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CURRENT CARRYING NON-NEWTONIAN FLUID PAST A NON-CONDUCTING SPHERE AT LOW REYNOLDS NUMBER

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In this paper the problem of non-conducting sphere moving slowly through a current carrying incompressible non-Newtonian fluid is studied. The problems on hydromagnetics are involved and simplifying plausible assumptions are made to obtain mathematical solutions. The method of matched asymptotic expansion is applied.

The drag coefficient is modified over that of Casewell and Schwarz where they have considered simply the non-conducting non-Newtonian incompressible fluid.

1. INTRODUCTION

Chow and Billings (1967) investigated the problem of creeping flow around a non-conducting sphere moving slowly through a Newtonian fluid carrying an otherwise uniform current. It was found that the effect of current is to hasten flow separation and to enlarge the reverse flow region. Stokes' and Oseen's expansions appropriate for the flow near and far from the sphere are constructed and matched as done by Proudman and Pearson (1957). Casewell and Schwarz (1962) have discussed the problem of slow flow stress equations for an incompressible Erickson and Rivlin (1955) fluid past a sphere by matching Stokes' and Oseen's expansions by the same method.

The aim of this investigation is to discuss the problem of Chow and Billings for Rivlin-Erickson fluid carrying an otherwise uniform current. For the case of the sphere polar coordinates (R, θ, ϕ) are chosen with the origin at the centre of the sphere and $\theta = 0$ in the upstream direction. It is convenient to write $\mu = \cos\theta$. All tensor quantities are expressed in terms of their physical components. Equations of continuity, motion and stress are first expressed in terms of Stokes variables given by

$$\left.\begin{aligned} & R = ar,\ U_r = Uu_r,\ U_\mu = Uu_\mu,\ T_{ik} = \frac{U\eta_0}{a}\, t_{ik} \\ & P = \frac{\eta_0 U}{a}\, p \end{aligned}\right\} \quad \ldots(1.1)$$

where a is the radius of the sphere, $\phi_1 = \eta_0$ the zero shear viscosity and U is the velocity of the uniform stream.

The stream function ψ can be obtained to satisfy the continuity equation

$$\frac{\partial}{\partial r}(r^2 u_r) + r\frac{\partial}{\partial \mu}(1-\mu^2)^{1/2} u_\mu = 0 \qquad ...(1.2)$$

giving

$$u_r = -\frac{1}{r^2}\frac{\partial\psi}{\partial\mu},\ u_\mu = \frac{1}{r(1-\mu^2)^{1/2}}\frac{\partial\psi}{\partial r}. \qquad ...(1.3)$$

Now consider the modified Navier-Stokes equation in non-dimensional coordinates following Chow and Billings (1967) and Casewell and Schwarz (1962).

$$\frac{1}{r^2}\frac{\partial(\psi, D^2\psi)}{\partial(r,\mu)} + \frac{2D^2\psi L\psi}{r^2} = \frac{1}{R_e}D^4\psi + \frac{1}{R_e}\Big[\frac{\partial}{\partial r}\Big\{\frac{1}{r^2}\frac{\partial}{\partial r}(r^3(1-\mu^2)^{1/2}\tau_{r\mu})$$
$$+ (1-\mu^2)^{1/2}\frac{\partial}{\partial\mu}((1-\mu^2)^{1/2}\tau_{\mu\mu}) + \mu\tau_{\phi\phi}\Big\}$$
$$- \frac{1-\mu^2}{r}\frac{\partial}{\partial\mu}\Big\{\frac{1}{r}\frac{\partial}{\partial r}(r^2\tau_{rr})$$
$$+ \frac{\partial}{\partial\mu}((1-\mu^2)^{1/2}\tau_{r\mu}) - \tau_{\mu\mu} - \tau_{\phi\phi}\Big]$$
$$+ \tfrac{3}{2}R_h\mu(1-\mu^2)\frac{1}{r^2}\left(1-\frac{1}{r^3}\right) \qquad ...(1.4)$$

where

$$\left.\begin{aligned} D^2 &= \frac{\partial^2}{\partial r^2} + \frac{1-\mu^2}{r^2}\frac{\partial^2}{\partial\mu^2},\ L = \frac{\mu}{1-\mu^2}\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial}{\partial\mu} \\ R_e &= \frac{Uad}{\eta_0}\ \text{(Reynolds number)} \\ R_h &= \frac{\mu_e J_0^2 a^2}{dU^2},\ \text{the magnetic pressure number} \end{aligned}\right\} \qquad ...(1.5)$$

μ_e being the magnetic permeability and J_0 the uniform current. The stress components $\tau_{r\mu}$, $\tau_{\mu\mu}$, etc. represent the non-Newtonian part of the stress matrix which is expressed in new variables as

$$T = \lambda(A_1^2 + \epsilon A_2) + \lambda^2[\epsilon_2 t_r(A_1)^2 A_1 + \epsilon_3 A_3 + \epsilon_4(A_1A_2 + A_2A_1)] \qquad ...(1.6)$$

where

$$\lambda = \frac{\phi_3 U}{a\eta_0},\ \epsilon_1 = \frac{\phi_2}{\phi_3},\ \epsilon_3 = \frac{\phi_6\eta_0}{\phi_3^2},\ \epsilon_3 = \frac{\eta_0\phi_4}{\phi_3^2},\ \epsilon_4 = \frac{\phi_5\eta_0}{\phi_3^2} \qquad ...(1.7)$$

Following Proudman and Pearson (1957) the stream function can be represented for small Reynolds number in the following form:

$$\psi(r,\mu) = \psi_0(r,\mu) + f_1(R_e)\psi_1(r,\mu) + \dots \qquad ...(1.8)$$

where

$$\frac{f_{n+1}(R_e)}{f_n(R_e)} \to 0 \quad \text{as } R_e \to 0. \qquad \ldots(1.9)$$

Substituting (1.8) in (1.4) we have for ψ_0

$$\begin{aligned} 0 = D^4\psi_0 + \Big\{\frac{\partial}{\partial r}\Big[\frac{1}{r^2}\frac{\partial}{\partial r}\,(r^3(1-\mu^2)^{1/2}\,\tau^0_{r\mu}) \\ + (1-\mu^2)^{1/2}\frac{\partial}{\partial \mu}\,(1-\mu^2)^{1/2}\,\tau^0_{\mu\mu}) \\ + \mu\tau^0_{\phi\phi}\Big] - \frac{1-\mu^2}{r}\frac{\partial}{\partial\mu}\Big\{\frac{1}{r}\frac{\partial}{\partial r}\,(r^2\tau^0_{rr}) \\ + \frac{\partial}{\partial\mu}\,((1-\mu^2)^{1/2}\,\tau^0_{r\mu}) - \tau^0_{\mu\mu} - \tau^0_{\phi\phi}\Big\} \end{aligned} \qquad \ldots(1.10)$$

where $\tau^0_{r\mu}$ etc. can be evaluated with the help of (1.6) in terms of ψ_0. Main aim of this problem is to obtain the solution of eqn. (1.10) which has been attempted in the next section.

Now we define Oseen's variables by

$$\rho = R_e r \quad \text{and} \quad \Psi = R_e^2\,\psi. \qquad \ldots(1.11)$$

Hence in terms of Oseen's variables eqn. (1.4) becomes

$$\frac{1}{\rho^2}\frac{\partial(\Psi, D^2\Psi)}{\partial(\rho,\mu)} + \frac{2D^2\Psi L\Psi}{\rho^2} = D^4\Psi + \tfrac{3}{2}\,R_h R_e \mu(1-\mu^2)\frac{1}{\rho^2} + R_e\,(\text{non-Newtonian terms}) \qquad \ldots(1.12)$$

where D^2 and L are the same operators as in (1.4) but ρ is substituted for r. We construct the Oseen's expansion of the form

$$\Psi = \Psi_0(\rho,\mu) + F_1(R_e)\,\Psi_1(\rho,\mu) + F_2(R_e)\,\Psi_2(\rho,\mu) + \ldots \qquad \ldots(1.13)$$

where

$$\frac{F_{n+1}(R_e)}{F_n(R_e)} \to 0 \quad \text{as} \quad R_e \to 0 \quad \text{and} \quad \rho \text{ is fixed} \qquad \ldots(1.14)$$

where $F_1(R_e) = R_e$ and the leading term must be uniform stream i.e.

$$\Psi_0(\rho,\mu) = \tfrac{1}{2}\,\rho^2(1-\mu^2). \qquad \ldots(1.15)$$

Now substituting for Ψ in (1.12) we have for Ψ_1

$$\left(D^2 - \mu\frac{\partial}{\partial\rho} - \frac{1-\mu^2}{\rho}\frac{\partial}{\partial\mu}\right)D^2\Psi_1 = -\tfrac{3}{2}\,R_h\rho^{-2}\mu(1-\mu^2). \qquad \ldots(1.16)$$

Thus we find that the flow at infinity is not influenced by nonlinear terms. This result is compatible with the tendency of non-Newtonian conducting fluids towards Newtonian behaviour for conducting fluids. The solution of (1.16) is found to be

$$\Psi_1 = \tfrac{1}{4} R_h(1-\mu^2)\,\rho + C_1(1+\mu)\{1-\exp(-\tfrac{1}{2}\rho(1-\mu)\} \qquad ...(1.17)$$

the last term in (1.17) being the classical Oseen's solution and C_1 being undetermined. Following Chow and Billings (1967) we find that

$$C_1 = -\tfrac{1}{2}(3 + R_h).$$

Hence the two terms of Oseen's expansion when rewritten in Stokes' variable can be expressed as

$$\psi \sim \tfrac{1}{2}\left(r^2 + \frac{R_h}{2} r\right)(1-\mu^2) - \tfrac{1}{2}(3+R_h)\frac{1}{R_e}(1+\mu)$$
$$\times \{1 - \exp(-\tfrac{1}{2} rR_e(1-\mu))\} \qquad ...(1.18)$$

as $R_e \to 0$, and this shows the disturbance field generated by the current and sphere at large distances.

2. The Leading Terms in the Stokes' Expansion

Following Casewell and Schwarz (1962) and Proudman and Pearson (1957) we have

$$\psi_0 = \chi_{00} + \lambda\chi_{01} + \lambda^2\chi_{02} + \cdots \qquad ...(2.1)$$

where $\lambda = \phi_2 U/a\eta_0 < 1$ and ϵ_i's are assumed to be of order unity whereas the non-Newtonian stress can be expressed as matrix representation

$$T^0 = \lambda\tau^{01} + \lambda^2\tau^{02} + \ldots \qquad ...(2.2)$$

with components

$$\tau^0_{rr} = \lambda\tau^{01}_{rr} + \lambda^2\tau^{02}_{rr} + \ldots \text{ etc.} \qquad ...(2.3)$$

Substituting expansions (2.1) and (2.2) in (1.10) and equating coefficients of each power of λ we get

$$D^2\chi_{00} = 0 \qquad ...(2.4)$$

$$0 = D^2\chi_{01} + \frac{\partial}{\partial r}\left\{\frac{1}{r^2}(r^3(1-\mu^2)^{1/2}\tau^{01}_{r\mu})\right.$$
$$+ (1-\mu^2)^{1/2}\frac{\partial}{\partial\mu}((1-\mu^2)^{1/2}\tau^{01}_{\mu\mu})$$
$$\left.+ \mu\tau^{01}_{\phi\phi}\right\} - \frac{1-\mu^2}{r}\frac{\partial}{\partial\mu}\left\{\frac{1}{r}\frac{\partial}{\partial r}(r^2\tau^{01}_{rr})\right.$$
$$\left.+\frac{\partial}{\partial\mu}((1-\mu^2)^{1/2}\tau^{01}_{r\mu}) - \tau^{01}_{\mu\mu} - \tau^{01}_{\phi\phi}\right\} \text{ etc.} \qquad ...(2.5)$$

The solution of (2.4) satisfying the no-slip condition at the surface and matched with the uniform stream is the well-known Stokes' solution

$$\chi_{00} = \tfrac{1}{2}\left(r^2 - \frac{3r}{2} + \frac{1}{2r}\right)(1-\mu^2). \qquad \ldots(2.6)$$

Equation (2.4) in this case is the same as in Casewell and Schwarz (1962) and χ_{00} has already been fully matched with the free stream and non-Newtonian terms should not contribute to Ψ_0. This also satisfies the requirement of Oseen's equation for Newtonian conducting behaviour at infinity. Thus following Casewell and Schwarz (1962) we have

$$\chi_{01} = -\tfrac{3}{8}(1+\epsilon_1)\left(1-\frac{1}{r}\right)^3 \mu(1-\mu^2)$$

satisfying the no slip condition at $r = 1$ and which does not contribute to ψ_0.

TABLE I

I	II	III	IV	V	VI
b_6	14	35	0	− 7	− 54
b_7	− 174	− 216	54	21/2	475/4
b_8	370	400	0	18	201/4
b_9	− 106	− 174	− 168	− 61/2	− 869/4
b_{10}	− 250	− 165	0	0	76
b_{11}	144	120	60	9	93/4
b_{12}	0	0	0	0	35/4
b_{13}	0	0	68	0	13
c_6	− 16	− 40	0	8	243/4
c_7	198	243	− 54	− 45/4	− 126
c_8	− 420	− 885/2	0	− 20	− 285/4
c_9	167	204	165	127/4	1943/8
c_{10}	162	138	0	0	− 459/8
c_{11}	− 87	− 102	− 42	− 17/2	− 54
c_{12}	0	0	0	0	− 5/2
c_{13}	0	0	− 30	0	− 45/4

$$\text{I} = (27/4)(1+\epsilon_1)\,\text{II} + (27/2)\,\epsilon_1(1+\epsilon_1)\,\text{III} + (27/2)(\epsilon_2+\epsilon_4)\,(\text{IV}) + 27\epsilon_4\,(\text{V}) + 6\epsilon_3\,(\text{VI}).$$

Obviously the equation for χ_{02} is analogous to (2.4) and contains the components τ_{rr}^{02} etc. of the non-Newtonian stress matrix.

$$T^{02} = A_1^{01} A_1^{00} + A_1^{00} A_1^{01} + \epsilon_1 A_2^{01} A_2^{00} + \epsilon_2 t_r (A_1^{00})^2 A_1^{00}$$
$$+ \epsilon_3 A_3^{00} + \epsilon_4 (A_1^{00} A_2^{00} + A_2^{00} A_1^{00}) \quad ...(2.7)$$

where A_1^{01}, A_1^{00}, A_2^{00} are evaluated in terms of χ_{00} and χ_{01}. Naturally the resulting differential equation for χ_{02} is

$$D^4 \chi_{02} = \sum_{n=6}^{13} [b_n + c_n(1 - \mu^2)]\, r^{-n}(1 - \mu^2) \quad ...(2.8)$$

where the coefficients b_n and c_n are given in Table I. The solution satisfying the no-slip condition at the surface and not contributing to free stream is

$$\chi_{02} = [\{(1 - \mu^2) - \tfrac{4}{5}\}\, \delta r^{-3} \log r$$
$$+ \sum_{\substack{m=-1 \\ m \neq 0}}^{9} \{\beta_m + \chi_m(1 - \mu^2)\}\, r^{-m}]\,(1 - \mu^2) \quad ...(2.9)$$

where β_m, δ, χ_m are shown in Table II.

TABLE II

I	II	III	IV	V	VI
δ	− 1.5714	− 1.9286	0.4286	0.0893	1.0000
β_{-1}	0.2419	0.2983	− 0.0431	− 0.0138	− 0.4570
β_1	− 0.5148	− 0.5568	0.1171	0.0049	1.5364
β_2	− 0.2500	− 0.6250	0	0.1250	− 0.5812
β_3	− 0.6948	− 0.3932	0.0947	− 0.1526	− 0.5129
β_4	1.4472	1.5389	0	0.0694	0.2281
β_5	− 0.1504	− 0.2132	− 0.1905	− 0.0354	− 0.2605
β_6	− 0.1150	− 0.0808	0	0	0.0362
β_7	0.0359	0.0319	0.0153	0.0024	0.0084
β_8	0	0	0	0	0.0012
β_9	0	0	0.0064	0	0.0013
χ_{-1}	0	0	0	0	0
χ_1	0.0392	− 0.0611	− 0.0400	0.0321	− 0.3624
χ_2	0.3333	0.8333	0	− 0.1667	− 0.1266
χ_3	1.1019	0.7670	− 0.1528	0.1802	0.5285
χ_4	− 1.7500	− 1.8437	0	− 0.0833	− 0.2969
χ_5	0.2109	0.2576	0.2083	0·0401	0.3067
χ_6	0.0900	0.0767	0	0	− 0.0319
χ_7	− 0.0253	− 0.0297	− 0.0122	− 0.0025	− 0.0157
χ_8	0	0	0	0	− 0.0004
χ_9	0	0	− 0.0032	0	− 0.0012

$$\text{I} = (27/4)\,(1 + \epsilon_1)\,(\text{II}) + (27/2)\,(1 + \epsilon_1)\,\epsilon_1\,(\text{III}) + (27/2)\,(\epsilon_2 + \epsilon_4)\,(\text{IV})$$
$$+ 27\,\epsilon_4\,(\text{V}) + 6\,\epsilon_3\,(\text{VI}).$$

3. Higher Terms in Oseen's and Stokes' Expansion

As we have seen that only the Newtonian terms are involved when matching procedure is adopted for inner and outer expansions, hence the solution to Oseen's equation will be as given by Chow and Billings (1967).

$$\Psi_1 = \tfrac{1}{4} R_h(1 - \mu^2)\, \rho - \tfrac{3}{2}\left(1 + \frac{R_h}{3}\right)(1 + \mu)\,\{1 - \exp\,(-\tfrac{1}{2}\rho(1 - \mu)\}. \qquad \ldots(3.1)$$

The coefficients $f_1(R_e) = R_e$ in the Stokes' expansion and the same procedure has been adopted for $F_1(R_e)$ in Oseen's expansion. The governing equation for Ψ_1 yields

$$D^2\Psi_1 + o(\lambda) = -\mu(1 - \mu^2)\left[\tfrac{9}{2}\left(\frac{1}{r^2} - \frac{3}{2r^3} + \frac{1}{2r^5}\right) + \frac{3R_h}{2r^2}\left(1 - \frac{1}{r^3}\right)\right] + o(\lambda). \qquad \ldots(3.2)$$

The terms $o(\lambda)$ on the left-hand side of the above equation are evaluated from the non-Newtonian matrix (1.4) making use of $\psi_0 = \chi_{00} + \lambda\chi_{01}$ and ψ_1, thus we find that the terms on the left-hand side will be linear in the derivatives of Ψ_1 with coefficients of r and μ. Similarly the terms on the right-hand side are evaluated from inertial terms of $o(\lambda)$ and obtained from χ_{01}.

Here the terms of $o(\lambda)$ in ψ_1 will be neglected by imposing the restriction

$$R_e \ll \lambda^2 \ll \lambda \qquad \ldots(3.3)$$

which is equivalent to

$$a^2 \ll \frac{\phi_3}{d} \qquad \ldots(3.4)$$

d being the density of the fluid. Condition (3.4) can be achieved by taking the radius of the sphere to be small.

Under these restrictions (3.2) reduces to Newtonian conducting case. Its solution properly matched with ψ_1 is given by Chow and Billings as

$$\chi_{10} = C_2\left(2r^2 - 3r + \frac{1}{r}\right)(1 - \mu^2) - \frac{3}{32}\left(2r^2 - 3r + 1 - \frac{1}{r} + \frac{1}{r^2}\right)\mu(1 - \mu^2) - \frac{R_h}{16}\left(r^2 - \tfrac{5}{2} + \frac{1}{r} + \frac{1}{2r^2}\right)\mu(1 - \mu^2) \qquad \ldots(3.5)$$

where $C_2 = \frac{1}{32}(3 + R_h)$ and ψ_1 has been put in the form

$$\psi_1 = \chi_{01} + \lambda\chi_{11} + \ldots .$$

4. The Stream Function

Now the stream function has been determined in the form

$$\psi = \chi_{00} + \lambda\chi_{01} + \lambda^2\chi_{02} + o(\lambda^3) + R_e\chi_{10} + o(\lambda R_e) + \ldots, \qquad \ldots(4.1)$$

or from (2.5), (2.6), (2.9) and (3.5)

$$\begin{aligned}\psi = (1-\mu^2)\Big[\tfrac{1}{4}(r-1)^2 \Big\{&(1 + \tfrac{1}{8}(3+R_h)R_e)\left(2+\frac{1}{r}\right) \\ &- \tfrac{3}{8}\left(2+\frac{1}{r}+\frac{1}{r^2}\right)\mu R_e \\ &+ \frac{2R_h}{3}\left(1+\frac{2}{r}+\frac{2}{r^2}\right) - \frac{3\lambda}{2r^3}(1+\epsilon_1)(r-1)\mu\Big\} \\ &+ \lambda^2\Big\{((1-\mu^2)-\tfrac{4}{5})\,\delta r^{-3}\log r + \sum_{\substack{m=-1\\ m\neq 0}}^{9}\left(\beta_m + \chi_m(1-\mu^2)\,r^{-m}\right)\Big\}\Big]\end{aligned} \qquad \ldots(4.2)$$

The stream function $\psi = 0$ consists of the surface of the sphere, the axis of symmetry and the surface

$$\mu = \frac{\left(\frac{8}{3R_e} + 1 + \frac{R_h}{3}\right)(2r^3 + r^2)}{2r^3\left(1+\frac{R_h}{3}\right) + \left(1 + \frac{4R_h}{3}\right)r^2 + (1 + \frac{1}{3}R_h)\,r + \frac{4\lambda}{R_e}(1+\epsilon_1)(r-1)} \qquad \ldots(4.3)$$

which is the approximated boundary of the standing eddies. The term in λ^2 was dropped from (4.2) to obtain (4.3) because of the restrictions on it. The minimum of (4.3) occurs when

$$\frac{2R_h}{3}r^3 + 4r^2\left(\alpha + 1 + \frac{R_h}{3}\right) + r\left(1 - 5\alpha + \frac{R_h}{3}\right) - 2\alpha = 0 \qquad \ldots(4.4)$$

where $\alpha = \frac{4}{R_e}(\epsilon_1 + 1)$.

The equation giving the end of Eddies lying downstream is given by

$$2r^3 + r^2\left(1 - R_h\frac{3R_e}{8}\right) - \frac{3R_e}{8}r\left(1 + \alpha + \frac{R_h}{3}\right) + \frac{3R_e}{8}\alpha = 0. \quad \ldots(4.5)$$

5. The Drag Force on the Sphere

The fluid will exert a drag force on the sphere which can be evaluated as in Casewell and Schwarz (1962)

$$D = 2\pi a\eta_0 U \int_{-1}^{1} [\mu(S_{rr})_{r=1} + (1-\mu^2)^{1/2}(S_{r\mu})_{r=1}]\, d\mu \qquad \ldots(5.1)$$

which when expressed in the notation of (4.1) becomes

$$\frac{D}{2\pi a\eta_0 U} = d_{00} + \lambda d_{01} + \lambda^2 d_{02} + \ldots + R_e d_{10} + \lambda R_e d_{11} + \ldots \qquad \ldots(5.2)$$

where the constants d_{ij} are given by

$$d_{ij} = \int_{-1}^{1} \{\mu(S^{ij}_{rr})_{r=1} + (1-\mu^2)^{1/2}(S^{ij}_{r\mu})_{r=1}\}\, d\mu \qquad \ldots(5.3)$$

in which S^{ij}_{rr}, $S^{ij}_{r\mu}$ are components of the stress tensor (1.6) expanded in similar manner. For $i = 0$, this gives $d_{00} = 3$, $d_{01} = 0$, and

$$d_{02} = (13.5)\,[4.5278\,(1 + 2.4528\,\epsilon_1)(1+\epsilon_1) + 3.0984\,(\epsilon_3 + \epsilon_4) - 1.0296\,\epsilon_4 - 5.6160\,\epsilon_4]. \qquad \ldots(5.4)$$

For $i = 1$,

$$d_{10} = \frac{3(3 + R_h)}{8}. \qquad \ldots(5.5)$$

Thus the final expression for the drag force is

$$D = 6\pi a\eta_0 U\left(1 + \frac{3+R_h}{8}R_e\right) + \frac{27\pi U^3}{a}\left[4.5278\left(1 + 2.4528\,\frac{\phi_2}{\phi_3}\right)\times\left(1 + \frac{\phi_1}{\phi_3}\right)\frac{\phi_3^2}{\eta_0} + 3.0984\,(\phi_5 + \phi_6 - 5.6160\,\phi_4 - 1.0296\,\phi_5)\right] \qquad \ldots(5.6)$$

which is Stokes' law with a small Reynolds number and magnetic pressure number correction and a term in U^3 whose coefficient involves the non-Newtonian parameters as given by Casewell and Schwarz (1962). When $R_h \to 0$ i.e. the liquid is non-conducting our results are in complete agreement with those of Casewell and Schwarz. In case $\lambda \to 0$ i.e. the liquid is non-Newtonian the results are in complete agreement with Chow and Billings. When $\lambda \to 0$, $R_h \to 0$ the results are well-known results of Proudman and Pearson (1957).

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