Indian J. pure appl. Math., **10**(6) : 761-762, June 1979

ANNULAR EFFECT IN POROUS MEDIA

N. CH. PATTABHI RAMACHARYULU

Department of Mathematics, Regional Engineering College, Warangal 506004

(*Received* 15 *June* 1978)

The aim of the present note is to examine the flow of a viscous fluid in a straight porous tube with an impermeable boundary, adopting a generalized Darcy's law. When the medium is highly porous, we notice an annular effect: the non-Darcian phenomenon is predominantly felt near the boundary and the classical Darcy-flow is realized only in a core very close to the axis of the tube.

Brinkman (see Bird *et al.* 1960) proposed the generalized Darcy's law

$$0 = -\nabla p - \frac{\mu}{k} \vec{V} + \mu \nabla^2 \vec{V} \qquad \text{...(1)}$$

to describe viscous flows through porous media where $\vec{V}$ and p stand for the velocity and pressure fields, μ the coefficient of viscosity of the fluid and k the permeability of the medium. The same equation was later derived analytically by Christopher (1969) to describe the viscous flow at low Reynold's numbers past a swarm of small spherical particles. This generalized law would give good results in the case of highly porous media such as pappus of dandelion and fibres.

Referred to a cylindrical coordinate system (r, θ, z), the steady flow of a viscous incompressible fluid in a highly porous cylinder under a constant pressure gradient $(-\partial p/\partial z = c)$ is characterized by the equation

$$\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr} - \frac{w}{k} = -\frac{1}{\mu}\frac{\partial p}{\partial z} = \frac{c}{\mu} \qquad \text{...(2)}$$

where $w(r)$ is the velocity down the tube and r is the distance measured from the tube-axis. The solution of this equation satisfying the no-slip condition on the tube wall, which is impermeable, can be seen to be

$$w(r) = \frac{ck}{\mu}\left[1 - \frac{I_0(r/\sqrt{k})}{I_0(a/\sqrt{k})}\right] \qquad \text{...(3)}$$

where a is the radius of the tube and $I_0(.)$ is the zeroth order modified Bessel function of the first kind.

For the large values of k,

$$I_0(r/\sqrt{k}) \doteq 1 + (r^2/4k)$$

and hence we have

$$w(r) = \frac{c}{4\mu}\left(1 + \frac{a^2}{k}\right)^{-1}(a^2 - r^2) \qquad \ldots(4)$$

which is the same as that in a clear (i.e., non-porous) medium but under a constant pressure gradient reduced in the ratio $(1 + a^2/k)^{-1}$.

When the permeability (k) is very low, as happens with highly porous media,

$$I_0(r/\sqrt{k}) \sim (2\pi r/\sqrt{k})^{-1/2} \exp(r/\sqrt{k}).$$

The velocity field (3), in such a case, reduces to

$$w(r) \sim \frac{ck}{\mu}[1 + \sqrt{a/r} \exp\{-(a - r)/\sqrt{k}\}]. \qquad \ldots(5)$$

The second term of this rapidly damps out as the distance $\eta (= a - r)$ measured from the tube wall increases while the first term remains constant. Thus far away from the tube wall, the velocity is almost $= ck/\mu$ which is the same as the velocity obtained under the framework of the classical Darcy's law:

$$0 = -\nabla p - \frac{\mu}{k}\vec{V} \qquad \ldots(6)$$

The non-Darcian effect is thus more predominant in a thin layer of thickness $\delta = O(\sqrt{k})$ around the tube wall and in a core very close to the axis, the flow is purely Darcian.

This phenomenon is similar to that noticed by Sexl (see Meksyn 1961) for the pulsating viscous flow in a circular tube and reported by Matunobu (1976) in relation to blood flow through vessels.

References

Bird, R. B., Stewart, W. E., and Lightfoot, E. N. (1960). Transport Phenomena. John Wiley & Sons, New York, p. 150.

Christopher, K. W. Tam (1969). The drag on a cloud of spherical particles in low Reynolds number flow. *J. Fluid Mech.*, **38**, 537.

Matunobu, Y. (1976). *Thrombosis Res.*, **8**, 287.

Meksyn, D. (1961). New Methods in Laminar Boundary Layer Theory. Pergamon Press, Oxford, p. 144.