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THE EFFECTS OF HALL CURRENTS ON THE OSCILLATORY MHD FLOW PAST A FLAT PLATE

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Hall effects on the hydromagnetic flow past an infinite vertical plate in the presence of a uniform transverse magnetic field has been analysed when the free stream oscillates in magnitude. The difference between the temperature of the plate and the free stream is moderately large causing the free convection currents. Approximate solutions for the coupled non-linear equations are derived. The effect of Hall currents, free convection currents and the oscillatory free stream on the flow field are studied. It is found that the secondary flow induced by the Hall currents suffers from a reverse flow at distances adjacent to the plate and extends to distances away from the plate when the Prandtl number increases. However, when the Grashoff number increases, the reverse flow gets confined to a thin layer near the plate.

1. INTRODUCTION

The oscillatory flow of a viscous incompressible fluid past an infinite vertical plate was studied by Soundalgekar (1973) making use of the method adopted by Stuart (1955) in studying the oscillatory flow past an infinite porous plate with constant suction. Soundalgekar (1975) also studied the problem of two dimensional unsteady flow of an electrically conducting fluid past an infinite vertical porous plate with uniform suction at the plate. He assumed that the difference between the temperaure of the plate and the free stream is moderately large causing the free convection currents, and studied the effect of the oscillatory free stream and the free convection currents on the flow.

When the strength of the magnetic field is very strong one cannot neglect the effects of Hall currents. Even though it is of considerable importance and interest to study how the results of the hydrodynamical problems get modified by the effect of Hall currents, the unsteady magnetohydrodynamic flow near an infinite flat plate with Hall currents has received considerably less attention. The object of the present paper is to study the problem of an oscillatory magnetohydrodynamic flow of a viscous incompressible fluid past an infinite vertical porous plate under the influence of a strong magnetic field. The fluid is subjected to uniform suction at the plate and the temperature of the plate differs from the temperature of the free stream, causing the flow of free convection current in the boundary layer. The free stream velocity oscillates in time about a constant mean. The Hall currents give rise to a cross flow making the flow three dimensional. A mathematical analysis is presented for this hydromagnetic flow and the effects of Hall currents, the free convection currents and the oscillatory free stream on the flow field are studied. The behaviour of the flow field is discussed for various values of the parameters that describe the flow.

2. Formulation of the Problem

We consider the unsteady flow of an electrically conducting incompressible viscous fluid past a vertical, porous, infinite flat plate $y = 0$, with the x-axis chosen along the plate. A uniform magnetic field H_0 is acting transverse to the plate. Since the plate is infinite in extent, all physical quantities, except pressure, are functions of y and t only. The fluid is subjected to constant suction at the plate and hence if (u, v, w) are the velocity components in the fluid, the equation of continuity gives $v = -v_0$. Using the relation $\nabla . \bar{H} = 0$ for the magnetic field $\bar{H} = (H_x, H_y, H_z)$, we obtain $H_y = H_0$ everywhere in the fluid (H_0 is a constant). If $\bar{J} = (J_x, J_y, J_z)$ is the current density, from the relation $\nabla . \bar{J} = 0$, we have $J_y =$ constant. Since the plate is non-conducting, $J_y = 0$ at the plate and hence zero everywhere. Assuming the magnetic Reynolds number to be small, we neglect the induced magnetic field in comparison with the applied field. The generalized Ohm's law, in the absence of the electric field (Meyer 1958), is

$$\bar{J} + \frac{\omega_e \tau_e}{H_0} \bar{J} \times \bar{H} = \sigma\left(\mu_e \bar{q} \times \bar{H} + \frac{1}{e n_e} \nabla p_e\right)$$

where σ, μ_e, ω_e, τ_e, e, n_e and p_e are respectively the electric conductivity, the magnetic permeability, the cyclotron frequency, the electron collision time, the electric charge, the number density of the electron and the electron pressure. Under the usual assumptions that the electron pressure (for a weakly ionized gas), the thermoelectric pressure and ion slip are negligible, we have from the Ohm's law

$$J_x - \omega_e \tau_e J_z = -\sigma \mu_e H_0 w$$

$$J_z + \omega_e \tau_e J_x = \sigma \mu_e H_0 u$$

from which we obtain

$$J_x = \frac{\sigma \mu_e H_0}{1 + m^2} (mu - w)$$

$$J_z = \frac{\sigma \mu_e H_0}{1 + m^2} (u + mw)$$

where $m = \omega_e \tau_e$ is the Hall parameter.

In accordance with the Boussinesq approximation, we assume that all fluid properties are considered constant except that the density variation with temperature is considered only in the body force term.

The equations of motion are

$$\rho\left(\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma\mu_e^2 H_0^2}{1+m^2}(u + mw) - \rho g \qquad \text{...(1)}$$

$$0 = -\frac{\partial p}{\partial y} \qquad \text{...(2)}$$

$$\rho\left(\frac{\partial w}{\partial t} - v_0 \frac{\partial w}{\partial y}\right) = -\frac{\partial p}{\partial z} + \mu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma\mu_e^2 H_0^2}{1+m^2}(mu - w). \qquad \text{...(3)}$$

The boundary conditions are

$$u = 0,\ T = T_w \quad \text{at} \quad y = 0 \qquad \text{...(4)}$$

$$u \to U(t) = U_0(1 + \epsilon e^{i\omega t}),\ T \to T_\infty \quad \text{as} \quad y \to \infty \qquad \text{...(5)}$$

where ω is the frequency of the fluctuating stream, U_0 the mean velocity and ϵU_0 the amplitude of the free stream fluctuations. From eqns. (1) and (3), we have for the free stream

$$\rho\frac{\partial U}{\partial t} = -\frac{\partial p}{\partial x} - \frac{\sigma\mu_e^2 H_0^2}{1+m^2} U - \rho_\infty g \qquad \text{...(6)}$$

$$0 = -\frac{\partial p}{\partial z} + \frac{\sigma\mu_e^2 H_0^2}{1+m^2} mU. \qquad \text{...(7)}$$

From eqns. (1) to (7), we obtain

$$\rho\left(\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y}\right) = \rho\frac{\partial U}{\partial t} + \mu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma\mu_e^2 H_0^2}{1+m^2}(u - U + mw) + g(\rho_\infty - \rho) \qquad \text{...(8)}$$

$$\rho\left(\frac{\partial w}{\partial t} - v_0 \frac{\partial w}{\partial y}\right) = \mu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma\mu_e^2 H_0^2}{1+m^2}(mu - mU - w). \qquad \text{...(9)}$$

From the equation of state, we have

$$g(\rho_\infty - \rho) = \beta' g\rho(T - T_\infty) \qquad \text{...(10)}$$

where β' is the coefficient of volume expansion.

We introduce the following non-dimensional variables and parameters:

$$y^* = yU_0/\nu,\ t^* = tU_0^2/\nu,\ \omega^* = \omega\nu/U_0^2,\ u^* = u/U_0$$

$$\lambda = v_0/U_0,\ U^* = U/U_0,\ \theta = (T - T_\infty)/(T_w - T_\infty)$$

$$G = \nu g\beta'(T_w - T_\infty)/U_0^3 \quad \text{(Grashoff number)}$$

$$P = \mu c_p/k \quad \text{(Prandtl Number)}$$

$$E = U_0^2/c_p(T_w - T_\infty) \quad \text{(Eckert number)}$$

$$M = \sigma\mu_e^2 \; H_0^2/\rho U_0^2 \quad \text{(magnetic parameter)}$$

$$\delta = M/(1 + m^2).$$

On using these non-dimensional quantities, we obtain the equations governing the motion, in the non-dimensional form as (on dropping the affix *)

$$\frac{\partial u}{\partial t} - \lambda \frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial y^2} - M(u - U + mw) - G\theta \qquad \text{...(11)}$$

$$\frac{\partial w}{\partial t} - \lambda \frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} + M(mu - mU - w) \qquad \text{...(12)}$$

$$\frac{\partial T}{\partial t} - \lambda \frac{\partial T}{\partial y} = \frac{1}{P}\frac{\partial T}{\partial y^2} + E\left(\frac{\partial u}{\partial y}\right)^2 + E\left(\frac{\partial w}{\partial y}\right)^2. \qquad \text{...(13)}$$

The boundary conditions reduce to

$$u = 0,\ w = 0,\ \theta = 1 \quad \text{at} \quad y = 0 \qquad \text{...(14)}$$

$$u \to U(t),\ w \to 0,\ \theta = 0 \quad \text{as} \quad y \to \infty. \qquad \text{...(15)}$$

To solve these coupled non-linear equations, we assume that the unsteady flow is superimposed on the mean steady flow. Hence we write in the neighbourhood of the plate

$$u(y, t) = u_0(y) + \epsilon e^{i\omega t} u_1(y) \qquad \text{...(16)}$$

$$w(y, t) = w_0(y) + \epsilon e^{i\omega t} w_1(y) \qquad \text{...(17)}$$

$$\theta(y, t) = \theta_0(y) + \epsilon e^{i\omega t} \theta_1(y) \qquad \text{...(18)}$$

and the free stream is given by

$$U = 1 + \epsilon e^{i\omega t}. \qquad \text{...(19)}$$

Substituting (16) to (19) in eqns. (11) to (13), we obtain governing differential for equations for u_0, w_0, θ_0, u_1, w_1, and θ_1 :

$$u_0'' - \delta\,[(u_0 - 1) + mw_0] + \lambda u_0' + G\theta_0 = 0 \qquad \text{...(20)}$$

$$w_0'' + \delta\,[m(u_0 - 1) - w_0] + \lambda w_0' = 0 \qquad \text{...(21)}$$

$$\theta_0'' + P\lambda\theta_0' + PE(u_0'^2 + w_0'^2) = 0 \quad ...(22)$$

$$u_1'' - \delta[(u_1 - 1) + mw_1] - i\omega(u_1 - 1) + \lambda u_1' + G\theta_1 = 0 \quad ...(23)$$

$$w_1'' + \delta[m(u_1 - 1) - w_1] - i\omega w_1 + \lambda w_1' = 0 \quad ...(24)$$

$$\theta_1'' + PE(2u_0' u_1' + 2w_0' w_1') + \lambda P\theta_1' - i\lambda P\theta_1 = 0 \quad ...(25)$$

where the prime denotes differentiation with respect to y. The boundary conditions (14) and (15) become

$$u_0 = 0, w_0 = 0, \theta_0 = 1 \text{ at } y = 0 \text{ and } u_0 \to 1, w_0 \to 0, \theta_0 = 0 \text{ as } y \to \infty \quad ...(26)$$

$$u_1 = w_1 = 0, \theta_1 = 0 \text{ at } y = 0 \text{ and } u_1 \to 1, w_1 \to 0, \theta_1 = 0 \text{ as } y \to \infty. \quad ...(27)$$

Equations (20) to (22) for u_0, w_0, and θ_0 and the eqns. (23) to (25) for u_1, w_1 and θ_1 are non-linear but coupled and hence difficult to solve. We expand the variables in powers of E, the Eckert number under the assumption $E \ll 1$ for incompressible fluids. We shall write

$$u_0(y) = u_{01}(y) + Eu_{02}(y) + o(E^2) \quad ...(28)$$

$$u_1(y) = u_{11}(y) + Eu_{12}(y) + o(E^2) \quad ...(29)$$

$$w_0(y) = w_{01}(y) + Ew_{02}(y) + o(E^2) \quad ...(30)$$

$$w_1(y) = w_{11}(y) + Ew_{12}(y) + o(E^2) \quad ...(31)$$

$$\theta_0(y) = \theta_{01}(y) + E\theta_{02}(y) + o(E^2) \quad ...(32)$$

$$\theta_1(y) = \theta_{11}(y) + E\theta_{12}(y) + o(E^2). \quad ...(33)$$

Substituting (28), (30) and (32) in eqns. (20) to (22), we obtain the following coupled differential equations for u_0, w_0 and θ_0.

$$u_{01}'' - \delta[(u_{01} - 1) + mw_{01}] + \lambda u_{01}' + G\theta_{01} = 0 \quad ...(34)$$

$$u_{02}'' - \delta[u_{02} + mw_{02}] + \lambda u_{02}' + G\theta_{02} = 0 \quad ...(35)$$

$$w_{01}'' + \delta[m(u_{01} - 1) - w_{01}] + \lambda w_{01}' = 0 \quad ...(36)$$

$$w_{02}'' + \delta[mu_{02} - w_{02}] + \lambda w_{02}' = 0 \quad ...(37)$$

$$\theta_{01}'' + P\lambda\theta_{01}' = 0 \quad ...(38)$$

$$\theta_{02}'' + P\lambda\theta_{02}' + P(u_{01}'^2 + w_{01}'^2) = 0. \quad ...(39)$$

Substituting (29), (31) and (33) in eqns. (23) to (25), we can obtain another set of coupled equations for u_{11}, u_{12}, w_{11}, w_{12}, θ_{11} and θ_{12}. In this paper we concentrate the attention on the steady part of the solution only. Hence we solve the coupled eqns. (34) - (39) subject to the boundary conditions obtained from (26) and (27), namely

$$u_{01} = 0, u_{02} = 0, w_{01} = 0, w_{02} = 0, \theta_{01} = 1, \theta_{02} = 0 \quad \text{at} \quad y = 0 \qquad \ldots(40)$$

$$u_{01} = 1, u_{02} = 0, w_{01} = 0, w_{02} = 0, \theta_{01} = 0, \theta_{02} = 0 \quad \text{at} \quad y = \infty. \qquad \ldots(41)$$

Solving eqns. (34) - (39) with the boundary conditions (40) and (41), we obtain

$$u_{01} = -(A+1)\,e^{-\alpha y}\cos\beta y - \frac{mA\delta}{f(P\lambda)}\,e^{-\alpha y}\sin\beta y + Ae^{-\lambda Py} + 1 \qquad \ldots(42)$$

$$w_{01} = -(A+1)\,e^{-\alpha y}\sin\beta y + \frac{mA\delta}{f(\lambda P)}\,(e^{-\alpha y}\cos\beta y - e^{-\lambda Py}) \qquad \ldots(43)$$

$$u_{02} = Be^{-\lambda Py} + Ce^{-(\alpha+\lambda P)y}\cos\beta y + De^{-(\alpha+\lambda P)y}\sin\beta y + He^{-\alpha y}\sin\beta y - Ke^{-\alpha y}\cos\beta y + Qe^{-2\lambda Py} + Re^{-2\alpha y} \qquad \ldots(44)$$

$$m\delta w_{02} = [Bf(P) - GE_1]\,e^{-\lambda Py} + [L(C,D) + GE_1]\,e^{-(\alpha+\lambda P)y}\cos\beta y + [M(C,D) + GF_1]\,e^{-(\alpha+\lambda P)y}\sin\beta y - mH\delta e^{-\alpha y}\cos\beta y - mK\delta e^{-\alpha y}\sin\beta y + Qf(2\lambda P)\,e^{-2\lambda Py} + Rf(2\alpha)\,e^{-2\alpha y} \qquad \ldots(45)$$

$$\theta_{01} = e^{-\lambda Py} \qquad \ldots(46)$$

$$\theta_{02} = D_1e^{-\lambda Py} + E_1e^{-(\alpha+\lambda P)y}\cos\beta y + F_1e^{-(\alpha+\lambda P)y}\sin\beta y + Q_1e^{-2\alpha y} + R_1e^{-2\lambda Py} \qquad \ldots(47)$$

where

$$\alpha = \frac{\lambda}{2} + \tfrac{1}{2}\left[\frac{(\lambda^2+4\delta) + [(\lambda^2+4\delta)^2 + 16m^2\delta^2]^{1/2}}{2}\right]^{1/2}$$

$$\beta = \tfrac{1}{2}\left[\frac{-(\lambda^2+4\delta) + [(\lambda^2+4\delta)^2 + 16m^2\delta^2]^{1/2}}{2}\right]^{1/2}$$

$$f(a) = a^2 - a\lambda - \delta$$

$$A = -Gf(\lambda P)/[f^2(\lambda P) + m^2\delta^2]$$

$$D_1 = -E_1 - \frac{A^2P}{2}\left(1 + \frac{m^2\delta^2}{f^2(\lambda P)}\right) - \frac{m^2\delta^2P(\alpha^2+\beta^2)}{(4\alpha^2 - 2\alpha P\lambda)}\left[\frac{A^2}{f^2(\lambda P)} + \frac{(A+1)^2}{m^2\delta^2}\right]$$

$$E_1 = \left[(\alpha+\lambda P)\left(1 + A + \frac{Am^2\delta^2}{f^2(\lambda P)}\right) - \frac{m\beta\delta}{f(\lambda P)}\right]\left[\frac{2\lambda AP^2}{(\alpha+\lambda P)^2 + \beta^2}\right]$$

$$F_1 = \left[-\frac{m\delta}{f(\lambda P)}(\alpha + \lambda P) - \beta\left(1 + A + \frac{Am^2\delta^2}{f^2(\lambda P)}\right)\right]\left[\frac{2\lambda AP^2}{(\alpha + \lambda P)^2 + \beta^2}\right]$$

$$Q_1 = \frac{Pm^2\delta^2}{4\alpha^2 - 2\alpha P\lambda}\left[\frac{A^2}{f^2(\lambda P)} + \frac{(A+1)^2}{m^2\delta^2}\right](\alpha^2 + \beta^2)$$

$$R_1 = \frac{PF^2}{2}\left(1 + \frac{m^2\delta^2}{f^2(\lambda P)}\right)$$

$$X = \lambda^2P^2\,[\lambda(P-1) + 2\alpha]^2 - 4\beta^2\lambda^2P^2 - 4m\lambda P\beta\delta$$

$$Y = 2\lambda P\,[\lambda(P-1) + 2\alpha]\,[2\beta\lambda P + m\delta]$$

$$L(a, b) = -a\beta^2 - 2b\beta(\alpha + \lambda P) + a(\alpha + \lambda P)^2 - a\lambda(\alpha + \lambda P) - a\delta + b\lambda\beta$$

$$M(a, b) = -b\beta^2 + 2a\beta(\alpha + \lambda P) + b(\alpha + \lambda P)^2 - b\lambda(\alpha + \lambda P) - b\delta - a\lambda P$$

$$B = -GD_1 f(\lambda P)/[f^2(\lambda P) + m^2\delta^2]$$

$$C = -G(XL + YM)/(X^2 + Y^2)$$

$$D = G(YL - XM)/(X^2 + Y^2)$$

$$Q = -\frac{GPA^2}{2}\left(1 + \frac{m^2\delta^2}{f^2(\lambda P)}\right)\frac{f(2\lambda P)}{f^2(2\lambda P) + m^2\delta^2}$$

$$R = \frac{-GQ_1 f(2\alpha)}{f^2(2\alpha) + m^2\delta^2}$$

$$H = [Bf(\lambda P) + L(C, D) + Qf(2\lambda P) + Rf(2\alpha)]/m\delta$$

$$K = B + C + Q + R.$$

3. Discussion of the Results

From the solution, we see that the Hall currents induce a cross flow in the z-direction. This flow can be visualised as a secondary flow. It is clear from the solution that the steady flow exhibits a boundary layer behaviour. Since the magnetic field is strong, for small values of the suction parameter ($\lambda^2 \ll \delta$), $\alpha = o(\sqrt{\delta})$ the exponential $e^{-\lambda P y}$ decays least rapidly than the other exponential terms and hence the thickness of the boundary layer is of order $1/\lambda P$ (assuming that P is less than one or order one). However when $\lambda P \gg \alpha$ or order α, $1/\alpha$ can be taken as a measure of the boundary layer thickness. In the later case, the boundary layer thickness decreases with the increase in the magnetic parameter and increases with the increase in the Hall parameter. When the Grashoff number G is small ($G \ll 1$), neglecting terms of order G in the solution, we have

$$u_0 \simeq 1 - e^{-\alpha y}\cos\beta y,\quad w_0 \simeq -e^{-\alpha y}\sin\beta y$$

and the boundary layer thickness is of order $1/\alpha$. Also the steady primary and secondary velocity distributions are in the form of a logarithmic spiral. Thus the effect of the Hall current is similar to that of rotation observed for a flow past a flat plate in a rotating fluid (Batchelor 1967). When the Grashoff number is of order 1 and if $\alpha \gg \lambda P$, outside a neighbourhood of the infinite plate (i.e. for distances y such that $\alpha y = O(1)$), the velocities are

$$u_0 \simeq 1 + Ae^{-\lambda P y}$$

$$w_0 \simeq \frac{mA\delta}{f(\lambda P)}\, e^{-\lambda P y}$$

and satisfy the linear relation $u_0 - 1 = Cw_0$ (C = constant).

From eqns. (42) and (43) it can also be seen that near the plate

$$u_0 \simeq \left[\alpha(A+1) - \lambda PA - \frac{mA\delta\beta}{f(\lambda P)}\right] y$$

$$w_0 \simeq \left[\frac{mA\delta}{f(\lambda P)}(\lambda P - \alpha) - \beta(A+1)\right] y.$$

The primary and secondary flow near the plate $y = 0$ are inclined at an angle

$$\tan^{-1}\left[\frac{mA\delta}{f(\lambda P)}(\lambda P - \alpha) - (A+1)\,\beta\right] \Big/ \left[\alpha(A+1) - \lambda PA - \frac{mA\delta\beta}{f(\lambda P)}\right]$$

to the direction of the free stream. When the Grashoff number $G = 0$, this angle reduces to $\tan^{-1}(-\beta/\alpha)$. Since $\beta < \alpha$, this angle lies between 0 and $-\pi/4$.

To study some more properties of the velocity distribution, curves are drawn for various values of the parameters (Figs. 1 – 7) that describe the flow.

(i) *The Primary Flow*

The primary velocity $u_0(y)$ is plotted with y in Fig. 1 when $M = 10$, $G = 5$, $E = 0.01$, $\lambda = 0.5$ and for various values of m and P. It is observed that as P increases the velocity u_0 decreases and the velocity u_0 takes less distance to reach unity when $P = 2.0$ than when $P = 0.5$. Also as the parameter m increases, the velocity increases.

Figure 2 shows the velocity distribution when $M = 10$, $E = 0.01$, $P = 2$, $m = 0.5$, $\lambda = 0.5$ and for values of $G = 0, 5, 10, -5$ and -10. The distance at which u_0 tends to unity is least when $G = 0$ in comparison with the distance observed when $G \neq 0$. When G is positive and increasing the velocity increases and the distance at which u_0 tends to unity also increases. When G is negative, the velocity decreases as G increases but the distance at which the decay in $(u_0 - 1)$

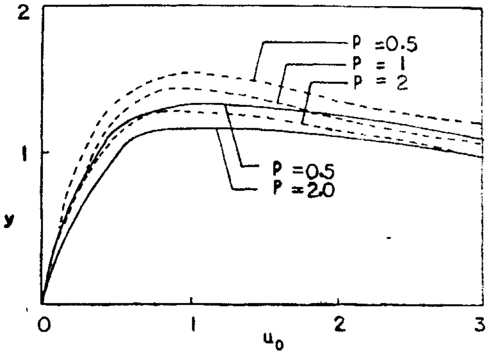

FIG. 1. Velocity u_0 plotted with y for various values of P - - - - $m = 2.5$, ——— $m = 0.5$.

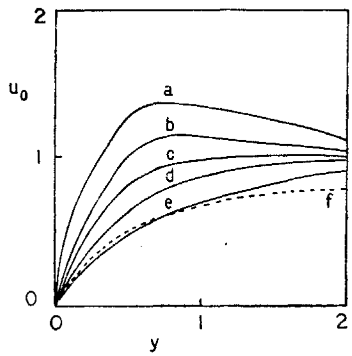

FIG. 2. Velocity u_0 when $m = 0.5$, $M = 10$, $E = 0.01$, $\lambda = 0.5$. (a) $G = 10$, $P = 2$; (b) $G = 5$, $P = 2$; (c) $G = 0$, $P = 2$; (d) $G = -5$, $P = 2$; (e) $G = -10$, $P = 2$; (f) $G = -5$, $P = 0.5$.

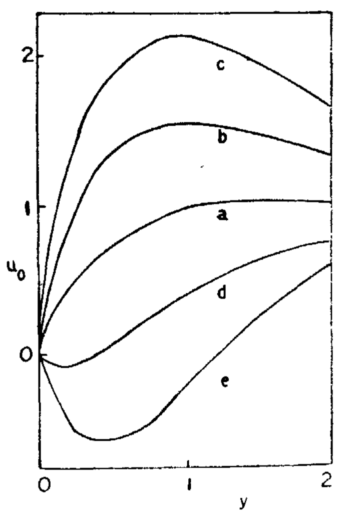

FIG. 3. Velocity u_0 when $m = 2.5$, $M = 10$, $E = 0.01$, $P = 0.5 = \lambda$. (a) $G = 0$, (b) $G = 5$, (c) $G = 10$, (d) $G = -5$, (e) $G = -10$.

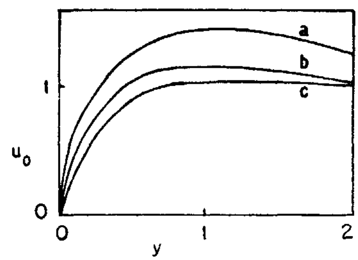

FIG. 4. Velocity u_0 plotted with y when $P = 2$, $M = 10$, $m = 2.5$, $G = 5$. (a) $\lambda = 0.5$, (b) $\lambda = 1$, (c) $\lambda = 2$.

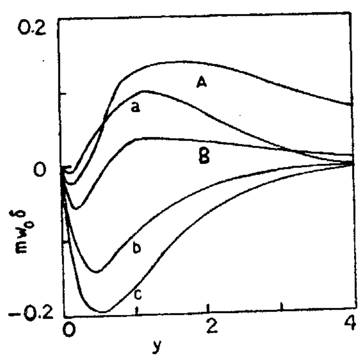

FIG. 5. The plot of $m\delta w_0$ when $m = 0.5$, $\lambda = 0.5$, $M = 10$. (A) $P = 0.5$, $G = 5$; (B) $P = 2$, $G = 5$; (a) $P = 2$, $G = 10$; (b) $P = 2$, $G = -5$; (c) $P = 2$, $G = -10$.

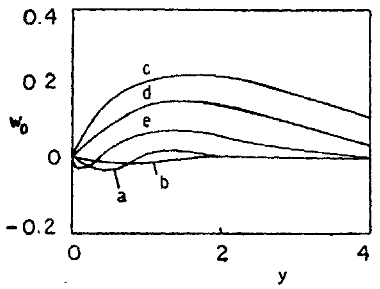

FIG. 6. Velocity w_0 when (a) $\lambda = 1$, $P = 2$, $m = 0.5$, $G = 5$; (b) $\lambda = 2$, $P = 2$, $m = 0.5$, $G = 5$; (c) $P = 0.5$, $\lambda = 0.5$, $m = 2.5$, $G = 5$; (d) $P = 1$, $\lambda = 0.5$, $m = 2.5$, $G = 5$; (e) $P = 2$, $\lambda = 0.5$, $m = 2.5$, $G = 5$.

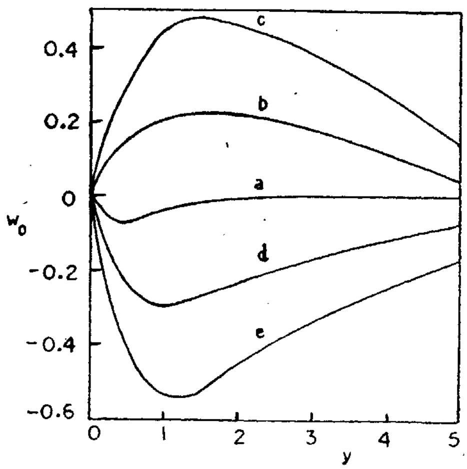

FIG. 7. Velocity w_0 when $m = 2.5$, $P = \lambda = 0.5$, $M = 10$. (a) $G = 0$, (b) $G = 5$, (c) $G = 10$, (d) $G = -5$, (e) $G = -10$.

occurs increases as G decreases. Further the velocity is always positive and no reverse type of flow occurs for any value of G.

Figure 3 shows the velocity distribution for various values of G when $m = 2.5$. The property that the velocity increases with G (when positive), observed for $m = 0.5$ remains valid for $m = 2.5$ also. But for negative values of G, we find that the velocity becomes negative in a region adjacent to the plate and ultimately turns positive. Thus, for larger values of m, when the plate is heated by the free convection currents, or due to greater viscous dissipative heat, the velocity profiles show that there is a reversed flow and the layer of the fluid adjacent to the plate in which the reversed flow occurs increases in thickness when G is negative and decreasing.

The velocity curves are drawn in Fig. 4 for various values of λ and it is observed that as λ increases the velocity decreases.

(ii) *The Secondary Flow*

In Figs. 5 and 6, the velocity w_0 is plotted with y. As P increases, the velocity w_0 decreases. But the secondary flow suffers from a reversed flow at distances adjacent to the plate and extends to distances away from the plate as P increases. There appears a reversed flow for values of Grashoff number $G > 0$ and as G increases the reversed flow gets confined to a thin layer near the plate.

This behaviour of the secondary flow does not persist for larger values of the Hall parameter m, as seen from Fig. 7 in which the curves for positive values of G show that the velocity is positive.

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