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THERMO-ELASTIC-PLASTIC TRANSITION IN ROTATING DISKS WITH STEADY STATE TEMPERATURE

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Seth's transition theory is applied to the problem of rotating disks with steady state temperature. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. If the additional condition of incompressibility is imposed, then the expressions for stresses correspond to those arising from Tresca yield criterion.

1. INTRODUCTION

Seth's transition theory has been successfully applied to a large number of problems in plasticity and creep (Seth 1964, 1966, 1972; Hulsurkar 1966; Borah 1970). The special feature of the transition theory is that it gives the results without assuming a yield criteria and a flow rule. Seth (1964) has shown that such assumptions are unnecessary and easily follow from the transition theory approach. Borah (1970) has solved the problem of thermo-elastic-plastic transitions in shells under uniform pressure. In this paper we derive the expressions for stresses in case of thermo-elastic-plastic transitions in rotating disks with steady state temperature. The expressions reduce to those arising from Tresca yield criteria if additional condition of incompressibility is imposed.

2. TRANSITION POINTS

Consider a circular disk of isotropic and homogeneous material with a cenral hole and rotating with an angular velocity ω. Let the internal and external radii of the disk be a and b respectively. We assume that a steady state temperature θ_0 is applied on the internal surface of the disk. We shall consider only the steady state deformation case and assume that the plane strain conditions exist. Above assumptions allows us to take the displacements, in cylindrical coordinates (r, ϕ, z), as

$$u = r(1 - \beta), \; v = 0, \; w = 0 \qquad \text{...(2.1)}$$

where $\beta = \beta(r)$.

The finite strains are given by (Seth 1964)

$$\left.\begin{aligned} e_{rr} &= \tfrac{1}{2}\{1 - (r\beta' + \beta)^2\} \\ e_{\phi\phi} &= \tfrac{1}{2}(1 - \beta^2) \\ e_{zz} &= e_{r\phi} = e_{\phi z} = e_{rz} = 0. \end{aligned}\right\} \quad \text{...(2.2)}$$

The stress-strain relations are

$$T_{ij} = \lambda \triangle \delta_{ij} + 2\mu e_{ij} - \xi\theta\delta_{ij} \quad \text{...(2.3)}$$

where λ, μ are Lame's constant, $\triangle = e_{ii}$, $\xi = \alpha(3\lambda + 2\mu)$ (α is the coefficient of thermal expansion) and θ is the rise of temperature. Further θ has to satisfy

$$\theta_{,ii} = 0. \quad \text{...(2.4)}$$

Substituting the expressions for strains from eqn. (2.2) in eqn. (2.3), we get

$$\left.\begin{aligned} T_{rr} &= \lambda \triangle + \mu\{1 - (r\beta' + \beta)^2\} - \xi\theta \\ T_{\phi\phi} &= \lambda \triangle + \mu(1 - \beta^2) - \xi\theta \\ T_{zz} &= \lambda \triangle - \xi\theta \\ T_{r\phi} &= T_{\phi z} = T_{rz} = 0 \end{aligned}\right\} \quad \text{...(2.5)}$$

where $\beta' = \dfrac{d\beta}{dr}$ and $\triangle = \tfrac{1}{2}\{2 - \beta^2 - (r\beta' + \beta)^2\}$.

The equation of equilibrium to be satisfied is

$$\frac{\partial T_{rr}}{\partial r} + \frac{T_{rr} - T_{\phi\phi}}{r} + \rho r\omega^2 = 0 \quad \text{...(2.6)}$$

where ρ is the density of the material.

The temperature field satisfying eqn. (2.4) and

$$\begin{aligned} \theta &= \theta_0 \quad \text{at} \quad r = a \\ &= 0 \quad \text{at} \quad r = b \end{aligned}$$

where θ_0 is constant, is given by

$$\theta = \theta_0 \frac{\log(r/b)}{\log(a/b)}. \quad \text{...(2.7)}$$

Equations (2.5), (2.6) and (2.7) lead to the following integro-differential equation for β

$$(r\beta' + \beta)^2 + (1 - c)\beta^2 + c\int \frac{\{(r\beta' + \beta)^2 - \beta^2 - (\xi\bar{\theta}_0/2\mu)\}}{r}\,dr - \frac{c\rho\omega^2 r^2}{2\mu} = K^2 \quad \text{...(2.8)}$$

where $c = 2\mu/(\lambda + 2\mu)$, $\bar{\theta}_0 = \theta_0/\log(a/b)$ and K^2 is a constant.

The substitutions (Seth 1963)

$$\frac{c\rho\omega^2}{2\mu} = \omega_0^2,\ s = \frac{r\omega_0}{K},\ y = \frac{\beta}{K},\ s = e^t, \qquad ...(2.9)$$

in eqn. (2.8) followed by the differentiation with respect to t lead to

$$(y + y')(y' + y'') + yy' + \frac{c\xi\bar{\theta}_0}{2\mu K^2} + \frac{c}{2} y'^2 = e^{2t}. \qquad ...(2.10)$$

Further substitutions (Seth 1963)

$$y = sP,\ P' + P = FP,\ P^{-2} = T \qquad ...(2.11)$$

simplifies eqn. (2.10) to yield

$$\left\{T - 2F - \left(2 + \frac{c}{2}\right)F^2 - F^3 - \frac{c\xi\bar{\theta}_0}{2\mu K^2}\left(\frac{T}{s^2}\right)\right\}\frac{dT}{dF} - 2T(1 - F^2) = 0. \qquad ...(2.12)$$

This shows that the transition points of T are

$$F = \pm 1,\ F = \pm \infty.$$

3. Thermo-Elastic-Plastic Transitions

We define the transition function R by

$$R = \{T_{\phi\phi} - \xi\bar{\theta}_0 \log(r/b)\}/\mu \qquad ...(3.1)$$

and use the transition point $F = \pm \infty$.

From eqns. (2.5) and (3.1), we get

$$R = \frac{K^2}{c} y^2 \left\{\frac{2 - c}{K^2} \cdot \frac{T}{s^2} - 1 - (1 - c)(1 + F)^2 \right\} \qquad ...(3.2)$$

after using the substitutions from eqns. (2.9) and (2.11).

Logarithmic differentiation of eqn. (3.2) and use of eqn. (2.10) leads to

$$\frac{d \log R}{d \log s} = \frac{-2F + 2(1 - c)\{T - F - (c/2)F^2 - (c\xi\bar{\theta}_0/2\mu)(T/s^2)\}}{\{(2 - c)/K^2\}(T/s^2) - 1 - (1 - c)(1 + F)^2}$$

which gives

$$\frac{d \log R}{d \log s} = c \quad \text{as} \quad F \to \pm \infty. \qquad ...(3.3)$$

Equations (3.1) and (3.3) give

$$T_{\phi\phi} = \xi\bar{\theta}_0 \log \frac{r}{b} + \left(\frac{2 - c}{3 - 2c}\right) \Upsilon Y r^c \qquad ...(3.4)$$

where γ is a constant and the relation $\mu = \{(2 - c)/(3 - 2c)\}\ Y$, Y the yield stress (Seth 1964) is utilized.

Equations (2.6) and (3.4) give

$$T_{rr} = \xi\bar{\theta}_0\left(\log\frac{r}{b} - 1\right) + \left(\frac{2-c}{3-2c}\right)\gamma Y\frac{r^c}{c+1} - \frac{\rho r^2\omega^2}{2} + \frac{B}{r} \quad \ldots(3.5)$$

where B is a constant.

Now the boundary conditions are

$$\left.\begin{aligned} T_{rr} &= 0 \quad \text{at} \quad r = a \\ &= \sigma_0 \quad \text{at} \quad r = b \end{aligned}\right\} \quad \ldots(3.6)$$

where σ_0 is the stress acting at the outer edge due to the external load.

The first of the boundary conditions (3.6) determines the constant B in eqn. (3.5) and is given by

$$B = -a\left\{\xi\bar{\theta}_0\left(\log\frac{a}{b} - 1\right) + \left(\frac{2-c}{3-2c}\right)\gamma Y\frac{a^c}{c+1} - \frac{\rho a^2\omega^2}{3}\right\}. \quad \ldots(3.7)$$

The boundary condition (3.6) leads to the following expression for critical angular velocity ω_1 at which the disk starts yielding :

$$\omega_1^2 = \frac{3}{\rho b^2}\left\{\left(\frac{2-c}{3-2c}\right)\gamma Y\frac{b^c}{c+1} + \frac{B}{b} - \xi\bar{\theta}_0 - \sigma_0\right\} \quad \ldots(3.8)$$

where B is given by eqn. (3.7).

The stresses given by eqns. (3.4), (3.5), (3.7) and (3.8) give the transition stresses. These also describe the behaviour of the materials which are not incompressible in the plastic state. For incompressible materials the expressions for stresses in fully plastic state are obtained by allowing $c \to 0$ in the above expressions. Therefore, the stresses in fully plastic state are

$$\left.\begin{aligned} T_{rr} &= \xi_0\bar{\theta}_0\left(\log\frac{r}{b} - 1\right) + \tfrac{2}{3}\gamma Y - \frac{\rho r^2\omega^2}{3} + \frac{B}{r} \\ T_{\phi\phi} &= \xi_0\bar{\theta}_0\log\frac{r}{b} + \tfrac{2}{3}\gamma Y \end{aligned}\right\} \quad \ldots(3.9)$$

where

$$B = -a\left\{\xi_0\bar{\theta}_0\left(\log\frac{a}{b} - 1\right) + \tfrac{2}{3}\gamma Y - \frac{\rho a^2\omega^2}{3}\right\}$$

$$\xi_0 = \lim_{c\to 0}\alpha(3\lambda + 2\mu).$$

It can be shown that ξ_0 is finite (Borah 1970). Note that these results correspond to Tresca yield condition.

The critical angular velocity ω_1^* is given by

$$(\omega_1^*)^2 = \frac{3}{\rho b^2}\left\{\tfrac{2}{3}\gamma Y + \frac{B}{b} - \xi_0\bar{\theta}_0 - \sigma_0\right\} \qquad \ldots(3.10)$$

with B given by eqn. (3.9). This gives the relation

$$(\omega_1^*)^2 = (\varpi_1)^2 - \frac{3}{\rho b^2}\xi_0\bar{\theta}_0 \qquad \ldots(3.11)$$

where ϖ_1 is the critical velocity in the absence of outcoming heat. It should be recalled that the expression for yield stress Y is qualitative and not exactly $(E/2)$ (Seth 1964). Hence in above expressions γY should be taken as yield stress. Equation (3.11) shows that the presence of the temperature hastens the yield. It also shows that the critical velocity ω_1^* depends on the radii of the disk and that the disk wtih small external radius yields much faster than the disk with larger external radius under similar conditions.

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