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A NOTE ON THE LAMINAR STEADY FLOW THROUGH A CURVED CROSS-SECTION

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Following Lal (1973, 1979) we have discussed in detail the fluid flow through a tube of uniform cross-section, formed by the intersection of a branch of hyperbola $y^2 - 0.18x^2 = 0.293$ and lines $y = \pm 2.4x$. The velocity variations have been shown graphically. It is further shown that the velocity at the centre of gravity is slightly less than the maximum velocity.

Using stream function method the expression of velocity distribution derived by Lal (1979) is

$$w = \frac{P}{(3.28 - 21.56a)\,\mu}\,[y^2 - 0.18x^2 - a] \times [1.18y^2 - 6.82x^2 - (2 - 6.82a)] \qquad \ldots(1)$$

For constant $a = 0.293$ eqn. (1) is reduced to the problem cited above and expression of flux Q; area of cross-section A; maximum velocity c; Boussinesq coefficients k, k' as calculated by Lal (1979) are

$$\left.\begin{aligned} Q &= \frac{P}{\mu}(0.00047),\ A = 0.122 \\ c &= \frac{P}{\mu}(0.0083),\ k = 0.032,\ k' = 0.062. \end{aligned}\right\} \qquad \ldots(2)$$

The coordinates of the centre of gravity give $\bar{x} = 0$, $\bar{y} = 0.3611$ and velocity expression $\bar{w}$ at $(\bar{x}, \bar{y})$ is

$$\bar{w} = 0.0082\,\frac{P}{\mu} \qquad \ldots(3)$$

which is also called the axial velocity.

The expressions of maximum velocity c_1, c_2, c_3, c_4 and c_5 for the flow through (i) circular pipe (Bateman *et al.* 1932) of radius a, (ii) elliptic tube (Bateman *et al.* 1932) of semi-axes (a_1, b_1), (iii) equilateral triangular cross-section (Lamb 1916) bounded by sides $x = a_2$, $x \pm \sqrt{3}y + 2a_2 = 0$, (iv) bounded by two circles (Lal 1965) $r = b$, $r = 2b\cos\theta$ and (v) bounded by two hyperbolas (Tripathi 1970) are given by

$$\left.\begin{aligned} c_1 &= \frac{Pa^2}{4\mu},\ c_2 = \frac{P}{2\mu}\frac{a_1^2\, b_1^2}{a_1^2 + b_1^2},\ c_3 = \frac{Pa_2^2}{3\mu} \\ c_4 &= \frac{P}{\mu}\,(0.1047)\, b^2 \text{ and } c_5 = \frac{P}{\mu}\,(0.125)\, d^2. \end{aligned}\right\} \quad \ldots(4)$$

In case (v) the equations of hyperbolas are

$$\left.\begin{aligned} [\sqrt{2} + 1]\, x^2 - [\sqrt{2} - 1]\, y^2 &= d^2 \\ [\sqrt{2} + 1]\, y^2 - [\sqrt{2} - 1]\, x^2 &= d^2. \end{aligned}\right\} \quad \ldots(5)$$

When their area of cross-sections are equal we compare the expressions of maximum velocity in above five cases.

The ratios of maximum velocity have been calculated when all the channels have equal area of cross-sections. Hence we get

$$c : c_1 = 1 : 1.1686$$

$$c : c_2 = 1 : 1.1686\left(1 - \frac{1}{8}\, e^4 - \frac{3}{16}\, e^6 \ldots\right.$$

(where e is ellipticity of the cross-section) ...(6)

$$c : c_3 = 1 : 0.9438$$

$$c : c_4 = 1 : 0.8048$$

$$c : c_5 = 1 : 1.0527.$$

From eqns. (6) we conclude that the maximum velocity in the present pipe is less than the maximum velocity in circular pipe, elliptic pipe and the channel bounded by two hyperbolas. It is more in the case of equilateral triangular pipe and a tube bounded by two circles.

Further the velocity at the centre of gravity (axial velocity) $\bar{w}$ is given by eqn. (3). Thus the ratio of $\bar{w} : c = 1 : 1.012$.

Here in this cross-section the maximum velocity is slightly higher than the axial velocity. In case of circular and elliptic cross-sections or in a hyperbolic region given by eqns. (4) both axial and maximum velocity are the same.

The expressions of discharge of flux per second for above cross-sections in the same order are

$$\left.\begin{aligned} Q_1 &= \pi a^4 P/8\mu, & Q_2 &= \pi a_1^3\, b_1^3 P/\mu(a_1^2 + b_1^2) \\ Q_3 &= 0.7796 P a_2^4/\mu, & Q_4 &= 0.0946 P b^4/\mu \\ Q_5 &= 0.0853 P d^4/\mu. \end{aligned}\right\} \quad \ldots(7)$$

Thus on comparison for equal area of cross-sections, we have the ratios

$$Q : Q_1 = 1 : 1.2578$$

$$Q : Q_2 = 1 : 1.2578 \left(1 - \frac{1}{8} e^4 - \frac{3}{10} e^6 \ldots\right)$$

$$Q : Q_3 = 1 : 0.9160$$

$$Q : Q_4 = 1 : 0.8193$$

$$Q : Q_5 = 1 : 0.8867. \qquad \ldots(8)$$

Thus it is concluded that the present cross-section is more efficient than a channel of equilateral cross-section, and a tube bounded by two circles and a channel bounded by two hyperbolas. It is less efficient than the circular and elliptic pipes when the cross-sectional areas are same.

Graphical Variations and Discussions

In Fig. 1, the variations are shown with respect to r for the velocity when $\theta = 70°, 75°, 80°, 85°, 90°$.

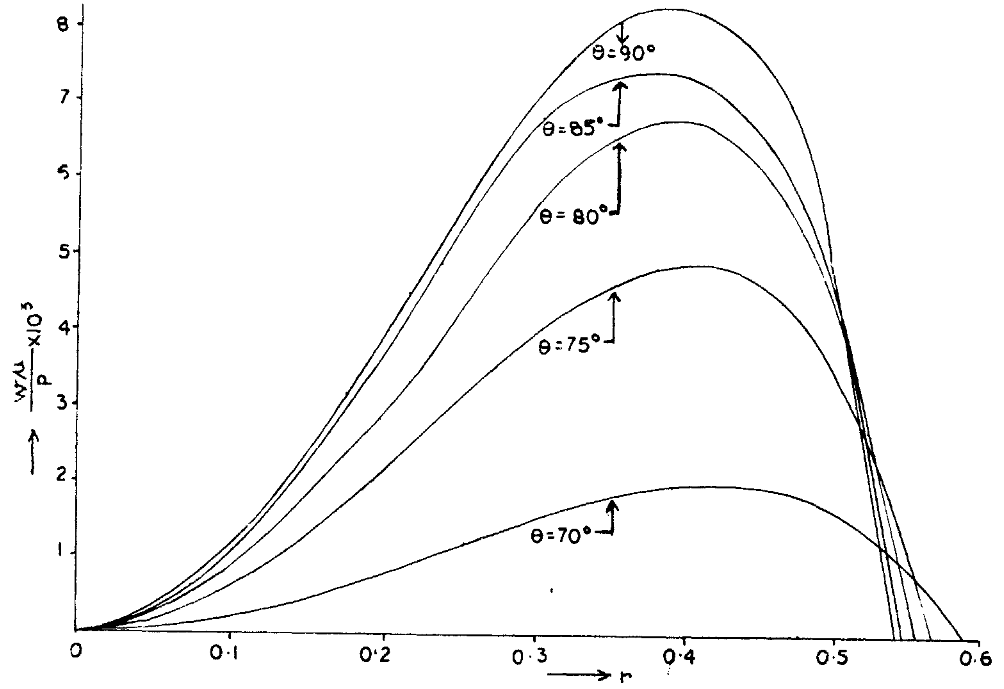

FIG. 1. Graph between w and r when $\theta = 70°, 75°, 80°, 85°, 90°$.

From calculations and also from Fig. 1 we see that when $\theta = 90°$ the velocity is higher at $r = 0.38$ than at $r = 0.4$. The variations are increasing sharply up to $r = 0.38$ and then they begin to decrease. The ratios can easily be compared as

$$\left.\begin{aligned}(w)_{0\cdot1} : (w)_{0\cdot2} &= 1 : 3.575 \\ (w)_{0\cdot5} : (w)_{0\cdot4} &= 1 : 1.978.\end{aligned}\right\} \quad \ldots(9)$$

For $\theta = 80°$, we see that velocity continues to increase up to $r = 0.39$ and then it begins to decrease. For ratios at different r, we have

$$\left.\begin{aligned}(w)_{0\cdot1} : (w)_{0\cdot2} &= 1 : 3.298 \\ (w)_{0\cdot5} : (w)_{0\cdot4} &= 1 : 1.713.\end{aligned}\right\} \quad \ldots(10)$$

For $\theta = 70°$, we see that the velocity continues to increase up to $r = 0.41$ and then it begins to decrease. The ratios in this case yield

$$\left.\begin{aligned}(w)_{0\cdot1} : (w)_{0\cdot2} &= 1 : 3.611 \\ (w)_{0\cdot5} : (w)_{0\cdot4} &= 1 : 1.483.\end{aligned}\right\} \quad \ldots(11)$$

The above comparison gives the idea of variation of velocity near the origin, away from the origin and near the hyperbolic section.

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