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A NOTE ON THE UNSTEADY FLOW OF VISCOELASTIC FLUIDS THROUGH TUBES WITH CROSS-SECTION AS A SECTOR OF A CIRCLE AND AS A SECTOR OF COAXIAL CIRCULAR DUCTS

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In this note, a study of the unsteady flow of viscoelastic fluids through tubes with cross-section as a sector of a circle and as a sector of coaxial circular ducts has been made, the main aim being to determine whether a viscoelastic fluid will behave in a similar manner or in a different manner if the flows are taken through different geometries.

1. INTRODUCTION

In viscometric flows, start-up problems are important (Huilgol 1975). The transitional velocity field as well as the final velocity field are of practical interest in the analysis of this flow. Thomas and Walters (1963) show that presence of elasticity in the liquid increases the rate of discharge of liquid in a curved pipe under a pressure gradient. In the case of stability of an elastico-viscous liquid film flowing down an inclined plane Lai (1967) shows that presence of elasticity in the liquid destabilizes the flow. Barnes and Walters (1969) show that the flow rate of liquid through straight and curved pipes is significantly enhanced by the presence of elasticity. In the problem of unsteady motion of a sphere in an elastico-viscous liquid, King and Waters (1972) show that maximum velocity is increased by increasing the elasticity of the liquid.

Now it is of interest to determine whether the viscoelastic fluids will behave in a similar fashion if the flows with different geometries are considered. In the present paper, we study the flow of viscoelastic fluids through geometries other than those studied by the previous authors and determine whether similar or different phenomena occur in our case. We take the flow through tubes with cross-section as a sector of a circle and as a sector of coaxial circular ducts. The rheological behaviour of the viscoelastic fluids are governed by the eight constant model (Oldroyd 1958).

2. EQUATIONS

Part A

We consider a slow shearing motion through a tube with sector of a circle as a cross-section which is bounded by the circle $r = a$ and two radii $\theta = \pm \alpha$. We assume that the flow is to be taken only along the length of the tube, so that the flow equation in the axial direction reduces to

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t}=k\left(1+\lambda_1\frac{\partial}{\partial t}\right)f(t)+\nu\left(1+\lambda_2\frac{\partial}{\partial t}\right)$$
$$\times\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) \quad \ldots(2.1)$$

where λ_1, λ_2, η_0 and ρ have their usual meanings and $\nu=\eta_0/\rho$ and $f(t)$, is any function of time. Here we have taken $-\frac{1}{\rho}\frac{\partial p}{\partial z}=kf(t)$ where k is some constant. The equation of continuity takes the form

$$\frac{\partial w}{\partial z}=0 \quad \ldots(2.2)$$

The boundary conditions are

(i) $w(r,\theta,t)=0$ when $r=a, t>0, -\alpha\leqslant\theta\leqslant\alpha$ ...(2.3)

(ii) $w(r,\theta,t)=0$ when $\theta=\pm\alpha, t>0, 0\leqslant r\leqslant a$ ...(2.4)

(iii) $w(r,\theta,t)=0$ when $t=0, 0\leqslant r\leqslant a$ and $-\alpha\leqslant\theta\leqslant\alpha$. ...(2.5)

The solution of the flow eqn. (2.1) under boundary conditions (2.3) to (2.5) using the integral transform technique has been found out. We consider the flow under three different categories of pressure gradient i.e. (i) exponential pressure gradient, (ii) harmonically oscillating pressure gradient (iii) constant pressure gradient.

Part B

In this part, we consider the unsteady flow of viscoelastic fluids through a tube with a sector of coaxial circular ducts as cross-section. Let a, b be the radii of the cross-section, where $a>b$.

Equation (2.1) still holds but we have to inegrate it under the following boundary conditions

(i) $w(r,\theta,t)=0$ when $r=a, t>0, -\alpha\leqslant\theta\leqslant\alpha$ ...(2.6)

(ii) $w(r,\theta,t)=0$ when $r=b, t>0, -\alpha\leqslant\theta\leqslant\alpha$ ...(2.7)

(iii) $w(r,\theta,t)=0$ when $\theta=\pm\alpha, t>0, b\leqslant r\leqslant a$ ...(2.8)

(iv) $w(r,\theta,t)=0$ when $t=0, b\leqslant r\leqslant a$ and $-\alpha\leqslant\theta\leqslant\alpha$. ...(2.9)

The solution of eqn. (2.1) under the boundary conditions (2.6) to (2.9) using the integral transform technique (Sneddon 1951, 1974) has been obtained in the three particular cases of the pressure gradient as explained above in Part A.

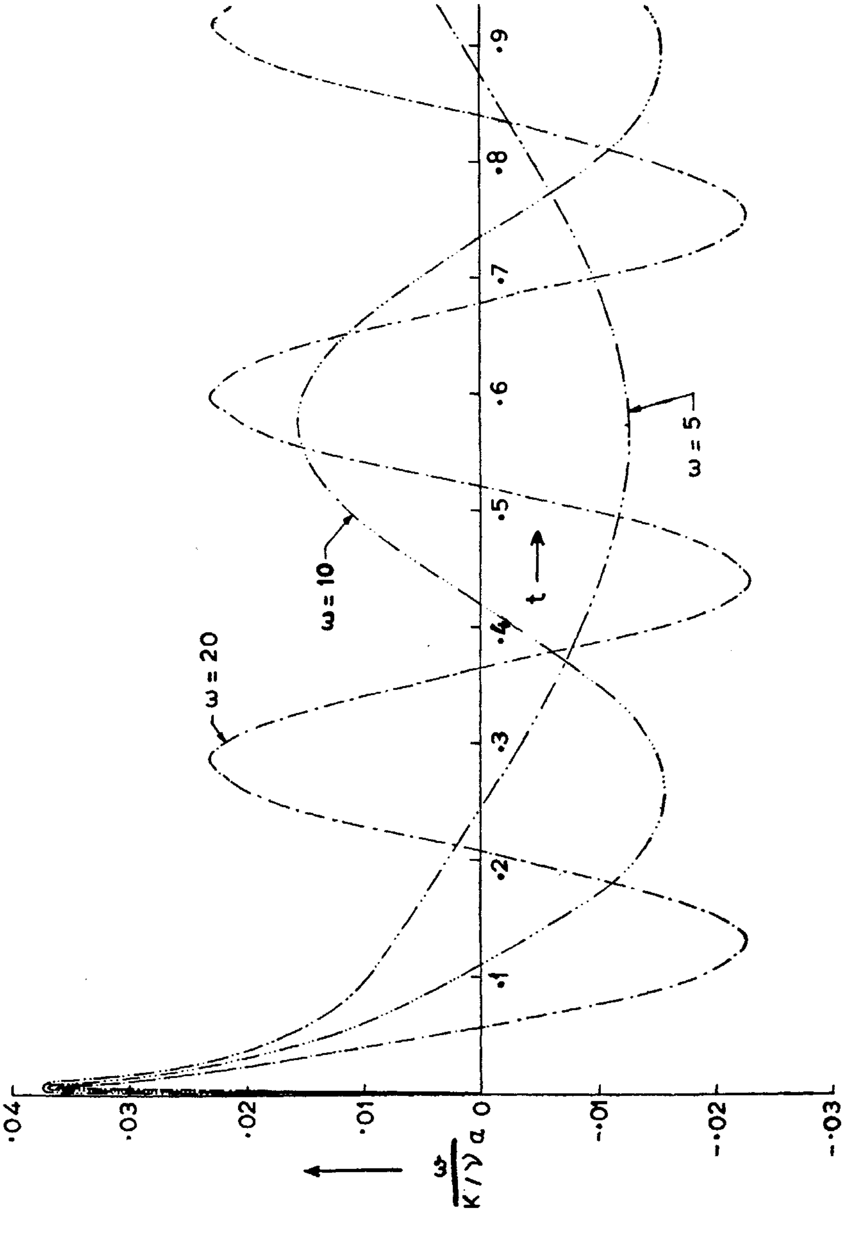

FIG. 1. Velocity under harmonically oscillating pressure gradient against time for different values of $\omega (\lambda_1 = 0.105,\ \lambda_2 = 0.0292,\ \nu = 60)$.

3. Numerical Results

The results obtained in Part A and Part B are shown graphically (Figs. 1 to 4 for Part A and Figs. 5 and 6 for Part B).

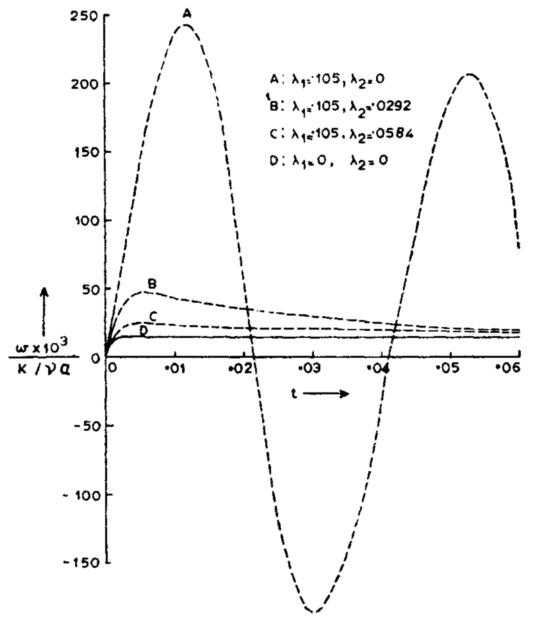

Fig. 2. Velocity profiles with constant pressure gradient plotted against t ($r = 0.7$, $\theta = \pi/12$, $\nu = 60$).

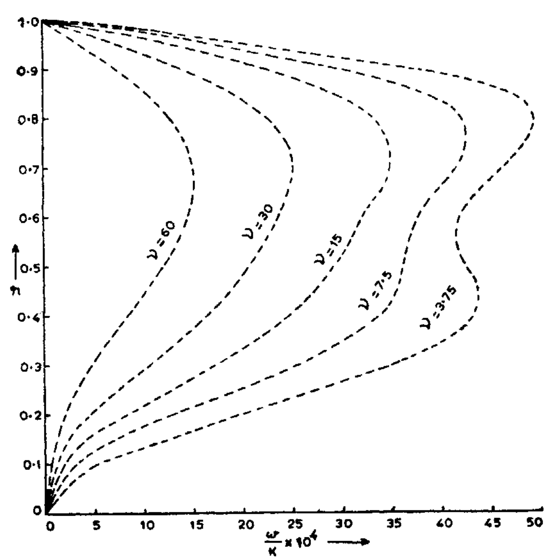

Fig. 3. Velocity under constant pressure gradient plotted against r for various values of ν ($\theta = \pi/12$, $t = 0.005$, $\lambda_1 = 0.105$, $\lambda_2 = 0.0292$).

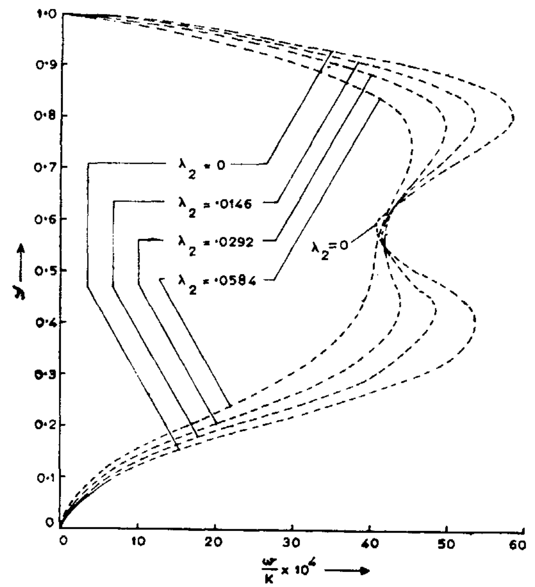

FIG. 4. Velocity under constant pressure gradient plotted against r for various values of λ_2 ($\lambda_1 = 0.105$, $\nu = 3.75$, $\theta = \pi/12$, $t = 0.005$).

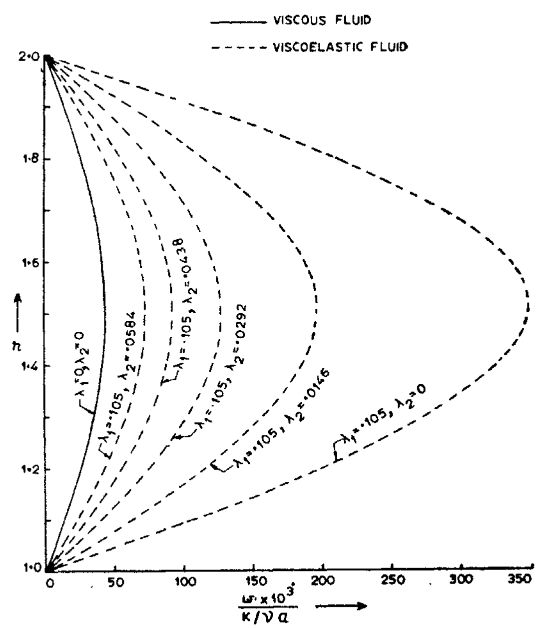

FIG. 5. Velocity under constant pressure gradient plotted against r for various values of λ_2 ($\theta = \pi/12$, $t = 0.012$, $\nu = 60$).

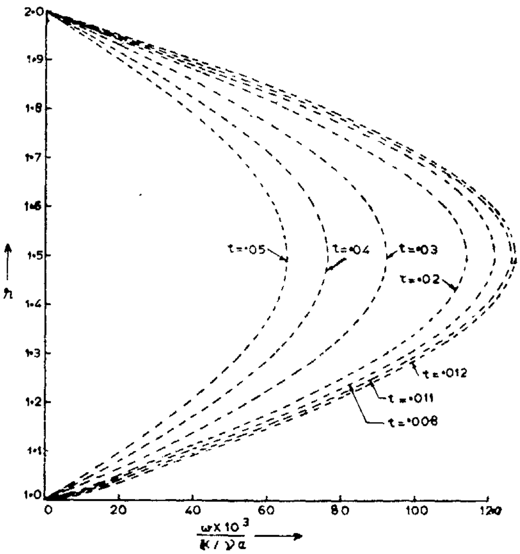

FIG. 6. Velocity of viscoelastic fluid under constant pressure gradient plotted against r for different values of t ($\theta = \pi/12$, $\lambda_1 = 0.105$, $\lambda_2 = 0.0292$, $\nu = 60$).

4. CONCLUSIONS

As with other geometries, in our problem also, we observe that:

(i) the elasticity of the fluid increases the velocity of the flow and hence destabilizes it;

(ii) in the beginning when the flow starts, the effect of decrease in frequency of oscillations increases the maximum amplitude but later on, decrease in frequency causes decrease in amplitude;

(iii) on account of elastico-viscous friction, boundary layers are formed and their width increases with the increase in elastic force in the laminar region.

Further we also find that:

(i) due to elasticity, parabolic flow is achieved earlier in comparison with the flow in the case of viscous force only;

(ii) in the case of non-oscillatory behaviour of the velocity, the time for reaching the terminal velocity increases as elasticity decreases.

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