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DECOMPOSITION OF NEO-RECURRENT CURVATURE TENSOR FIELD OF THE FIRST ORDER

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Takano (1967) studied the decomposition of curvature tensor field in recurrent space. The object of the present paper is to decompose the neo-recurrent curvature tensor field of the first order and to study the properties of such decomposition.

1. INTRODUCTION

Let F_m be a subspace of an n-dimensional Finsler space $F_n (n > m)$. The tangential component $A^\alpha (\alpha = 1, 2, \ldots, m)$ of the curvature vector of a curve C in F_m is given by

$$A^\alpha = \frac{d^2 u^\alpha}{ds^2} + F^\alpha_{\beta\gamma} \frac{du^\beta}{ds} \frac{du^\gamma}{ds}$$

where

$$F^\alpha_{\beta\gamma} (u, \dot{u}) \overset{def}{=} P^{*\alpha}_{\beta\gamma} (u, \dot{u}) + \sum_{\nu, \tau} \bar{C}^*_{\nu\tau\,|}\ \Omega^*_{\nu\,|\,\beta\gamma} \left(t^{*\alpha}_{\tau\,|} - \sec \phi_{\tau\,|} \frac{du^\alpha}{ds} \right)$$

is the neo-connection of F_m and $P^{*\alpha}_{\beta\gamma}$ is the induced connection (Chandra 1972).

The neo-covariant differentiation of the vector $Y^\alpha (u, \dot{u})$ with respect to u^β is given by (Chandra 1972)

$$\overset{n}{\underset{\beta}{\nabla}} Y^\alpha = \partial_\beta Y^\alpha + \dot{\partial}_\delta Y^\alpha \partial_\beta u^\delta + F^\alpha_{\beta\gamma} Y^\gamma \qquad \ldots(1.1)$$

where $\partial_\beta Y^\alpha$ and $\dot{\partial}_\delta Y^\alpha$ represents the partial differentiation of Y^α with respect to u^β and $\dot{u}^\delta$ respectively and $\overset{n}{\nabla}$ is the notation for the neo-covariant differentiation.

Now differentiating (1.1) neo-covariantly with respect to u^γ, and commuting the indices β and γ we have

$$\overset{n}{\underset{\beta\gamma}{\nabla}} Y^\alpha - \overset{n}{\underset{\gamma\beta}{\nabla}} Y^\alpha = N^\alpha_{\delta\beta\gamma} Y^\delta \qquad \ldots(1.2)$$

where

$$N^{\alpha}_{\delta\beta\gamma} = \left(\frac{\partial}{\partial u^{\gamma}} F^{\alpha}_{\beta\delta} - \frac{\partial}{\partial u^{\beta}} F^{\alpha}_{\gamma\delta}\right) + \left(\frac{\partial}{\partial \dot{u}^{\phi}} F^{\alpha}_{\beta\delta} \frac{\partial \dot{u}^{\phi}}{\partial u^{\gamma}} - \frac{\partial}{\partial \dot{u}^{\phi}} F^{\alpha}_{\gamma\delta} \frac{\partial \dot{u}^{\phi}}{\partial u^{\beta}}\right)$$
$$+ (F^{\alpha}_{\gamma\phi} F^{\phi}_{\beta\delta} - F^{\alpha}_{\beta\phi} F^{\phi}_{\gamma\delta})$$

is the neo-curvature tensor (Chandra 1972) which is skew symmetric in β and γ. The neo-curvature tensor $N^{\alpha}_{\delta\beta\gamma}$ obviously satisfies the relation

$$N^{\alpha}_{\delta\beta\gamma} + N^{\alpha}_{\beta\gamma\delta} + N^{\alpha}_{\gamma\delta\beta} = 0 \qquad \text{...(1.3)}$$

and the Bianchi identities

$$\overset{n}{\underset{\phi}{\nabla}} N^{\alpha}_{\delta\beta\gamma} + \overset{n}{\underset{\beta}{\nabla}} N^{\alpha}_{\delta\gamma\phi} + \overset{n}{\underset{\gamma}{\nabla}} N^{\alpha}_{\delta\phi\beta} = 0.$$

If $T_{\alpha\beta}(u, \dot{u})$ be a tensor field, then using (1.2), the commutation formulae involving the neo-curvature tensor field may be written as

$$\overset{n}{\underset{\gamma\delta}{\nabla}} T_{\alpha\beta} - \overset{n}{\underset{\delta\gamma}{\nabla}} T_{\alpha\beta} = - T_{\alpha\phi} N^{\phi}_{\beta\gamma\delta} - T_{\phi\beta} N^{\phi}_{\alpha\gamma\delta}. \qquad \text{...(1.4)}$$

The neo-recurrent curvature tensor field of the first order is defined by

$$\overset{n}{\underset{\phi}{\nabla}} N^{\alpha}_{\delta\beta\gamma} = v_{\phi} N^{\alpha}_{\delta\beta\gamma} \qquad \text{...(1.5)}$$

where v_{ϕ} is a non-zero neo-recurrence vector field (Prasad and Srivastava 1979).

Using (1.5) in Bianchi identities, we have

$$v_{\phi} N^{\alpha}_{\delta\beta\gamma} + v_{\beta} N^{\alpha}_{\delta\gamma\phi} + v_{\gamma} N^{\alpha}_{\delta\phi\beta} = 0. \qquad \text{...(1.6)}$$

2. Decomposition of Neo-recurrent Curvature Tensor Fields

Let us assume the decomposition of the neo-recurrent curvature tensor field $N^{\alpha}_{\delta\beta\gamma}$ in the form

$$N^{\alpha}_{\delta\beta\gamma} = U^{\alpha} M_{\delta\beta\gamma}(u, \dot{u}) \qquad \text{...(2.1)}$$

where $M_{\delta\beta\gamma}$ is a tensor field and U^{α} is a non-zero vector field such that

$$U^{\alpha} v_{\alpha} = \sigma \qquad \text{...(2.2)}$$

The following theorems can be stated obviously :

Theorem 2.1 — The tensor field $M_{\delta\beta\gamma}(u, \dot{u})$ satisfies the identities

$$M_{\delta\beta\gamma} + M_{\delta\gamma\beta} = 0 \qquad \text{...(2.3)}$$

and

$$M_{\delta\beta\gamma} + M_{\beta\gamma\delta} + M_{\gamma\delta\beta} = 0. \qquad \text{...(2.4)}$$

Theorem 2.2 — Under the decomposition (2.1), the Bianchi identities for the neo-curvature tensor field take the form

$$v_\phi M_{\delta\beta\gamma} + v_\beta M_{\delta\gamma\phi} + v_\gamma M_{\delta\phi\beta} = 0. \qquad \text{...(2.5)}$$

Theorem 2.3 — Under the decomposition (2.1), the neo-curvature tensor field satisfies the relation

$$v_\alpha N^\alpha_{\delta\beta\gamma} = (v_\gamma \bar{M}_{\delta\beta} - v_\beta \bar{M}_{\delta\gamma}) \qquad \text{...(2.6)}$$

where $M_{\delta\beta\gamma} U^\gamma = \bar{M}_{\delta\beta}$ is any tensor field.

Now we shall prove the following theorems :

Theorem 2.4 — The necessary and sufficient condition that the tensor field $M_{\delta\beta\gamma}$ behaves like a neo-recurrence tensor field of the first order is that U^α be neo-covariant constant.

PROOF : Taking neo-covariant differentiation of (2.1) with respect to u^ϕ we get

$$\overset{n}{\underset{\phi}{\nabla}} N^\alpha_{\delta\beta\gamma} = (\overset{n}{\underset{\phi}{\nabla}} U^\alpha)\, M_{\delta\beta\gamma} + U^\alpha (\overset{n}{\underset{\phi}{\nabla}} M_{\delta\beta\gamma}). \qquad \text{...(2.7)}$$

Using (1.5) and (2.1) in eqn. (2.7), we get

$$v_\phi U^\alpha M_{\delta\beta\gamma} = (\overset{n}{\underset{\phi}{\nabla}} U^\alpha)\, M_{\delta\beta\gamma} + U^\alpha (\overset{n}{\underset{\phi}{\nabla}} M_{\delta\beta\gamma})$$

which may be written as

$$U^\alpha (\overset{n}{\underset{\phi}{\nabla}} M_{\delta\beta\gamma} - v_\phi M_{\delta\beta\gamma}) = (\overset{n}{\underset{\phi}{\nabla}} U^\alpha)\, M_{\delta\beta\gamma}. \qquad \text{...(2.8)}$$

From (2.8), the necessary and sufficient condition follows.

This proves the theorem.

Theorem 2.5 — If the vector U^α be neo-covariant constant, then under the decomposition (2.1) the vector field v_ϕ satisfies the relation

$$\overset{n}{\underset{\epsilon}{\nabla}} (\overset{n}{\underset{\theta}{\nabla}} v_\phi - \overset{n}{\underset{\phi}{\nabla}} v_\theta) = v_\epsilon (\overset{n}{\underset{\theta}{\nabla}} v_\phi - \overset{n}{\underset{\phi}{\nabla}} v_\theta) \qquad \text{...(2.9)}$$

PROOF : Since $\overset{n}{\underset{\phi}{\nabla}} U^{\alpha} = 0$, then from eqn. (2.8), we get

$$\overset{n}{\underset{\phi}{\nabla}} M_{\delta\beta\gamma} = v_{\phi} M_{\delta\beta\gamma}. \qquad \ldots(2.10)$$

Differentiating (2.10) neo-covariantly with respect to u^{θ}, we get

$$\overset{n}{\underset{\phi\theta}{\nabla}} M_{\delta\beta\gamma} = (\overset{n}{\underset{\theta}{\nabla}} v_{\phi})\, M_{\delta\beta\gamma} + v_{\phi}(\overset{n}{\underset{\theta}{\nabla}} M_{\delta\beta\gamma}) \qquad \ldots(2.11)$$

Using (2.10) in (2.11), we get

$$\overset{n}{\underset{\phi\theta}{\nabla}} M_{\delta\beta\gamma} = (\overset{n}{\underset{\theta}{\nabla}} v_{\phi})\, M_{\delta\beta\gamma} + v_{\phi} v_{\theta} M_{\delta\beta\gamma}. \qquad \ldots(2.12)$$

Commuting the indices ϕ and θ in eqn. (2.12), we get

$$\overset{n}{\underset{\phi\theta}{\nabla}} M_{\delta\beta\gamma} - \overset{n}{\underset{\theta\phi}{\nabla}} M_{\delta\beta\gamma} = (\overset{n}{\underset{\theta}{\nabla}} v_{\phi} - \overset{n}{\underset{\phi}{\nabla}} v_{\theta})\, M_{\delta\beta\gamma}. \qquad \ldots(2.13)$$

Using the commutation formula (1.4) in eqn. (2.13), we get

$$(\overset{n}{\underset{\theta}{\nabla}} v_{\phi} - \overset{n}{\underset{\phi}{\nabla}} v_{\theta})\, M_{\delta\beta\gamma} = - M_{\psi\beta\gamma} N^{\psi}_{\delta\phi\theta} - M_{\delta\psi\gamma} N^{\psi}_{\beta\phi\theta} - M_{\delta\beta\psi} N^{\psi}_{\gamma\phi\theta}\,. \qquad \ldots(2.14)$$

Again differentiating (2.14) neo-covariantly with respect to u^{ϵ}, we get

$$\begin{aligned} &\overset{n}{\underset{\epsilon}{\nabla}} (\overset{n}{\underset{\theta}{\nabla}} v_{\phi} - \overset{n}{\underset{\phi}{\nabla}} v_{\theta})\, M_{\delta\beta\gamma} + (\overset{n}{\underset{\theta}{\nabla}} v_{\phi} - \overset{n}{\underset{\phi}{\nabla}} v_{\theta})(\overset{n}{\underset{\epsilon}{\nabla}} M_{\delta\beta\gamma}) \\ &= - (\overset{n}{\underset{\epsilon}{\nabla}} M_{\psi\beta\gamma})\, N^{\psi}_{\delta\phi\theta} - (\overset{n}{\underset{\epsilon}{\nabla}} M_{\delta\psi\gamma})\, N^{\psi}_{\beta\phi\theta} - (\overset{n}{\underset{\epsilon}{\nabla}} M_{\delta\beta\psi})\, N^{\psi}_{\gamma\phi\theta} \\ &\quad - M_{\psi\beta\gamma}(\overset{n}{\underset{\epsilon}{\nabla}} N^{\psi}_{\delta\phi\theta}) - M_{\delta\psi\gamma}(\overset{n}{\underset{\epsilon}{\nabla}} N^{\psi}_{\beta\phi\theta}) - M_{\delta\beta\psi}(\overset{n}{\underset{\epsilon}{\nabla}} N^{\psi}_{\gamma\phi\theta}). \end{aligned} \qquad \ldots(2.15)$$

Using (1.5), (2.10) and (2.14) in (2.15) and simplifying we get (2.9).

Hence the proof.

Theorem 2.6 — If the vector U^{α} be neo-covariant constant, then under the decomposition (2.1) the vector field v_{ϕ} satisfies the relation

$$v_{\phi}(\overset{n}{\underset{\psi}{\nabla}} v_{\theta} - \overset{n}{\underset{\theta}{\nabla}} v_{\psi}) + v_{\theta}(\overset{n}{\underset{\phi}{\nabla}} v_{\psi} - \overset{n}{\underset{\psi}{\nabla}} v_{\phi}) + v_{\psi}(\overset{n}{\underset{\theta}{\nabla}} v_{\phi} - \overset{n}{\underset{\phi}{\nabla}} v_{\theta}) = 0. \qquad \ldots(2.16)$$

PROOF : Neo-covariant differentiation of (2.10) with respect to u^θ, gives

$$\overset{n}{\underset{\phi\theta}{\nabla}} M_{\delta\beta\gamma} = (\overset{n}{\underset{\theta}{\nabla}} v_\phi) M_{\delta\beta\gamma} + v_\phi(\overset{n}{\underset{\theta}{\nabla}} M_{\delta\beta\gamma}). \quad ...(2.17)$$

Using (2.10) in (2.17), we get

$$\overset{n}{\underset{\phi\theta}{\nabla}} M_{\delta\beta\gamma} = (\overset{n}{\underset{\theta}{\nabla}} v_\phi) M_{\delta\beta\gamma} + v_\phi v_\theta M_{\delta\beta\gamma}. \quad ...(2.18)$$

Taking neo-covariant differentiation of (2.18) with respect to u^ψ, and in the resulting equation using equation (2.10), we get

$$\overset{n}{\underset{\phi\theta\psi}{\nabla}} M_{\delta\beta\gamma} = [\overset{n}{\underset{\theta\psi}{\nabla}} v_\phi + v_\psi(\overset{n}{\underset{\theta}{\nabla}} v_\phi) + v_\phi v_\theta v_\psi + v_\phi(\overset{n}{\underset{\psi}{\nabla}} v_\theta)$$
$$+ v_\theta(\overset{n}{\underset{\psi}{\nabla}} v_\phi)] M_{\delta\beta\gamma}. \quad ...(2.19)$$

Commuting the indices θ and ψ in eqn. (2.19), we get

$$(\overset{n}{\underset{\phi\theta\psi}{\nabla}} M_{\delta\beta\gamma} - \overset{n}{\underset{\phi\psi\theta}{\nabla}} M_{\delta\beta\gamma})$$
$$= [(\overset{n}{\underset{\theta\psi}{\nabla}} v_\phi - \overset{n}{\underset{\psi\theta}{\nabla}} v_\phi) + v_\phi(\overset{n}{\underset{\psi}{\nabla}} v_\theta - \overset{n}{\underset{\theta}{\nabla}} v_\psi)] M_{\delta\beta\gamma} \quad ...(2.20)$$

The eqn. (2.20) can be written as

$$\overset{n}{\underset{\theta\psi}{\nabla}} (\overset{n}{\underset{\phi}{\nabla}} M_{\delta\beta\gamma}) - \overset{n}{\underset{\psi\theta}{\nabla}} (\overset{n}{\underset{\phi}{\nabla}} M_{\delta\beta\gamma}) = [(\overset{n}{\underset{\theta\psi}{\nabla}} v_\phi - \overset{n}{\underset{\psi\theta}{\nabla}} v_\phi)$$
$$+ v_\phi(\overset{n}{\underset{\psi}{\nabla}} v_\theta - \overset{n}{\underset{\theta}{\nabla}} v_\psi)] M_{\delta\beta\gamma} \quad ...(2.21)$$

Using the commutation formula (1.4) in eqn. (2.21), we get

$$-(\overset{n}{\underset{\phi}{\nabla}} M_{\alpha\beta\gamma}) N^{\alpha}_{\delta\theta\psi} - (\overset{n}{\underset{\phi}{\nabla}} M_{\delta\alpha\gamma}) N^{\alpha}_{\beta\theta\psi} - (\overset{n}{\underset{\phi}{\nabla}} M_{\delta\beta\alpha}) N^{\alpha}_{\gamma\theta\psi}$$
$$= - v_\alpha N^{\alpha}_{\phi\theta\psi} M_{\delta\beta\gamma} + v_\phi(\overset{n}{\underset{\psi}{\nabla}} v_\theta - \overset{n}{\underset{\theta}{\nabla}} v_\psi) M_{\delta\beta\gamma}. \quad ...(2.22)$$

Using (2.10) in (2.22), this takes the form

$$- v_\phi(M_{\alpha\beta\gamma}N^{\alpha}_{\delta\theta\psi} + M_{\delta\alpha\gamma}N^{\alpha}_{\beta\theta\psi} + M_{\delta\beta\alpha}N^{\alpha}_{\gamma\theta\psi})$$
$$= - v_\alpha N^{\alpha}_{\phi\theta\psi} M_{\delta\beta\gamma} + v_\phi(\overset{n}{\underset{\psi}{\nabla}} v_\theta - \overset{n}{\underset{\theta}{\nabla}} v_\psi) M_{\delta\beta\gamma}. \quad ...(2.23)$$

Cyclic permutation of the indices ϕ, θ and ψ in (2.23) gives two more relations. On adding these three relations, we get

$$- M_{\alpha\beta\gamma} \{v_\phi N^{\alpha}_{\delta\theta\psi} + v_\theta N^{\alpha}_{\delta\psi\phi} + v_\psi N^{\alpha}_{\delta\phi\theta}\} - M_{\delta\alpha\gamma} \{v_\phi N^{\alpha}_{\beta\theta\psi}$$

$$+ v_\theta N^{\alpha}_{\beta\psi\phi} + v_\psi N^{\alpha}_{\beta\phi\theta}\} - M_{\delta\beta\alpha} \{v_\phi N^{\alpha}_{\gamma\theta\psi} + v_\theta N^{\alpha}_{\gamma\psi\phi} + v_\psi N^{\alpha}_{\gamma\phi\theta}\}$$

$$= - v_\alpha M_{\delta\beta\gamma} \{N^{\alpha}_{\phi\theta\psi} + N^{\alpha}_{\theta\psi\phi} + N^{\alpha}_{\psi\phi\theta}\} M_{\delta\beta\gamma} \{v_\phi(\overset{n}{\underset{\psi}{\nabla}} v_\theta - \overset{n}{\underset{\theta}{\nabla}} v_\psi)$$

$$+ v_\theta(\overset{n}{\underset{\phi}{\nabla}} v_\psi - \overset{n}{\underset{\psi}{\nabla}} v_\phi) + v_\psi(\overset{n}{\underset{\theta}{\nabla}} v_\phi - \overset{n}{\underset{\phi}{\nabla}} v_\theta)\}. \qquad ...(2.24)$$

Now using eqns. (1.3) and (1.6) in (2.24), we get (2.16). This proves the theorem.

Theorem 2.7 — If the vector U^α be neo-covariant constant, then under the decomposition (2.1) the tensor field $M_{\delta\beta\gamma}$ satisfies the relation

$$(\overset{n}{\underset{\phi\theta\psi}{\nabla}} M_{\delta\beta\gamma} - \overset{n}{\underset{\phi\psi\theta}{\nabla}} M_{\delta\beta\gamma}) + (\overset{n}{\underset{\theta\psi\phi}{\nabla}} M_{\delta\beta\gamma} - \overset{n}{\underset{\theta\phi\psi}{\nabla}} M_{\delta\beta\gamma})$$

$$+ (\overset{n}{\underset{\psi\phi\theta}{\nabla}} M_{\delta\beta\gamma} - \overset{n}{\underset{\psi\theta\phi}{\nabla}} M_{\delta\beta\gamma}) = 0. \qquad ...(2.25)$$

PROOF : Using the commutation formula (1.4) in eqn. (2.20), we get

$$(\overset{n}{\underset{\phi\theta\psi}{\nabla}} M_{\delta\beta\gamma} - \overset{n}{\underset{\phi\psi\theta}{\nabla}} M_{\delta\beta\gamma}) = - v_\alpha N^{\alpha}_{\phi\theta\psi} M_{\delta\beta\gamma}$$

$$+ v_\phi(\overset{n}{\underset{\psi}{\nabla}} v_\theta - \overset{n}{\underset{\theta}{\nabla}} v_\psi) M_{\delta\beta\gamma} \qquad ...(2.26)$$

Cyclically permuting ϕ, θ and ψ in (2.26), we get two more relations. On adding these three relations and using eqns. (1.3) and (2.16) we get (2.25).

Now decomposing the tensor field $M_{\delta\beta\gamma}$ in the form.

$$M_{\delta\beta\gamma} = v_\delta P_{\beta\gamma} \qquad ...(2.27)$$

we may establish the following theorems.

Theorem 2.8 — If the vector U^α be the neo-covariant constant then under the decompositions (2.1) and (2.27) the tensor field $P_{\phi\theta}$ can be expressed in the form

$$P_{\phi\theta} = \overset{n}{\underset{\phi}{\nabla}} v_\theta - \overset{n}{\underset{\theta}{\nabla}} v_\phi. \qquad ...(2.28)$$

PROOF : Differentiating (1.5) neo-covariantly with respect to u^θ and commuting the indices ϕ and θ in the resulting expression we get

$$\overset{n}{\underset{\phi\theta}{\nabla}} N^{\alpha}_{\delta\beta\gamma} - \overset{n}{\underset{\theta\phi}{\nabla}} N^{\alpha}_{\delta\beta\gamma} = (\overset{n}{\underset{\theta}{\nabla}} v_\phi - \overset{n}{\underset{\phi}{\nabla}} v_\theta) N^{\alpha}_{\delta\beta\gamma} . \qquad ...(2.29)$$

Using the commutation formula (1.4) in (2.29), we get

$$(\overset{n}{\underset{\theta}{\nabla}} v_\phi - \overset{n}{\underset{\phi}{\nabla}} v_\theta) N^{\alpha}_{\delta\beta\gamma} = N^{\psi}_{\delta\beta\gamma} N^{\alpha}_{\psi\phi\theta} - N^{\alpha}_{\psi\beta\gamma} N^{\psi}_{\delta\phi\theta} - N^{\alpha}_{\delta\psi\gamma} N^{\psi}_{\beta\phi\theta} - N^{\alpha}_{\delta\beta\psi} N^{\psi}_{\gamma\phi\theta} . \qquad ...(2.30)$$

By virtue of the decompositions (2.1) and (2.27) and the relation $M_{\delta\beta\gamma}U^\gamma = \bar{M}_{\delta\beta}$, we get

$$(\overset{n}{\underset{\theta}{\nabla}} v_\phi - \overset{n}{\underset{\phi}{\nabla}} v_\theta) M_{\delta\beta\gamma} = (v_\beta \bar{M}_{\delta\gamma} - v_\gamma \bar{M}_{\delta\beta}) P_{\phi\theta}. \qquad ...(2.31)$$

From (2.6), we have $M_{\delta\beta\gamma} = (1/\sigma)(v_\gamma \bar{M}_{\delta\beta} - v_\beta \bar{M}_{\delta\gamma})$. Using this in (2.31), we get the required result.

Theorem 2.9 — Under the decompositions (2.1) and (2.27), if the vector U^α is neo-covariant constant then the tensor field $P_{\beta\gamma}$ behaves like neo-recurrent tensor field provided σ is constant.

PROOF : Differentiating (2.27) neo-covariantly with respect to u^ϕ, we get

$$\overset{n}{\underset{\phi}{\nabla}} M_{\delta\beta\gamma} = (\overset{n}{\underset{\phi}{\nabla}} v_\delta) P_{\beta\gamma} + v_\delta(\overset{n}{\underset{\phi}{\nabla}} P_{\beta\gamma}). \qquad ...(2.32)$$

Since U^α is neo-covariant constant, we have from the theorem (2.4),

$$\overset{n}{\underset{\phi}{\nabla}} M_{\delta\beta\gamma} = v_\phi M_{\delta\beta\gamma}. \qquad ...(2.33)$$

Using (2.33) in (2.32) and in the resulting equation using (2.27) we get

$$v_\phi v_\delta P_{\beta\gamma} = (\overset{n}{\underset{\phi}{\nabla}} v_\delta) P_{\beta\gamma} + v_\delta(\overset{n}{\underset{\phi}{\nabla}} P_{\beta\gamma}). \qquad ...(2.34)$$

Transvecting (2.34) by U^δ and using (2.2), we get

$$\overset{n}{\underset{\phi}{\nabla}} P_{\beta\gamma} = v_\phi P_{\beta\gamma}.$$

This proves the theorem.

Theorem 2.10 — In a neo-recurrent Finsler space, under the decompositions (2.1) and (2.27), the necessary and sufficient condition for $\overset{n}{\underset{\psi}{\nabla}} v_\theta = \overset{n}{\underset{\theta}{\nabla}} v_\psi$, is that $\sigma=0$.

PROOF : From (2.27) and (2.23), we have

$$[P_{\beta\gamma}(\overset{n}{\underset{\psi}{\nabla}} v_\theta - \overset{n}{\underset{\theta}{\nabla}} v_\psi) + \{P_{\theta\psi}(v_\alpha P_{\beta\gamma} + v_\beta P_{\alpha\gamma} + v_\gamma P_{\beta\alpha}) - v_\alpha P_{\beta\gamma}\} U^\alpha] = 0. \quad ...(2.35)$$

Now, with the help of (2.4) and (2.27), eqn. (2.35) becomes

$$[P_{\beta\gamma}(\overset{n}{\underset{\psi}{\nabla}} v_\theta - \overset{n}{\underset{\theta}{\nabla}} v_\psi) + P_{\theta\psi} U^\alpha v_\alpha P_{\beta\gamma}] = 0. \quad ...(2.36)$$

Using (2.2) in (2.36), we get

$$(\overset{n}{\underset{\psi}{\nabla}} v_\theta - \overset{n}{\underset{\theta}{\nabla}} v_\psi) + \sigma P_{\theta\psi} = 0. \quad ...(2.37)$$

From (2.37), the necessary and sufficient conditions follow.

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