orbital chain of groups:

$$U_{3(A-1)} \supset \times \begin{cases} SU_3 \supset R_3 \supset R_2 \\ U_{A-1} \supset O_{A-1} \supset S_A \end{cases} \quad \text{...(2.4)}$$

The IR N, $\{\rho\}$, (ν), $[f]$, L, and M belong to the groups $U_{3(A-1)}$, U_{A-1} (and SU_3 simultaneously), O_{A-1}, S_A, R_3 and R_2 respectively. In the relation (2.3), the quantum number α is introduced to distinguish between the different states, that may arise from the classifications, having the same IR. Direct methods for defining the IR characterizing the chain of groups (2.4) are given by Vanagas (1971).

3. Two-Particle Orbital FPC

The orbital function $| A\Gamma \rangle$ assumes the following two-particle fractional parentage decomposition :

$$| A\Gamma \rangle = \sum_{\bar{\Gamma}\Gamma_a} \langle A\Gamma \mid A - 2\bar{\Gamma}; 2\Gamma_a \rangle \mid A - 2\bar{\Gamma} \rangle \mid 2\Gamma_a \rangle \qquad \text{...(3.1)}$$

where

$$\bar{\Gamma} \equiv \bar{N}\,\{\bar{\rho}\}\,\{\bar{\bar{\rho}}\}\,(\bar{\bar{\nu}})\,[\bar{\bar{f}}]\,\bar{L}\bar{M},\ \Gamma_a \equiv \{\epsilon\}\,(\nu_a)\,[f_a]\,lm,\ (\bar{N} + \epsilon = N) \qquad \text{...(3.2)}$$

are the sets of all orbital quantum numbers characterizing the states of $A - 2$ and 2 particles, respectively, and $\langle A\Gamma \mid A - 2\bar{\Gamma}; 2\Gamma_a \rangle$ are the two-particle orbital FPC.

The bare in the above equations indicates that one quasi particle of coordinates

$$\vec{\xi}_a = \frac{1}{\sqrt{2}}\,(\vec{r}_{A-1} - \vec{r}_A) \equiv \frac{1}{\sqrt{2}}\,\vec{r}$$

has been separated from the orbital wave function of the A-particle state. The double bares indicate that two quasi particles of coordinates $\vec{\xi}_a$ and

$$\vec{\xi}_s = \sqrt{\frac{A-2}{2A}}\left[\frac{2}{A-2}\sum_{t=1}^{A-2}\vec{r}_t - (\vec{r}_{A-1} + \vec{r}_A)\right]$$

have been separated.

From the transformation properties of the functions of eqn. (3.1) it follows that the two-particle FPC assumes the following factorization (Doma and Machabeli 1975) :

$$\langle A\Gamma \mid A - 2\bar{\Gamma}; 2\Gamma_a \rangle = \sum_{\beta} \langle \{\bar{\rho}\}\,\bar{L}\bar{M};\,\{\epsilon\}\,lm \mid \beta\{\rho\}\,LM \rangle$$

$$\times \langle AN\,\{\rho\}\,(\nu)\,[f]\,\beta \mid A - 2\bar{N}\,\{\bar{\rho}\}\,\{\bar{\bar{\rho}}\}(\bar{\bar{\nu}})\,[\bar{\bar{f}}];\,2\,\{\epsilon\}\,(\nu_a)\,[f_a] \rangle \qquad \text{...(3.3)}$$

where β is a repetition quantum number that may arise in the direct product $\{\bar{\rho}\} \times \{\epsilon\} \rightarrow \{\rho\}$, and $\langle \{\bar{\rho}\}\,\bar{L}\bar{M};\ \{\epsilon\}\,lm \mid \beta\,\{\rho\}\,LM \rangle$ is a Clebsh-Gordan coefficient (CGC) of the group SU_3. The CGC of the SU_3 group are factorized as follows :

$$\langle \{\bar{\rho}\}\, \bar{L}\bar{M};\, \{\epsilon\}\, lm \mid \{\rho\}\, LM \rangle = (\bar{L}\bar{M},\, lm \mid LM)\, \langle \{\bar{\rho}\}\, \bar{L};\, \{\epsilon\}\, l \,\|\, \{\rho\}\, L \rangle \qquad \ldots(3.4)$$

where $(\bar{L}\bar{M}, lm \mid LM)$ is a CGC of the rotational group R_3, and $\langle \{\bar{\rho}\}\, \bar{l}, \{\epsilon\}\, l \,\|\, \{\rho\}\, L \rangle$ is an isoscalar factor of the SU_3 group. Explicit algebraic expressions for the isoscalar factors of the SU_3 group are given by Hecht (1965), Iosifescu and Stancu (1967), Vergados (1968) and Ališauskas (1969).

Let us, for simplicity, recall the second term in the right-hand side of eqn. (3.3) the two-particle orbital FPC. In terms of a new coefficient $\mathcal{A}$, introduced by Vanagas (1971) the two particle orbital FPC can be calculated as follows :

$$\langle AN\, \{\rho\}\, (\nu)\, [f] \mid A - 2\bar{N}\, \{\bar{\rho}\}\, \{\bar{\bar{\rho}}\}\, (\bar{\bar{\nu}})\, [\bar{\bar{f}}];\, 2\, \{\epsilon\}\, (\nu_a)\, [f_a] \rangle$$

$$= \sum_{[\bar{f}]\{\bar{\rho}'\}\{\rho_{12}\}} \mathcal{A}^{(\{\rho\}[\bar{\bar{f}}])}_{\{\bar{\rho}'\}\{\bar{\bar{\rho}}\}(\bar{\bar{\nu}}),\,(\nu)[f][\bar{f}]}\, \mathcal{U}^{([f][\bar{\bar{f}}])}_{[\bar{f}],\,[f_a]}$$

$$\times D^{j}_{m'm}(a)\, \langle ((\{\bar{\bar{\rho}}\}\, (\epsilon_2\epsilon_1)\, \{\rho_{12}\})\, \{\rho\} \mid (((\{\bar{\bar{\rho}}\}\, \epsilon_2)\, \{\bar{\rho}'\}\, \epsilon_1)\, \{\rho\} \rangle$$

$$\times \langle (((\{\bar{\bar{\rho}}\}\, \epsilon_s)\, \{\bar{\rho}\}\, \epsilon_a)\, \{\rho\} \mid (\{\bar{\bar{\rho}}\}\, (\epsilon_s\epsilon_a)\, \{\rho_{12}\})\, \{\rho\} \rangle \qquad \ldots(3.5)$$

where

$$\epsilon_1 = \rho - \bar{\rho}',\ \epsilon_2 = \bar{\rho}' - \bar{\bar{\rho}},\ \epsilon_s = \bar{\rho} - \bar{\bar{\rho}},\ \epsilon_a = \rho - \bar{\rho},$$

$$\{\rho_{12}\} = \{\rho_1\rho_2\} \subset \{\epsilon_2\} \dot{\times} \{\epsilon_1\},\ j = \tfrac{1}{2}(\rho_1 - \rho_2),\ m' = \tfrac{1}{2}(\epsilon_s - \epsilon_a),$$

and $m = \frac{1}{2}(\epsilon_2 - \epsilon_1)$. The quantum numbers $[\bar{f}]$, $\{\bar{\rho}'\}$, $\{\rho_{12}\}$ are IR of the groups S_{A-1}, U_{A-2}, SU_3, respectively a is the determinant

$$a = \begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix} = \begin{vmatrix} \sqrt{(A)/2(A-1)} & -\sqrt{(A-2)/2(A-1)} \\ \sqrt{(A-2)/2(A-1)} & \sqrt{(A)/2(A-1)} \end{vmatrix}$$

and

$$D^{j}_{m'm}(a) = \sum_{i} \frac{[(j+m')!\,(j-m')!\,(j+m)!\,(j-m)!]^{1/2}}{i!\,(j-m'-i)!\,(j+m-i)!\,(i-m+m')!}$$

$$\times\, a_1^{j+m-i}\, a_2^{i}\, a_3^{i-m+m'}\, a_4^{j-m'-i} \qquad \ldots(3.6)$$

are the matrix elements of the IR of the group SU_2. The matrix $\mathcal{U}^{([f][\bar{\bar{f}}])}$ of eqn. (3.5) is defined as (Vanagas 1971) :

$$\mathcal{U}^{([f][\bar{\bar{f}}])} = \begin{array}{c|cc} [\bar{f}] \backslash [f_a] & [2] & [11] \\ \hline [\bar{f}] & \sqrt{(g-1)/2g} & \sqrt{(g+1)/2g} \\ [\bar{f}'] & \sqrt{(g+1)/2g} & -\sqrt{(g-1)/2g} \end{array} \qquad \text{...(3.7)}$$

Here $\bar{f} \rangle \bar{f}'$, in the sense that the first different rows, having non-equal number of squares, in the two IR $[\bar{f}]$ and $[\bar{f}']$ have number of squares in $[\bar{f}]$ greater than the corresponding in $[\bar{f}']$, and g is the axial distance for the IR $[\bar{f}']$, given by Vanagas (1971) :

$$g \equiv g^{[f]}_{[\bar{f}'][\bar{\bar{f}}]} = \Lambda([f]) - 2\Lambda([\bar{f}']) + \Lambda([\bar{\bar{f}}]), \qquad \text{...(3.8)}$$

where
$$\Lambda([f]) = \tfrac{1}{2} \sum_{i=1}^{A} f_i(f_i - 2i + 1); \; [f] = [f_1, \ldots, f_i, \ldots, f_A], \text{ etc.} \qquad \text{...(3.9)}$$

The last two elements in the right-hand side of eqn. (3.5) are the recoupling-matrix elements, the first of which is factorized in terms of products of $6j$-symbols of the SU_3 group, and the second is equivalent to a $9j$-symbol of the SU_3 group. Explicit algebraic expressions for these recoupling matrices are given by Ališauskas (1972).

Finally a recurrence relations for the coefficients $\mathcal{A}$ and tables of the two-particle arbital FPC, eqn. (3.5), for $3 \leqslant A \leqslant 6$, and $N \leqslant 3$ are given by Vanagas (1971). General and direct method for calculating the coefficients $\mathcal{A}$ and tables of the two-particle orbital FPC for $A = 6$, and $2 \leqslant N \leqslant 4$ are given by Doma and Machabeli (1975). This direct method was used to calculate the coefficients $\mathcal{A}$ for nuclei with $A = 3$ and $0 \leqslant N \leqslant 10$. Then after the two-particle orbital FPC are calculated for these nuclei.

4. Construction of the Tables

In order to construct the tables of the two-particle orbital FPC for a given mass number A and number of quanta of excitations N one must find all the IR of the following chain of groups :

$$\left.\begin{array}{lllll} U_{3(A-1)} \supset SU_3 \dot{\times} U_{A-1} \supset & & O_{A-1} \supset S_A \\ \cup \qquad\qquad \cup & & \cup \qquad \cup \\ U_{3(A-2)} \supset SU_3 \dot{\times} U_{A-2} \supset U_{A-3} \supset & & O_{A-3} \supset S_{A-2} \\ \dot{\times} \qquad\qquad \dot{\times} & & \dot{\times} \qquad \dot{\times} \\ U_3 \quad \supset SU_3 \dot{\times} U_1 \quad \supset & & O_1 \quad \supset S_2. \end{array}\right\} \qquad \text{...(4.1)}$$

The first row in this chain belongs to the set of A particles, after its centre of mass has been separated, the second and the third rows belong to the set of $A - 2$, and two particles, respectively.

The problem of finding the IR of the chain of groups (4.1) was solved completely by Vanagas (1971). We are now interested in the case of $A = 3$ and $0 \leqslant N \leqslant 10$. In such a case the IR characterizing the groups U_0, O_0, S_1 in the second chain of groups are simply : $\{0\}$, (0), and $[1]$. Hence in all the tables $[\bar{\bar{f}}] = [1]$, $\{\bar{\bar{\rho}}\} = \{0\}$, $(\bar{\bar{\nu}}) = (0)$, and so they will be omitted from the tables.

From Pauli exclusion principle for the two-particle states ϵ must be even number if $[f_a] = [2]$ and odd number if $[f_a] = [11]$. Also, since $\bar{\rho} + \epsilon = N$, $\bar{\rho}$ is even number if N is even, $[f_a] = [2]$, or if N is odd, $[f_a] = [11]$, and $\bar{\rho}$ is odd number if N is even, $[f_a] = [11]$, or if N is odd, $[f_a] = [2]$. Finally, since $[\bar{f}] = [f_a]$ for $A = 3$ the IR $[\bar{f}]$ will be omitted from the tables.

To be very clear we illustrate the following example :

Let us construct the tables of the two-particle orbital FPC for $A = 3$, $N = 6$, and $\{\rho\} = \{51\}$. According to the chain of groups (4.1) one can obtain the following two corressponding sets of IR :

$\{\bar{\rho}\}$	$\{\bar{\bar{\rho}}\}$	$(\bar{\bar{\nu}})$	$[\bar{\bar{f}}]$
{5}	{0}	(0)	[1]
{4}	{0}	(0)	[1]
{3}	{0}	(0)	[1]
{2}	{0}	(0)	[1]
{1}	{0}	(0)	[1]

(ν)	$[f]$	$[\bar{f}]$	$[f_a]$
(0)*	[111]	[11]	[11]
(2)	[21]	[2]	[2]
(2)	[21]	[11]	[11]
(4)	[21]	[2]	[2]
(4)	[21]	[11]	[11]

Hence the matrix of $\mathcal{A}$ will be of the form

$\{\bar{\rho}\}\{\bar{\bar{\rho}}\}(\bar{\bar{\nu}})$ \ $(\nu)[f][\bar{f}]$	(0)* [111] [11]	(2) [21] [2]	(2) [21] [11]	(4) [21] [2]	(4) [21] [11]
{5}{0}(0)					
{4}{0}(0)					
{3}{0}(0)					
{2}{0}(0)					
{1}{0}(0)					

So that we have the following two matrices for the two-particle orbital FPC

$\{\rho\} = \{51\}$, $[f_a] = [11]$

$(\nu)[f]$ \ $\{\rho\}$	$(0)^*$ [111]	(2) [21]	(4) [21]
{5}			
{3}			
{1}			

$\{\rho\} = \{51\}$, $[f_a] = [2]$

$(\nu)[f]$ \ $\{\bar{\rho}\}$	(2) [21]	(4) [21]
{4}		
{2}		

5. Results and Conclusions

In Tables I–XI we present the two-particle orbital FPC for $A = 3$ and $0 \leqslant N \leqslant 10$. These coefficients are orthogonal and normalized. The coefficients corresponding to even N belong to even-parity states and that corresponding to odd N belong to odd-parity states.

The functions resulting after the separation of the states of $A - 2$ particles, by means of the two-particle orbital FPC, do not depend on the coordinates $\vec{r}_{A-1}$ and $\vec{r}_A$ but depend on the radius vector : $\vec{r} = \vec{r}_{A-1} - \vec{r}_A$. It follows, then, that these functions are useful in calculating the matrix elements of any kind of the two-particle central operators :

$$V(\mid \vec{r}_{A-1} - \vec{r}_A \mid) \equiv V(r).$$

Table I : $N = 0$

$\{\rho\} = \{0\}$, $[f_a] = [2]$

$(\nu)[f]$ \ $\{\bar{\rho}\}$	(0) [3]
{0}	1

TABLE II : $N = 1$

$\{\rho\} = \{1\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) $[f]$	(1) [21]
{1}	1

$\{\rho\} = \{1\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) $[f]$	(1) [21]
{0}	-1

TABLE III : $N = 2$

$\{\rho\} = \{2\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) $[f]$	(0) [3]	(2) [21]
{2}	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
{0}	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$

$\{\rho\} = \{2\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) $[f]$	(2) [21]
{1}	1

$\{\rho\} = \{11\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) $[f]$	(0)* [111]
{1}	1

TABLE IV : $N = 3$

$\{\rho\} = \{3\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) $[f]$	(1) [21]	(3) [3]
{3}	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
{1}	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$

$\{\rho\} = \{3\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) $[f]$	(1) [21]	(3) [111]
{2}	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
{0}	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

TABLE IV (*contd.*)

$\{\rho\} = \{21\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]
{1}	− 1

$\{\rho\} = \{21\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]
{2}	− 1

TABLE V : $N = 4$

$\{\rho\} = \{4\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0) [3]	(2) [21]	(4) [21]
{4}	$\frac{\sqrt{6}}{4}$	$\frac{2\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$
{2}	$\frac{2}{4}$	0	$-\frac{2\sqrt{3}}{4}$
{0}	$\frac{\sqrt{6}}{4}$	$-\frac{2\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$

$\{\rho\} = \{4\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(2) [21]	(4) [21]
{3}	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
{1}	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$

$\{\rho\} = \{31\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(2) [21]
{2}	− 1

$\{\rho\} = \{31\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0)* [111]	(2) [21]
{3}	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
{1}	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$

$\{\rho\} = \{22\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0) [3]
{2}	1

TABLE VI : $N = 5$

$\{\rho\} = \{5\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [3]	(5) [21]
{5}	$\frac{\sqrt{10}}{4}$	$\frac{\sqrt{5}}{4}$	$\frac{1}{4}$
{3}	$\frac{2}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{10}}{4}$
{1}	$\frac{\sqrt{2}}{4}$	$-\frac{3}{4}$	$\frac{\sqrt{5}}{4}$

$\{\rho\} = \{5\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [111]	(5) [21]
{4}	$-\frac{\sqrt{2}}{4}$	$\frac{3}{4}$	$\frac{\sqrt{5}}{4}$
{2}	$-\frac{2}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{10}}{4}$
{0}	$-\frac{\sqrt{10}}{4}$	$-\frac{\sqrt{5}}{4}$	$\frac{1}{4}$

$\{\rho\} = \{41\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) (f)	(1) [21]	(3) [3]
{3}	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
{1}	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

$\{\rho\} = \{41\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [111]
{4}	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
{2}	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$

$\{\rho\} = \{32\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]
{3}	-1

$\{\rho\} = \{32\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]
{2}	1

TABLE VII : $N = 6$

$\{\rho\} = \{6\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0) [3]	(2) [21]	(4) [21]	(6) [3]
{6}	$\frac{\sqrt{20}}{8}$	$\frac{\sqrt{30}}{8}$	$\frac{\sqrt{12}}{8}$	$\frac{\sqrt{2}}{8}$
{4}	$\frac{\sqrt{12}}{8}$	$\frac{\sqrt{2}}{8}$	$-\frac{\sqrt{20}}{8}$	$-\frac{\sqrt{30}}{8}$
{2}	$\frac{\sqrt{12}}{8}$	$-\frac{\sqrt{2}}{8}$	$-\frac{\sqrt{20}}{8}$	$\frac{\sqrt{30}}{8}$
{0}	$\frac{\sqrt{20}}{8}$	$-\frac{\sqrt{30}}{8}$	$\frac{\sqrt{12}}{8}$	$-\frac{\sqrt{2}}{8}$

$\{\rho\} = \{6\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(2) [21]	(4) [21]	(6) [111]
{5}	$\frac{\sqrt{20}}{8}$	$-\frac{\sqrt{32}}{8}$	$\frac{\sqrt{12}}{8}$
{3}	$\frac{\sqrt{24}}{8}$	0	$-\frac{\sqrt{40}}{8}$
{1}	$\frac{\sqrt{20}}{8}$	$\frac{\sqrt{32}}{8}$	$\frac{\sqrt{12}}{8}$

$\{\rho\} = \{51\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(2) [21]	(4) [21]
{4}	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
{2}	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$

$\{\rho\} = \{51\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0)* [111]	(2) [21]	(4) [21]
{5}	$\frac{\sqrt{3}}{2\sqrt{2}}$	$\frac{2}{2\sqrt{2}}$	$-\frac{1}{2\sqrt{2}}$
{3}	$\frac{\sqrt{2}}{2\sqrt{2}}$	0	$\frac{\sqrt{6}}{2\sqrt{2}}$
{1}	$\frac{\sqrt{3}}{2\sqrt{2}}$	$\frac{-2}{2\sqrt{2}}$	$\frac{-1}{2\sqrt{2}}$

$\{\rho\} = \{42\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0) [3]	(2) [21]
{4}	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
{2}	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$

TABLE VII (*contd.*)

$\{\rho\} = \{42\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(2) [21]
{3}	-1

$\{\rho\} = \{33\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0)* [111]
{3}	1

TABLE VIII : $N = 7$

$\{\rho\} = \{7\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [3]	(5) [21]	(7) [21]
{7}	$\frac{\sqrt{35}}{8}$	$\frac{\sqrt{21}}{8}$	$\frac{\sqrt{7}}{8}$	$\frac{1}{8}$
{5}	$\frac{\sqrt{15}}{8}$	$-\frac{1}{8}$	$-\frac{3\sqrt{3}}{8}$	$-\frac{\sqrt{21}}{8}$
{3}	$\frac{3}{8}$	$-\frac{\sqrt{15}}{8}$	$-\frac{\sqrt{5}}{8}$	$\frac{\sqrt{35}}{8}$
{1}	$\frac{\sqrt{5}}{8}$	$-\frac{3\sqrt{3}}{8}$	$\frac{5}{8}$	$-\frac{\sqrt{7}}{8}$

$\{\rho\} = \{7\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [111]	(5) [21]	(7) [21]
{6}	$-\frac{\sqrt{5}}{8}$	$\frac{3\sqrt{3}}{8}$	$\frac{5}{8}$	$-\frac{\sqrt{7}}{8}$
{4}	$-\frac{3}{8}$	$\frac{\sqrt{15}}{8}$	$-\frac{\sqrt{5}}{8}$	$\frac{\sqrt{35}}{8}$
{2}	$-\frac{\sqrt{15}}{8}$	$\frac{1}{8}$	$-\frac{3\sqrt{3}}{8}$	$-\frac{\sqrt{21}}{8}$
{0}	$-\frac{\sqrt{35}}{8}$	$-\frac{\sqrt{21}}{8}$	$\frac{\sqrt{7}}{8}$	$\frac{1}{8}$

TABLE VIII (*contd.*)

$\{\rho\} = \{61\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [21]	(5) [21]
{5}	$-\frac{\sqrt{2}}{4}$	$-\frac{3}{4}$	$-\frac{\sqrt{5}}{4}$
{3}	$-\frac{2}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{10}}{4}$
{1}	$-\frac{\sqrt{10}}{4}$	$\frac{\sqrt{5}}{4}$	$-\frac{1}{4}$

$\{\rho\} = \{61\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [111]	(5) [21]
{6}	$-\frac{\sqrt{10}}{4}$	$\frac{\sqrt{5}}{4}$	$\frac{1}{4}$
{4}	$-\frac{2}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{10}}{4}$
{2}	$-\frac{\sqrt{2}}{4}$	$-\frac{3}{4}$	$\frac{\sqrt{5}}{4}$

$\{\rho\} = \{52\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [3]
{5}	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
{3}	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

$\{\rho\} = \{52\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [111]
{4}	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
{2}	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

$\{\rho\} = \{43\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]
{3}	1

$\{\rho\} = \{43\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]
{4}	1

TABLE IX : $N = 8$

$\{\rho\} = \{8\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) $[f]$	(0) [3]	(2) [21]	(4) [21]	(6) [3]	(8) [21]
{8}	$\frac{\sqrt{70}}{16}$	$\frac{\sqrt{112}}{16}$	$\frac{\sqrt{56}}{16}$	$\frac{4}{16}$	$\frac{\sqrt{2}}{16}$
{6}	$\frac{\sqrt{40}}{16}$	$\frac{4}{16}$	$-\frac{\sqrt{32}}{16}$	$-\frac{\sqrt{112}}{16}$	$-\frac{\sqrt{56}}{16}$
{4}	$\frac{6}{16}$	0	$-\frac{\sqrt{80}}{16}$	0	$\frac{\sqrt{140}}{16}$
{2}	$\frac{\sqrt{40}}{16}$	$-\frac{4}{16}$	$-\frac{\sqrt{32}}{16}$	$\frac{\sqrt{112}}{16}$	$-\frac{\sqrt{56}}{16}$
{0}	$\frac{\sqrt{70}}{16}$	$-\frac{\sqrt{112}}{16}$	$\frac{\sqrt{56}}{16}$	$-\frac{4}{16}$	$\frac{\sqrt{2}}{16}$

$\{\rho\} = \{8\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) $[f]$	(1) [21]	(4) [21]	(6) [111]	(8) [21]
{7}	$\frac{\sqrt{56}}{16}$	$-\frac{\sqrt{112}}{16}$	$\frac{\sqrt{72}}{16}$	$\frac{4}{16}$
{5}	$\frac{\sqrt{72}}{16}$	$-\frac{4}{16}$	$-\frac{\sqrt{56}}{16}$	$-\frac{\sqrt{112}}{16}$
{3}	$\frac{\sqrt{72}}{16}$	$\frac{4}{16}$	$-\frac{\sqrt{56}}{16}$	$\frac{\sqrt{112}}{16}$
{1}	$\frac{\sqrt{56}}{16}$	$\frac{\sqrt{112}}{16}$	$\frac{\sqrt{72}}{16}$	$-\frac{4}{16}$

$\{\rho\} = \{71\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) $[f]$	(2) [21]	(4) [21]	(6) [3]
{6}	$-\frac{\sqrt{20}}{8}$	$-\frac{\sqrt{32}}{8}$	$-\frac{\sqrt{12}}{8}$
{4}	$-\frac{\sqrt{24}}{8}$	0	$\frac{\sqrt{40}}{8}$
{2}	$-\frac{\sqrt{20}}{8}$	$\frac{\sqrt{32}}{8}$	$-\frac{\sqrt{12}}{8}$

TABLE IX (*contd.*)

$\{\rho\} = \{71\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0)* [111]	(2) [21]	(4) [21]	(6) [111]
{7}	$\frac{\sqrt{20}}{8}$	$\frac{\sqrt{30}}{8}$	$-\frac{\sqrt{12}}{8}$	$\frac{\sqrt{2}}{8}$
{5}	$\frac{\sqrt{12}}{8}$	$\frac{\sqrt{2}}{8}$	$\frac{\sqrt{20}}{8}$	$-\frac{\sqrt{30}}{8}$
{3}	$\frac{\sqrt{12}}{8}$	$-\frac{\sqrt{2}}{8}$	$\frac{\sqrt{20}}{8}$	$\frac{\sqrt{30}}{8}$
{1}	$\frac{\sqrt{20}}{8}$	$-\frac{\sqrt{30}}{8}$	$-\frac{\sqrt{12}}{8}$	$-\frac{\sqrt{2}}{8}$

$\{\rho\} = \{62\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0) [3]	(2) [21]	(4) [21]
{6}	$\frac{\sqrt{3}}{2\sqrt{2}}$	$\frac{-2}{2\sqrt{2}}$	$\frac{-1}{2\sqrt{2}}$
{4}	$\frac{\sqrt{2}}{2\sqrt{2}}$	0	$\frac{\sqrt{6}}{2\sqrt{2}}$
{2}	$\frac{\sqrt{3}}{2\sqrt{2}}$	$\frac{2}{2\sqrt{2}}$	$\frac{-1}{2\sqrt{2}}$

$\{\rho\} = \{62\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(2) [21]	(4) [21]
{5}	$\frac{-1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
{3}	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$

$\{\rho\} = \{53\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(2) [21]
{4}	1

TABLE IX (*contd.*)

$\{\rho\} = \{53\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0)* [111]	(2) [21]
{5}	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
{3}	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$

$\{\rho\} = \{44\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0) [3]
{4}	− 1

TABLE X : $N = 9$

$\{\rho\} = \{9\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [3]	(5) [21]	(7) [21]	(9) [3]
{9}	$\frac{\sqrt{126}}{16}$	$\frac{\sqrt{84}}{16}$	$\frac{6}{16}$	$\frac{3}{16}$	$\frac{1}{16}$
{7}	$\frac{\sqrt{56}}{16}$	0	$-\frac{8}{16}$	$-\frac{10}{16}$	$-\frac{6}{16}$
{5}	$\frac{6}{16}$	$-\frac{\sqrt{24}}{16}$	$-\frac{\sqrt{56}}{16}$	$\frac{\sqrt{14}}{16}$	$\frac{\sqrt{126}}{16}$
{3}	$\frac{\sqrt{24}}{16}$	$-\frac{8}{16}$	0	$\frac{\sqrt{84}}{16}$	$-\frac{\sqrt{84}}{16}$
{1}	$\frac{\sqrt{14}}{16}$	$-\frac{\sqrt{84}}{16}$	$\frac{10}{16}$	$-\frac{7}{16}$	$\frac{3}{16}$

$\{\rho\} = \{9\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [111]	(5) [21]	(7) [21]	(9) [111]
{8}	$-\frac{\sqrt{14}}{16}$	$\frac{\sqrt{84}}{16}$	$\frac{10}{16}$	$-\frac{7}{16}$	$\frac{3}{16}$
{6}	$-\frac{\sqrt{24}}{16}$	$\frac{8}{16}$	0	$\frac{\sqrt{84}}{16}$	$-\frac{\sqrt{84}}{16}$
{4}	$-\frac{6}{16}$	$\frac{\sqrt{24}}{16}$	$-\frac{\sqrt{56}}{16}$	$\frac{\sqrt{14}}{16}$	$\frac{\sqrt{126}}{16}$
{2}	$-\frac{\sqrt{56}}{16}$	0	$-\frac{8}{16}$	$-\frac{10}{16}$	$-\frac{6}{16}$
{0}	$-\frac{\sqrt{126}}{16}$	$-\frac{84}{16}$	$\frac{6}{16}$	$\frac{3}{16}$	$\frac{1}{16}$

TABLE X (*contd.*)

$\{\rho\} = \{81\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [3]	(5) [21]	(7) [21]
{7}	$-\frac{\sqrt{5}}{8}$	$-\frac{\sqrt{27}}{8}$	$-\frac{5}{8}$	$-\frac{\sqrt{7}}{8}$
{5}	$-\frac{3}{8}$	$-\frac{\sqrt{15}}{8}$	$\frac{\sqrt{5}}{8}$	$\frac{\sqrt{35}}{8}$
{3}	$-\frac{\sqrt{15}}{8}$	$-\frac{1}{8}$	$\frac{\sqrt{27}}{8}$	$-\frac{\sqrt{21}}{8}$
{1}	$-\frac{\sqrt{35}}{8}$	$\frac{\sqrt{21}}{8}$	$-\frac{\sqrt{7}}{8}$	$\frac{1}{8}$

$\{\rho\} = \{81\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [111]	(5) [21]	(7) [21]
{8}	$-\frac{\sqrt{35}}{8}$	$\frac{\sqrt{21}}{8}$	$\frac{\sqrt{7}}{8}$	$-\frac{1}{8}$
{6}	$-\frac{\sqrt{15}}{8}$	$-\frac{1}{8}$	$-\frac{\sqrt{27}}{8}$	$-\frac{\sqrt{21}}{8}$
{4}	$-\frac{3}{8}$	$-\frac{\sqrt{15}}{8}$	$-\frac{\sqrt{5}}{8}$	$-\frac{\sqrt{35}}{8}$
{2}	$-\frac{\sqrt{5}}{8}$	$-\frac{\sqrt{27}}{8}$	$\frac{5}{8}$	$\frac{\sqrt{7}}{8}$

$\{\rho\} = \{72\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [3]	(5) [21]
{7}	$-\frac{\sqrt{10}}{4}$	$-\frac{\sqrt{5}}{4}$	$-\frac{1}{4}$
{5}	$-\frac{2}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{10}}{4}$
{3}	$-\frac{\sqrt{2}}{4}$	$\frac{3}{4}$	$-\frac{\sqrt{5}}{4}$

TABLE X (*contd.*)

$\{\rho\} = \{72\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [111]	(5) [21]
{6}	$\frac{\sqrt{2}}{4}$	$\frac{3}{4}$	$-\frac{\sqrt{5}}{4}$
{4}	$\frac{2}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{10}}{4}$
{2}	$\frac{\sqrt{10}}{4}$	$-\frac{\sqrt{5}}{4}$	$-\frac{1}{4}$

$\{\rho\} = \{63\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [3]
{5}	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
{3}	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

$\{\rho\} = \{63\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]	(3) [111]
{6}	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
{4}	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

$\{\rho\} = \{54\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]
{5}	1

$\{\rho\} = \{54\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(1) [21]
{4}	− 1

TABLE XI : $N = 10$

$\{\rho\} = \{10\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0) [3]	(2) [21]	(4) [21]	(6) [3]	(8) [21]	(10) [21]
{10}	$\frac{\sqrt{252}}{32}$	$\frac{\sqrt{420}}{32}$	$\frac{\sqrt{240}}{32}$	$\frac{\sqrt{90}}{32}$	$\frac{\sqrt{20}}{32}$	$\frac{\sqrt{2}}{32}$
{8}	$\frac{\sqrt{140}}{32}$	$\frac{\sqrt{84}}{32}$	$-\frac{\sqrt{48}}{32}$	$-\frac{\sqrt{338}}{32}$	$-\frac{\sqrt{324}}{32}$	$-\frac{\sqrt{90}}{32}$
{6}	$\frac{\sqrt{120}}{32}$	$\frac{\sqrt{8}}{32}$	$-\frac{\sqrt{224}}{32}$	$-\frac{\sqrt{84}}{32}$	$\frac{\sqrt{168}}{32}$	$\frac{\sqrt{420}}{32}$
{4}	$\frac{\sqrt{120}}{32}$	$-\frac{\sqrt{8}}{32}$	$-\frac{\sqrt{224}}{32}$	$\frac{\sqrt{84}}{32}$	$\frac{\sqrt{168}}{32}$	$-\frac{\sqrt{420}}{32}$
{2}	$\frac{\sqrt{140}}{32}$	$-\frac{\sqrt{84}}{32}$	$-\frac{\sqrt{48}}{32}$	$\frac{\sqrt{338}}{32}$	$-\frac{\sqrt{324}}{32}$	$\frac{\sqrt{90}}{32}$
{0}	$\frac{\sqrt{252}}{32}$	$-\frac{\sqrt{420}}{32}$	$\frac{\sqrt{240}}{32}$	$-\frac{\sqrt{90}}{32}$	$\frac{\sqrt{20}}{32}$	$-\frac{\sqrt{2}}{32}$

$\{\rho\} = \{10\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(2) [21]	(4) [21]	(6) [111]	(8) [21]	(10) [21]
{9}	$\frac{\sqrt{42}}{16}$	$-\frac{\sqrt{96}}{16}$	$\frac{9}{16}$	$\frac{\sqrt{32}}{16}$	$-\frac{\sqrt{5}}{16}$
{7}	$\frac{\sqrt{56}}{16}$	$-\frac{\sqrt{32}}{16}$	$-\frac{\sqrt{12}}{16}$	$-\frac{\sqrt{96}}{16}$	$\frac{\sqrt{60}}{16}$
{5}	$\frac{\sqrt{60}}{16}$	0	$-\frac{\sqrt{70}}{16}$	0	$-\frac{\sqrt{126}}{16}$
{3}	$\frac{\sqrt{56}}{16}$	$\frac{\sqrt{32}}{16}$	$-\frac{\sqrt{12}}{16}$	$\frac{\sqrt{96}}{16}$	$\frac{\sqrt{60}}{16}$
{1}	$\frac{\sqrt{42}}{16}$	$\frac{\sqrt{96}}{16}$	$\frac{9}{16}$	$-\frac{\sqrt{32}}{16}$	$-\frac{\sqrt{5}}{16}$

TABLE XI (*contd.*)

$\{\rho\} = \{91\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(2) [21]	(4) [21]	(6) [3]	(8) [21]
{8}	$-\frac{\sqrt{56}}{16}$	$-\frac{\sqrt{112}}{16}$	$-\frac{\sqrt{72}}{16}$	$-\frac{4}{16}$
{6}	$-\frac{\sqrt{72}}{16}$	$-\frac{4}{16}$	$\frac{\sqrt{56}}{16}$	$\frac{\sqrt{112}}{16}$
{4}	$-\frac{\sqrt{72}}{16}$	$\frac{4}{16}$	$\frac{\sqrt{56}}{16}$	$-\frac{\sqrt{112}}{16}$
{2}	$-\frac{\sqrt{56}}{16}$	$\frac{\sqrt{112}}{16}$	$-\frac{\sqrt{72}}{16}$	$\frac{4}{16}$

$\{\rho\} = \{91\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0)* [111]	(2) [21]	(4) [21]	(6) [111]	(8) [21]
{9}	$\frac{\sqrt{70}}{16}$	$\frac{\sqrt{112}}{16}$	$-\frac{\sqrt{56}}{16}$	$\frac{4}{16}$	$\frac{\sqrt{2}}{16}$
{7}	$\frac{\sqrt{40}}{16}$	$\frac{4}{16}$	$\frac{\sqrt{32}}{16}$	$-\frac{\sqrt{112}}{16}$	$-\frac{\sqrt{56}}{16}$
{5}	$\frac{6}{16}$	0	$\frac{\sqrt{80}}{16}$	0	$\frac{\sqrt{140}}{16}$
{3}	$\frac{\sqrt{40}}{16}$	$-\frac{4}{16}$	$\frac{\sqrt{32}}{16}$	$\frac{\sqrt{112}}{16}$	$-\frac{\sqrt{56}}{16}$
{1}	$\frac{\sqrt{70}}{16}$	$-\frac{\sqrt{112}}{16}$	$-\frac{\sqrt{56}}{16}$	$-\frac{4}{16}$	$\frac{\sqrt{2}}{16}$

$\{\rho\} = \{82\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0) [3]	(2) [21]	(4) [21]	(6) [3]
{8}	$\frac{\sqrt{20}}{8}$	$-\frac{\sqrt{30}}{8}$	$-\frac{\sqrt{12}}{8}$	$-\frac{\sqrt{2}}{8}$
{6}	$\frac{\sqrt{12}}{8}$	$-\frac{\sqrt{2}}{8}$	$\frac{\sqrt{20}}{8}$	$\frac{\sqrt{30}}{8}$
{4}	$\frac{\sqrt{12}}{8}$	$\frac{\sqrt{2}}{8}$	$\frac{\sqrt{20}}{8}$	$-\frac{\sqrt{30}}{8}$
{2}	$\frac{\sqrt{20}}{8}$	$\frac{\sqrt{30}}{8}$	$-\frac{\sqrt{12}}{8}$	$\frac{\sqrt{2}}{8}$

TABLE XI (*contd.*)

$\{\rho\} = \{82\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(2) [21]	(4) [21]	(6) [111]
{7}	$-\frac{\sqrt{20}}{8}$	$\frac{\sqrt{32}}{8}$	$\frac{\sqrt{12}}{8}$
{5}	$-\frac{\sqrt{24}}{8}$	0	$-\frac{\sqrt{40}}{8}$
{3}	$-\frac{\sqrt{20}}{8}$	$-\frac{\sqrt{32}}{8}$	$\frac{\sqrt{12}}{8}$

$\{\rho\} = \{73\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(2) [21]	(4) [21]
{6}	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
{4}	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$

$\{\rho\} = \{73\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0)* [111]	(2) [21]	(4) [21]
{7}	$\frac{\sqrt{6}}{4}$	$-\frac{\sqrt{8}}{4}$	$\frac{\sqrt{2}}{4}$
{5}	$\frac{2}{4}$	0	$-\frac{\sqrt{12}}{4}$
{3}	$\frac{\sqrt{6}}{4}$	$\frac{\sqrt{8}}{4}$	$\frac{\sqrt{2}}{4}$

$\{\rho\} = \{64\}$, $[f_a] = [2]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0) [3]	(2) [21]
{6}	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
{4}	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$

$\{\rho\} = \{64\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(2) [21]
{5}	1

$\{\rho\} = \{55\}$, $[f_a] = [11]$

$\{\bar{\rho}\}$ \ (ν) [f]	(0)* [111]
{5}	1

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