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DUSTY VISCOUS FLOW THROUGH A CYLINDER OF ELLIPTIC SECTION UNDER TIME-DEPENDENT PRESSURE GRADIENT

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Unsteady laminar flow of a dusty, viscous, incompressible fluid through a cylindrical tube of elliptic section is discussed when the pressure gradient varies (i) exponentially and (ii) periodically with time. The velocity fields for the fluid and the dust particles have been obtained. Results in particular cases are derived. Expressions for flux and drag arc obtained and compared with the corresponding expressions for the clean fluid. Some numerical results are presented graphically.

1. INTRODUCTION

Based on the theoretical model proposed by Saffman (1962), several workers have solved a number of dusty gas flow problems recently.

Here, we have studied the flow of a dusty viscous fluid through a cylindrical tube of elliptic section under exponential and periodic pressure gradients. The velocity fields for the fluid and dust particles are obtained and particular cases of interest are deduced. Expressions for flux and drag are obtained and comparison is made with the clean fluid results. Some numerical work is done and the results are presented graphically.

2. GOVERNING EQUATIONS

Taking the axis of the infinite cylinder in Z-direction and assuming axisymmetry and the number density N to be constant (N_0), there remain components of velocity of fluid and of dust particles in Z-direction only i.e. w and w^* respectively and these are independent of Z. Saffman's (1962) equations for the present problem reduce to

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu \nabla^2 w + \frac{KN_0}{\rho}(w^* - w) \qquad ...(2.1)$$

$$m\frac{\partial w^*}{\partial t} = K(w - w^*) \qquad ...(2.2)$$

where

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$; and the symbols have their usual meaning.

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Eliminating w^* from (2.5) with the help of (2.6) we get

$$\tau \frac{\partial^2 w}{\partial t^2} = - \frac{1}{\rho} \frac{\partial}{\partial z} \left(\tau \frac{\partial p}{\partial t} + p \right) + [\nu\tau \nabla^2 - (1 + f)] \frac{\partial w}{\partial t} + \nu \nabla^2 w \qquad \ldots(2.3)$$

where

$\tau \left(= \frac{m}{K} \right)$ and $f = \left(\frac{mN_0}{\rho} \right)$ are respectively the relaxation time and the mass concentration of dust.

3. Solution of the Problem: Exponential Pressure Gradient

Assuming

$$- \frac{1}{\rho} \frac{\partial p}{\partial z} = \alpha e^{\sigma t} \qquad \ldots(3.1)$$

$$w = \phi(x, y)\, e^{\sigma t} \qquad \ldots(3.2)$$

$$w^* = \phi^*(x, y)\, e^{\sigma t} \qquad \ldots(3.3)$$

where α is a real constant, and $\sigma > 0$; and substituting in (2.2) and (2.3), we obtain

$$\phi^* = - \frac{1}{1 + \sigma\tau} \phi \qquad \ldots(3.4)$$

and

$$\nabla^2 \phi - \lambda^2 \phi + \frac{\alpha}{\nu} = 0 \qquad \ldots(3.5)$$

where

$$\lambda^2 = \frac{\sigma}{\nu} \left(\frac{1 + f + \sigma\tau}{1 + \sigma\tau} \right). \qquad \ldots(3.6)$$

Now, putting

$$\phi = \psi + (\alpha/\nu\lambda^2) \qquad \ldots(3.7)$$

we get

$$\nabla^2 \psi - \lambda^2 \psi = 0. \qquad \ldots(3.8)$$

Introducing the elliptic coordinates ξ, η defined by

$$x + iy = c \cosh (\xi + i\eta) \qquad \ldots(3.9)$$

$$c = (a^2 - b^2)^{1/2}; \ (0 \leqslant \xi < \infty,\ 0 \leqslant \eta < 2\pi),$$

eqn. (3.8) transforms to

$$\frac{\partial^2\psi}{\partial\xi^2} + \frac{\partial^2\psi}{\partial\eta^2} - 2k^2 (\cosh 2\xi - \cos 2\eta) \psi = 0 \qquad ...(3.10)$$

where

$$4k^2 = \lambda^2 c^2 = \frac{c^2\sigma}{\nu} \left(\frac{1 + f + \sigma\tau}{1 + \sigma\tau}\right). \qquad ...(3.11)$$

Separating variables, we obtain

$$\frac{d^2X}{d\xi^2} - (a + 2k^2 \cosh 2\xi) X = 0 \qquad ...(3.12)$$

$$\frac{d^2Y}{d\eta^2} + (a + 2k^2 \cos 2\eta) Y = 0. \qquad ...(3.13)$$

The general solution may be put as

$$\phi = \left(-\frac{\alpha}{\nu\lambda^2}\right) + \sum_{n=0}^{\infty} C_{2n} Ce_{2n}(\xi, -k^2)\, ce_{2n}(\eta, -k^2) \qquad ...(3.14)$$

where (McLachlan 1947)

$$ce_{2n}(\eta, -k^2) = (-1)^n \sum_{r=0}^{\infty} (-1)^r A_{2r}^{(2n)} \cos 2r\eta \qquad ...(3.15)$$

$$Ce_{2n}(\xi, -k^2) = (-1)^n \sum_{r=0}^{\infty} (-1)^r A_{2r}^{(2n)} \cosh 2r\xi \qquad ...(3.16)$$

the coefficients being functions of k^2.

If $\xi = \xi_0$ defines the boundary of the section of the cylinder, we must have

$$\phi = 0 \quad \text{when} \quad \xi = \xi_0 \qquad ...(3.17)$$

which implies the vanishing of velocities of fluid as well as of dust particles. Making use of this boundary condition, the constants C_{2n} in (3.14) are evaluated as

$$C_{2n} = \frac{2(-1)^{n-1} A_0^{(2n)}}{Ce_{2n}(\xi_0, -k^2)} \cdot \frac{\alpha}{\nu\lambda^2} \qquad ...(3.18)$$

using orthogonality relations and normalization (McLachlan 1947).

We finally obtain

$$w = \frac{\alpha}{\nu\lambda^2} e^{\sigma t} + e^{\sigma t} \sum_{n=0}^{\infty} C_{2n} Ce_{2n}(\xi, -q)\, ce_{2n}(\eta, -q) \qquad ...(3.19)$$

$$w^* = \left(\frac{e^{\sigma t}}{1 + \sigma t}\right)\left[\frac{\alpha}{\nu\lambda^2} + \sum_{n=0}^{\infty} C_{2n}Ce_{2n}(\xi, -q)\, ce_{2n}(\eta, -q)\right]. \qquad ...(3.20)$$

For small $|q|$, we obtain

$$w = \frac{\alpha c^2 e^{\sigma t}}{8\nu} E + \frac{\alpha c^4 e^{\sigma t}}{192\nu} F \qquad ...(3.21)$$

which can be put in the form

$$w = \left[K^* e^{\sigma t}\left\{1 + \left(\frac{24\nu E}{\sigma c^2 F}\right)\right\}\right]_c + \left[K^* e^{\sigma t}\left(\frac{f}{1 + \sigma\tau}\right)\right]_d \qquad ...(3.22)$$

where

$$K^* = \frac{F\alpha\sigma c^4}{192\nu^2}$$

$$E = \cosh 2\xi_0 - \cosh 2\xi - \cos 2\eta + \frac{\cosh 2\xi \cos 2\eta}{\cosh 2\xi_0}$$

$$F = \frac{(\cosh 2\xi \cos 2\eta + 3\cosh^2 2\xi - 2)(\cosh 2\xi + \cos 2\eta)}{\cosh 2\xi_0}$$

$$+ \frac{(2 - \cosh^2 2\xi_0)\cosh 2\xi \cos 2\eta}{\cosh 2\xi_0} - \tfrac{3}{4}(\cosh^2 2\xi + \cos^2 2\eta$$

$$+ 3\cosh^2 2\xi_0) + \tfrac{9}{8}$$

and $[\ ]_c$, $[\ ]_d$ denote respectively the clean and dusty flow parts of the expression.

If we retain terms up to $O(q)$ only in the above approximations, we get

$$w = \frac{\alpha c^2 e^{\sigma t}}{8\nu} E = \frac{\alpha e^{\sigma t}}{2\nu}\frac{a^2 b^2}{a^2 + b^2}\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right) \qquad ...(3.23)$$

in Cartesian coordinates, where $c \cosh \xi_0 = a$, $c \sinh \xi_0 = b$. Equation (3.23) gives the well-known velocity field for the viscous fluid flow through the elliptic tube under exponential pressure gradient. We see from (3.23) that the effect of dust is not found up to $O(q)$.

For large $|q|$, the velocity field of the fluid can be put as

$$w = \frac{\alpha e^{\sigma t}}{\nu\lambda^2} - \frac{2\alpha e^{\sigma t}}{\nu\lambda^2} A_0^{(0)} \frac{\cosh \frac{1}{2}\xi_0}{\cosh \frac{1}{2}\xi} \exp\{-\epsilon(1 + \delta)(c\cosh \xi_0 - c\cosh \xi)\}$$

$$\times\, ce_0(\eta, -q) \qquad ...(3.24)$$

where

$$\epsilon = (\sigma/\nu)^{1/2} \quad \text{and} \quad \delta = f/2(1 + \sigma\tau).$$

In the case of a circular cylinder (3.24) reduces to

$$w = \frac{\alpha e^{\sigma t}}{\nu\lambda^2} - \frac{\alpha e^{\sigma t}}{\nu\lambda^2}\left(\frac{a}{r}\right)^{1/2} \exp\{-\epsilon(1+\delta)(a-r)\}. \quad ...(3.25)$$

The case of exponentially decreasing pressure gradient has also been worked out but the details are omitted here to save space.

Flux and Drag

The volume of fluid discharged per unit time is given by

$$Q = \int_0^{\xi_0}\int_0^{2\pi} w(\xi, \eta)\, h_1 d\xi\, h_2 d\eta \quad ...(3.26)$$

where $h_1 = h_2 = \frac{c}{\sqrt{2}}(\cosh 2\xi - \cos 2\eta)$; and the drag per unit length of the cylinder is given by

$$D = \mu \int_0^{2\pi} \left(\frac{\partial w}{\partial \xi}\right)_{\xi=\xi_0} d\eta. \quad ...(3.27)$$

Using the expression (3.21) of velocity of fluid for small $|q|$, we obtain the expressions for flux and drag, which can be put in forms

$$Q = Q_c + Q_d = \left[e^{\sigma t} AG\left(\frac{192\nu H}{c^2\sigma G} - 1\right)\right]_c + \left[AG\frac{fe^{\sigma t}}{1+\sigma\tau}\right]_d \quad ...(3.28)$$

$$D = D_c + D_d = [B \sinh 2\xi_0 e^{\sigma t}(1 - P^{-1})]_c + \left[-BP^{-1}\sinh 2\xi_0 \frac{fe^{\sigma t}}{1+\sigma\tau}\right]_d \quad ...(3.29)$$

where

$$A = \frac{\pi\sigma\alpha c^6}{6144\nu^2}, \quad B = -\tfrac{1}{2}\rho\alpha\pi c^2$$

$$G = \sinh 2\xi_0\,[1 + 4(\cosh^2 2\xi_0 + \operatorname{sech}^2 2\xi_0)]$$

$$H = \tanh 2\xi_0 \sinh^2 2\xi_0, \quad P^{-1} = \frac{\sigma c^2}{16\nu}\tanh 2\xi_0 \sinh 2\xi_0.$$

4. Solution of the Problem : Periodic Pressure Gradient

Assuming the pressure gradient to be periodic with period $2\pi/\sigma$ we take

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = \alpha e^{i\sigma t} \quad ...(4.1)$$

$$w = \phi(x, y)\, e^{i\sigma t} \qquad \ldots(4.2)$$

$$w^* = \phi^*(x, y)\, e^{i\sigma t} \qquad \ldots(4.3)$$

where α is real; and the real parts of the right-hand side expressions are to be taken. Proceeding as in the previous case we obtain the velocity of the fluid and that of the dust particles respectively as

$$w = \mathrm{Re}\, e^{i\sigma t}\left[\frac{\alpha}{\nu\lambda^2} + \sum_{n=0}^{\infty} C_{2n} Ce_{2n}(\xi, -q)\, ce_{2n}(\eta, -q)\right] \qquad \ldots(4.4)$$

$$w^* = \mathrm{Re}\, \frac{e^{i\sigma t}}{1 + i\sigma t}\left[\frac{\alpha}{\nu\lambda^2} + \sum_{n=0}^{\infty} C_{2n} Ce_{2n}(\xi, -q)\, ce_{2n}(\eta, -q)\right] \qquad \ldots(4.5)$$

where q has been written for k^2; and

$$\lambda^2 = \frac{i\sigma}{\nu}\, \frac{(1 + f + i\sigma\tau)}{(1 + i\sigma\tau)} = \frac{4k^2}{c^2}. \qquad \ldots(4.6)$$

For small frequency we obtain

$$w = \frac{\alpha c^2}{8\nu} E \cos \sigma t + \frac{\alpha c^4}{192\nu} F\, [\mathrm{Re}\, (\lambda^2 e^{i\sigma t})] \qquad \ldots(4.7)$$

where E and F are same as in section 3.

We may put (4.7) as

$$w = [K^* M \cos (\sigma t + L)]_c + [K^* f \beta^{-1} \cos (\sigma t + \gamma)]_d \qquad \ldots(4.8)$$

where

$$K^* = \frac{F\alpha\sigma c^4}{92\nu^2}, \qquad \sigma\tau = \beta \cos \gamma,$$

$$\frac{24E\nu}{F\sigma c^2} = M \cos L, \qquad 1 = \beta \sin \gamma,$$

$$1 = M \sin L, \qquad \beta = (1 + \sigma^2\tau^2)^{1/2},$$

$$M = \left(1 + \frac{576E^2\nu^2}{F^2\sigma^2 c^4}\right)^{1/2}, \quad \gamma = \cot^{-1} \sigma\tau,$$

$$L = \cot^{-1}\left(\frac{24E\nu}{F\sigma c^2}\right).$$

If we retain terms up to $O(q)$ only in the above approximations we get

$$w = \frac{\alpha c^2}{8} E \cos \sigma t = \frac{\alpha}{2\nu}\, \frac{a^2 b^2}{a^2 + b^2}\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right) \cos \sigma t \qquad \ldots(4.9)$$

in Cartesian coordinates, where $c \cosh \xi_0 = a$, $c \sinh \xi_0 = b$. Equation (4.9) gives the well-known velocity field for the viscous flow through the elliptic tube under periodic pressure gradient. We see from (4.9) that the effect of dust is not found up to $O(q)$. When the frequency of the pressure gradient is large we may put

$$w = \Big[\lambda_r - \frac{\alpha}{\sigma} \frac{\cosh \frac{1}{2}\xi_0}{\cosh \frac{1}{2}\xi} e^{-\zeta} \{(\Gamma_2 \cos \theta + \Gamma_1 \sin \theta) + f\beta^{-1}(\Gamma_1 \cos (\theta + \gamma) - \Gamma_2 \sin (\theta + \gamma))\}\Big] \text{ if } f < |1 + i\sigma\tau| \quad \ldots(4.10)$$

where

$$\lambda_r = \mathrm{Re} \frac{\alpha e^{i\sigma t}}{\nu\lambda^2} = \frac{\alpha}{\sigma} \sin \sigma t + \frac{\alpha}{\sigma} f\beta^{-1} \cos (\sigma t + \gamma),$$

$$\zeta = \epsilon g(c \cosh \xi_0 - c \cosh \xi),\ \epsilon = \left(\frac{\sigma}{2\nu}\right)^{1/2},$$

$$\theta = \sigma t - \epsilon h(c \cosh \xi_0 - c \cosh \xi),\ g = 1 + \tfrac{1}{2} f\left(\frac{1 + \sigma\tau}{1 + \sigma^2\tau^2}\right),$$

$$\Gamma_1 = \mathrm{Re}\, ce_0(\eta, -q),\ h = 1 + \tfrac{1}{2} f\left(\frac{1 - \sigma\tau}{1 + \sigma^2\tau^2}\right),$$

$$\Gamma_2 = \mathrm{Im}\, ce_0(\eta, -q),\ 2k = \epsilon c(g + ih).$$

In the case of a circular cylinder this reduces to

$$w = \lambda_r - \frac{\alpha}{\sigma}\left(\frac{a}{r}\right)^{1/2} \exp\{-\epsilon g(a - r)\} [\sin\{\sigma t - \epsilon h(a - r)\} + f\beta^{-1} \cos\{\sigma t + \gamma - \epsilon h(a - r)\}]. \quad \ldots(4.11)$$

Flux and Drag

Using the expression (4.7) of velocity of fluid for small $|q|$ we obtain flux and drag which may be put as

$$Q = Q_c + Q_d = [AGR \cos (\sigma t + T)]_c + [AGf\beta^{-1} \cos (\sigma t + \gamma)]_d \quad \ldots(4.12)$$

$$D = D_c + D_d = [B \sinh 2\xi_0 \cos (\sigma t - S)]_c + [-BP^{-1} \sinh 2\xi_0 f\beta^{-1} \cos (\sigma t + \gamma)]_d \quad \ldots(4.13)$$

where A, B, G, H and P are as defined in section 3 and

$$\frac{192\nu H}{\sigma c^2 G} = R \cos T,\ R = \left[1 + \left(\frac{192\nu H}{\sigma c^2 G}\right)^2\right]^{1/2},$$

$$1 = R \sin T,\ T = \cot^{-1}\left(\frac{192\nu H}{\sigma c^2 G}\right),$$

$$P = J \cos S, \quad J = (1 + P^2)^{1/2},$$

$$1 = J \sin S, \quad S = \cot^{-1} P.$$

5. Discussion of Results

(1) Removal of dust from the expressions by putting $f = 0$ results in all the flow quantities of clean viscous fluid being recovered.

(2) We observe from eqns. (3.28) - (3.29) that in the case of exponential pressure gradient if q is small, the effect of the presence of dusty particles in the fluid is to decrease the flux and to increase the drag on the walls of the cylinder by the values

$$- AGfe^{\sigma t}/(1 + \sigma\tau) \qquad ...(5.1)$$

$$- BP^{-1} (\sinh 2\xi_0) fe^{\sigma t}/(1 + \sigma\tau) \qquad ...(5.2)$$

respectively as can be expected from physical considerations qualitatively. The corresponding expressions for the case of periodic pressure gradient, as may be seen from eqns. (4.12) - (4.13) are

$$- AGf\beta^{-1} \cos(\sigma t + \gamma) \qquad ...(5.3)$$

$$- BP^{-1} (\sinh 2\xi_0) f\beta^{-1} \cos(\sigma t + \gamma). \qquad ...(5.4)$$

Here, it may be observed* that if we change σ in (5.1) into $i\sigma$ and $-i\sigma$ respectively and add the resulting expressions, we get

$$-AGf\left(\frac{e^{i\sigma t}}{1 + i\sigma\tau} + \frac{e^{-i\sigma t}}{1 - i\sigma\tau}\right) = -\frac{AG}{1 + \sigma^2\tau^2}(2 \cos \sigma t + 2\sigma\tau \sin \sigma t)$$

$$= -2AGf\beta^{-1} \cos(\sigma t + \gamma) \qquad ...(5.5)$$

which is the additional drag corresponding to the periodic pressure gradient proportional to $2 \cos(\sigma t + \gamma)$.

Further, if $\sigma\tau < 1$ we have

$$(1 + \sigma\tau)^{-1} \simeq 1 - \sigma\tau \qquad ...(5.6)$$

which shows that the additional drag due to the presence of dust decreases as τ increases up to a critical level given by $\tau_{crit} = 1/\sigma$ in the case of exponential pressure gradient. Correspondingly, we have for small values of τ, in the case of periodic pressure gradient,

$$\beta^{-1} = (1 + \sigma^2\tau^2)^{-1/2} \simeq 1 - \tfrac{1}{2}\sigma^2\tau^2 \qquad ...(5.7)$$

*This has been pointed out by Prof. B. R. Seth.

which also shows that the additional drag decreases as τ increases up to a critical level given by $\tau_{crit} = \sqrt{2}/\sigma$. This is in conformity with the observation of Saffman that when τ is small, which implies that the dust is fine, the effective kinematic viscosity is reduced. Further, we observe that in the case of the periodic pressure gradient the critical value of τ is $\sqrt{2}$ times greater than that of τ in the case of exponential pressure gradient.

Again for large τ we have in respect of the exponential and periodic pressure gradients

$$f(1 + \sigma\tau)^{-1} \simeq f(\sigma\tau)^{-1}\left(1 - \frac{1}{\sigma\tau}\right) = s\sigma^{-1}\,[1 - O(\sigma\tau)^{-1}] \qquad \text{...(5.8)}$$

and

$$f\beta^{-1} = f(\sigma\tau)^{-1}\left(1 + \frac{1}{\sigma^2\tau^2}\right)^{-1/2} \simeq f(\sigma\tau)^{-1}\left(1 - \frac{1}{2\sigma^2\tau^2}\right)$$

$$= s\sigma^{-1}\,[1 - O((\sigma\tau)^{-2})] \qquad \text{...(5.9)}$$

respectively, where $s = f\tau^{-1} = KN_0/\rho$. The additional drag varies with s in the case of coarse dust.

From eqns. (3.24) and (4.10) we infer that in the central portion of the cylinder $\epsilon(1 + \delta)\,(c \cosh \xi_0 - c \cosh \xi)$ in the case of exponential pressure gradient and ζ in the case of periodic pressure gradient are large, while the second terms are small, but near the boundary of the cylinder they are appreciable. The motion has therefore the boundary layer character. It varies directly with the mass concentration f and inversely with the relaxation time τ.

The boundary layer effects noticeable for large $|q|$ in the case of increasing exponential pressure gradient are not to be found in the case of decreasing exponential pressure gradient.

We can write (3.28) as

$$Q(\sigma) = A_1\sigma G e^{\sigma t}\left(\frac{192\nu H}{c^2\sigma G} - 1\right) + A_1\sigma G e^{\sigma t}\left(\frac{f}{1 + \sigma\tau}\right) \qquad \text{...(5.10)}$$

$$Q(\sigma') = A_1\sigma' G e^{\sigma' t}\left(\frac{192\nu H}{c^2\sigma' G} - 1\right) \qquad \text{...(5.11)}$$

where

$$A_1 = \frac{\pi\alpha c^6}{6144\nu^2}.$$

Expanding the exponential function and equating $Q_c(\sigma') = Q(\sigma)$, we obtain the relations

$$\sigma' = \sigma\left(1 - \frac{f}{1 + \sigma\tau}\right) \quad \ldots(5.12)$$

$$\sigma' - \sigma' t\left(\frac{192\nu H}{c^2 G} - \sigma'\right) = \sigma\left(1 - \frac{f}{1 + \sigma\tau}\right) - \sigma t\left[\frac{192\nu H}{c^2 G} - \sigma\left(1 - \frac{f}{1 + \sigma\tau}\right)\right] \quad \ldots(5.13)$$

... etc.

At any time, for a given 'σ', these relations give the corresponding σ' (and therefore the pressure gradient $e^{\sigma' t}$) for which the clean fluid flux $Q_c(\sigma')$ equals the dusty fluid flux $Q(\sigma)$.

6. Numerical Results

To examine the effect of the presence of dust on the flux and drag numerical work has been carried out for small $| q |$. Figs. 1 to 5 relate to the case of exponential pressure gradient and Figs. 6 to 9 relate to the case of periodic pressure gradient.

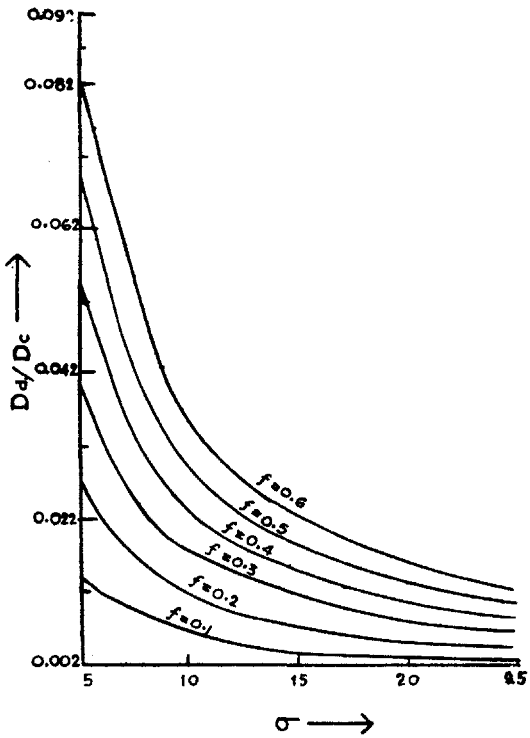

FIG. 1. Drag ratio D_d/D_c against σ in the case of pressure gradient increasing exponentially with time for different mass concentrations of dust f. ($a = 5$, $b = 3$, $\tau = 2$, $\nu = 3$).

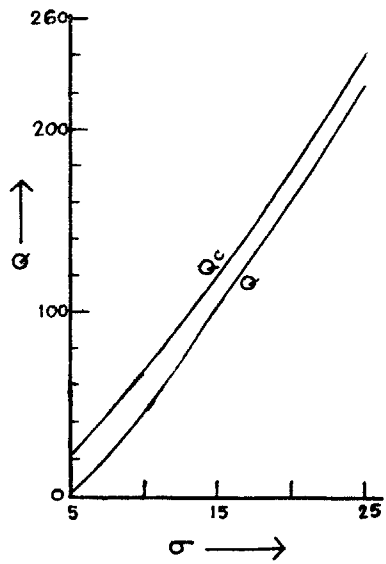

FIG. 2. Flux of clean and dusty fluids under exponential pressure gradient increasing with time plotted against σ. ($a = 5$, $b = 3$, $\tau = 2$, $\nu = 3$, $\alpha = -1$, $t = 0.01$ sec., $f = 0.04$).

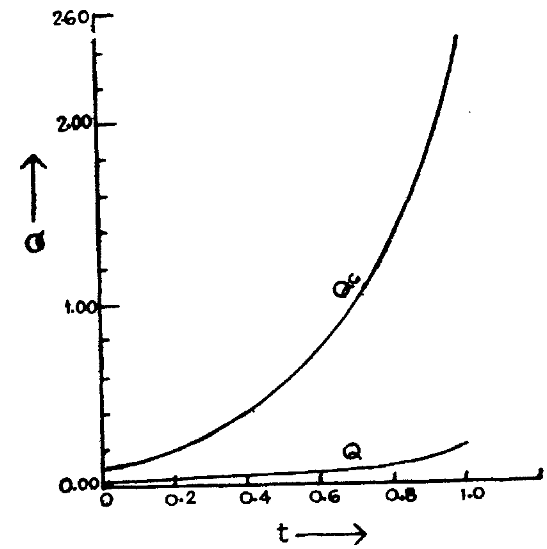

FIG. 3. Flux of clean and dusty fluids under exponential pressure gradient increasing with time plotted against time. ($a = 5$, $b = 3$, $\tau = 15$, $\nu = 3$, $\alpha = -1$, $\sigma = 3$, $f = 0.20$).

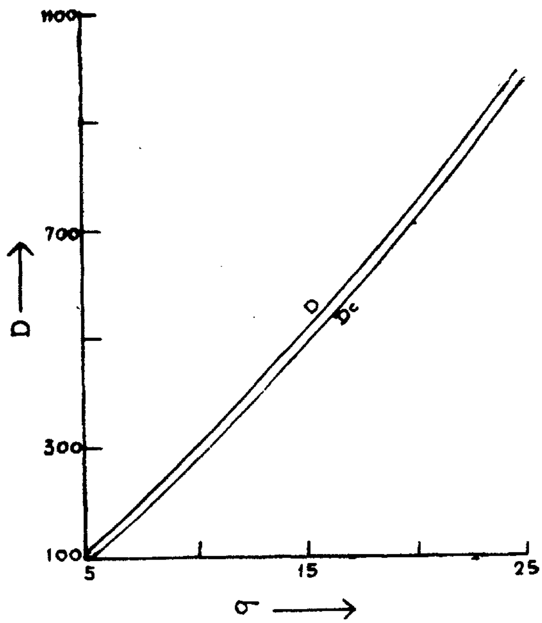

FIG. 4. Curves of clean drag and dusty drag against σ under exponential pressure gradient increasing with time. ($a = 5$, $b = 3$, $\alpha = 1$, $\tau = 2$, $\nu = 3$, $\rho = 1.26$, $f = 0.8$, $t = 0.01$ sec.).

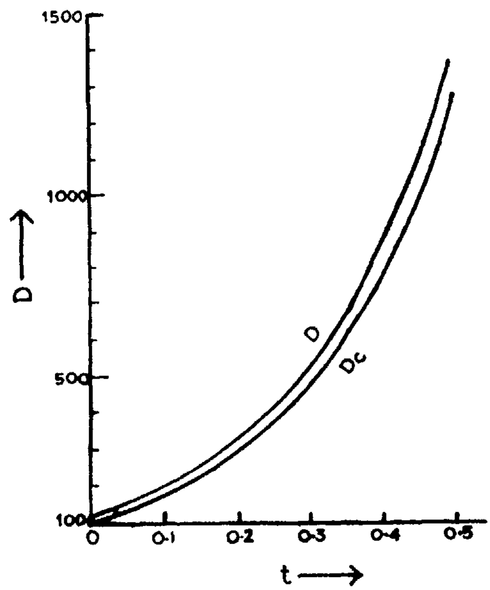

FIG. 5. Curves of clean drag and dusty drag against time under exponential pressure gradient increasing with time. ($a = 5$, $b = 3$, $\tau = 2$, $\nu = 3$, $\rho = 1.26$, $\sigma = 5$, $\alpha = 1$, $f = 0.8$).

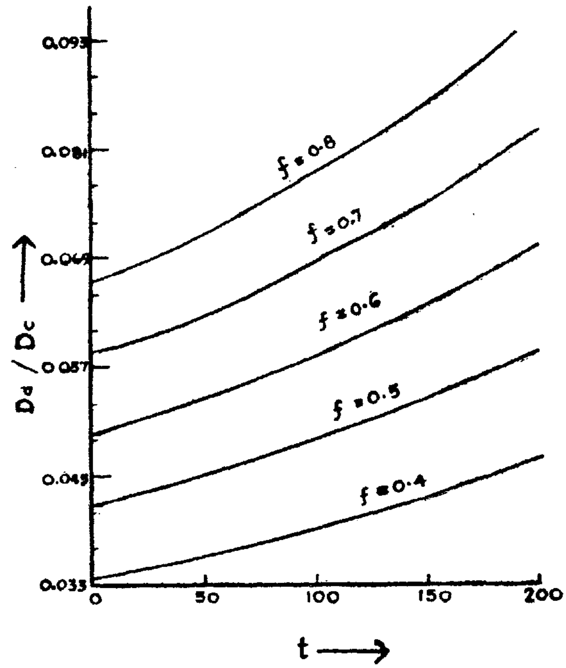

FIG. 6. Drag-ratio D_d/D_c against time t under periodic pressure gradient for different mass concentrations of dust f. ($a = 5$, $b = 3$, $\tau = 6$, $\nu = 3$, $\sigma = 0.5$).

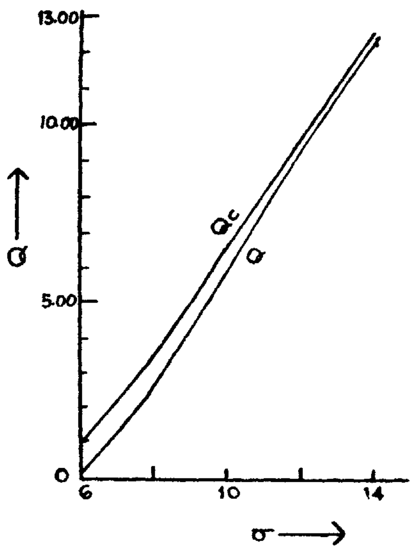

FIG. 7. Flux of clean and dusty fluids under periodic pressure gradient plotted against σ. ($a = 5$, $b = 3$, $\alpha = -1$, $\nu = 3$, $\tau = 0.5$, $t = 0.1$ sec., $f = 0.8$).

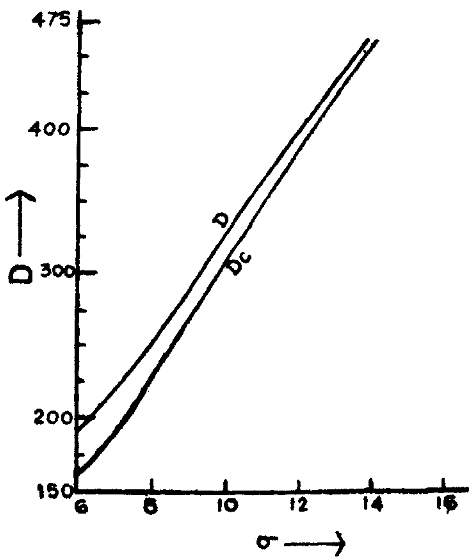

FIG. 8. Curves of clean and dusty drag against σ under periodic pressure gradient. ($a = 5$, $b = 3$, $\alpha = -1$, $\nu = 3$, $\tau = 6$, $\rho = 1.26$, $f = 0.8$, $t = 0.1$ sec.).

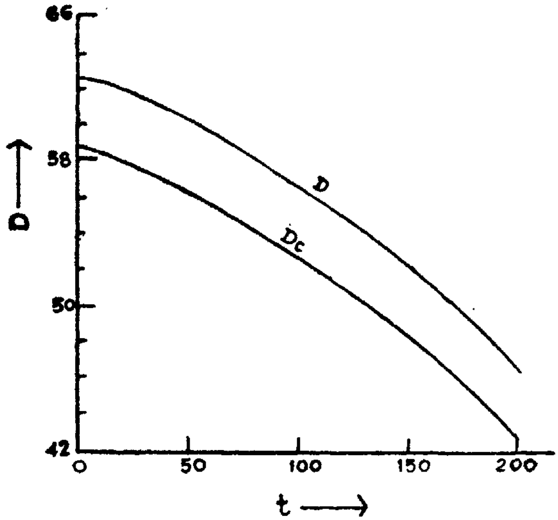

FIG. 9. Curves of clean drag and dusty drag against time under periodic pressure gradient. ($a = 5$, $b = 3$, $\nu = 3$, $\tau = 6$, $\rho = 1.26$, $\alpha = -1$, $f = 0.8$, $\sigma = 0.5$).

Fig. 1 shows that the drag ratio D_d/D_c increases with mass concentration f. Further, as σ increases, the drag-ratio falls, the fall between $\sigma = 5$ and $\sigma = 10$ being steep while it is not so between $\sigma = 20$ and $\sigma = 25$. Fig. 2 indicates that the flux due to clean fluid is greater than that due to dusty fluid and that both the clean flux and the dusty flux increase almost uniformly with σ. From Fig. 3 we observe that the dusty fluid flux and clean fluid flux increase with time; and that while the dusty fluid flux increases very slowly, the clean fluid flux rises steeply. In Figs. 4 and 5 are plotted the variations of the drag against σ and t respectively. The drag of the dusty fluid is greater than that of the clean fluid as can be seen from Figs. 4 and 5 and they increase almost uniformly with σ and t.

From Fig. 6 we see that the drag-ratio increases with mass concentration f. It also increases with time t which implies that the dusty part of the drag D_d increases with time. Fig. 7 indicates that the flux due to clean fluid is greater than that due to dusty fluid in the interval $\sigma = 6$ to $\sigma = 14$. In Figs. 8 and 9 are plotted the variations of drag against σ and t respectively. The drag of the dusty fluid is greater than that of the clean fluid for the values of σ and t. Fig. 9 reveals that D and D_c decrease with time and the increases at any time in the drag due to dust particles is almost uniform.

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