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VISCOUS DISSIPATION EFFECTS ON THE UNSTEADY FREE CONVECTIVE FLOW OF AN ELASTICO-VISCOUS FLUID PAST AN INFINITE VERTICAL PLATE WITH CONSTANT SUCTION

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An analysis of the effects of the viscous dissipative heat on the two-dimensional unsteady free convective flow of an elastico-viscous fluid (Walters' liquid B') past an infinite, vertical, porous plate is carried out under the following assumptions: (1) constant suction at the plate, (2) plate temperature oscillates in magnitude about a non-zero constant mean. Approximate solutions to the coupled non-linear equations are derived for the mean velocity, the mean temperature, the mean skin friction and the mean rate of heat transfer, the transient velocity, the transient temperature, the amplitude and the phase of the skin friction and the rate of heat transfer. The velocity and the temperature profiles are shown graphically and the values of the amplitude and the phase of the skin friction and the rate of heat transfer are tabulated. During the course of discussion, the effects of G (the Grashof number), P (the Prandtl number), E (the Eckert number), k (the elastic parameter) and ω (the frequency) have been presented.

NOTATION

$\mid B \mid$	amplitude of the skin friction
c_p	specific heat at constant pressure
E	Eckert number
g	acceleration due to gravity
G	Grashof number
k_0	short memory coefficient
k	non-dimensional elastic parameter
M_r, M_i	fluctuating parts of velocity profile
$N(\tau)$	distribution function of relaxation times τ
P	Prandtl number

q' rate of heat transfer

q_m mean rate of heat transfer

$|Q|$ amplitude of the rate of heat transfer

T' temperature in the boundary layer

T'_∞ temperature of fluid far away from plate

T_r, T_i fluctuating part of temperature

t' time

u', v' velocity components in x, y directions respectively

v_0 suction velocity

$P'_{x'y'}$ skin friction

Greek symbols :

α phase of the skin friction

β phase of the rate of heat transfer

β_1 coefficient of volume expansion

ϵ positive constant

η_0 viscosity

λ' thermal conductivity

ρ' fluid density

ν kinematic viscosity

ω' frequency

θ non-dimensional temperature

τ non-dimensional skin friction

τ_m non-dimensional mean skin friction.

1. Introduction

Unsteady free convective flow of a Newtonian fluid past an infinite porous plate was studied by Nanda and Sharma (1962) for the case of suction varying as $t^{-1/2}$. The effects of viscous dissipative heat on the steady free convective flow of a viscous Newtonian fluid were discussed by Gebhart (1962) and Gebhart and Mollendorf (1969) and in case of an unsteady flow, it was discussed by Soundalgekar (1972). Without viscous dissipation, the study of an unsteady free convective flow of an elastico-viscous fluid was presented, for the case of constant suction, by Soundalgekar (1971). In this paper, the fluid was assumed to be Walters' liquid B'. This fluid has been found to be of use in a number of technological applications. The viscous dissipative heat is important, even in case of free convective flows, when the fluid is of high Prandtl number. As Walters' liquid B' is of high Prandtl number, it is essential to consider the effects of the viscous dissipative heat on the unsteady free convective flows. Hence, it is the object of the present paper to study the unsteady free convective flows of Walters' liquid B' on taking into account the viscous dissipative heat.

In section 2, the mathematical analysis has been presented and approximate solutions to the mean velocity, the mean temperature, the mean skin friction and the mean rate of heat transfer, the transient velocity, the amplitude and the phase of the skin friction and the rate of heat transfer have been derived. The numerical values of the mean skin friction, the mean rate of heat transfer, the amplitude, the phase of the skin friction and the rate of heat transfer are tabulated whereas the others are shown graphically.

2. Mathematical Analysis

A two-dimensional flow of an elastico-viscous fluid past an infinite porous plate is considered. The x'-axis is taken along the plate in the upward direction and the y'-axis is taken normal to the plate. Hence, the flow variables are functions of y' and t' only. Then under the usual Boussinesq's approximation, the unsteady flow of an elastico-viscous fluid (Walters' liquid B') (Walters 1962) is governed by the following equations of continuity, momentum and energy :

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = g\beta_1(T' - T'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{k_0}{\rho'} \left(\frac{\partial^3 u'}{\partial y'^2 \partial t'} + v' \frac{\partial^3 u'}{\partial y'^3} \right) \quad ...(1)$$

$$0 = -\frac{1}{\rho'} \frac{\partial p'}{\partial y'} \quad ...(2)$$

and

$$\frac{\partial v'}{\partial y'} = 0 \quad ...(3)$$

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{\lambda'}{\rho' C_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{C_p} \left(\frac{\partial u'}{\partial y'} \right)^2 - \frac{k_0}{\rho' C_p} \frac{\partial u'}{\partial y'} \left\{ \frac{\partial^2 u'}{\partial t' \partial y'} + v' \frac{\partial^2 u'}{\partial y'^2} \right\}. \quad ...(4)$$

The boundary conditions are :

$$\left. \begin{aligned} u' = 0,\ T' = T'_w(t') \quad &\text{at} \quad y' = 0 \\ u' = 0,\ T' \to T'_\infty \quad &\text{as} \quad y' \to \infty. \end{aligned} \right\} \quad ...(5)$$

For constant suction, eqn. (3) integrates to

$$v' = -v_0 \quad ...(6)$$

where v_0 is constant suction velocity.

On introducing the following non-dimensional quantities

$$\left.\begin{aligned} & y = y'v_0/\nu,\ t = t'v_0^2/4\nu,\ u = u'/v_0,\ k = k_0 v_0^2/\nu \\ & \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ \omega = 4\nu\omega'/v_0'^2,\ G = \nu g \beta_1 (T'_w - T'_\infty)/v_0^3 \\ & P = \eta_0 C'_p/\lambda',\ E = v_0^2/C_p(T'_w - T'_\infty) \end{aligned}\right\} \qquad \ldots(7)$$

in (1) and (4), and taking account of (6), we get

$$\tfrac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = G\theta + \frac{\partial^2 u}{\partial y^2} - k\left[\tfrac{1}{4}\frac{\partial^3 u}{\partial y^2 \partial t} - \frac{\partial^3 u}{\partial y^3}\right] \qquad \ldots(8)$$

$$\frac{P}{4}\frac{\partial \theta}{\partial t} - P\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + PE\left(\frac{\partial u}{\partial y}\right)^2 - kPE\frac{\partial u}{\partial y}\left[\tfrac{1}{4}\frac{\partial^2 u}{\partial t \partial y} - \frac{\partial^2 u}{\partial y^2}\right] \qquad \ldots(9)$$

with the corresponding boundary conditions as

$$\left.\begin{aligned} & u = 0,\ \theta = \theta_w(t) = 1 + \epsilon e^{i\omega t} \quad \text{at } y = 0 \\ & u = 0,\ \theta \to 0 \quad \text{as } y \to \infty. \end{aligned}\right\} \qquad \ldots(10)$$

Here G is the Grashof number, P the Prantl number, E the Eckert number. Also, physically the boundary condition on θ in (10) implies that the plate temperature oscillates with time about a non-zero constant mean. Following Soundalgekar (1971, 1972) the solutions are derived. The procedure being straightforward, the details are omitted to save space. The mean velocity and the mean temperature are shown in Figs. 1 and 2 for different values of G, P, E and k. The values of G and P are chosen arbitrarily whereas those of E are always very small for an incompressible fluid. It is rather interesting to see the effects of the elastic property of the fluid on the mean velocity. The mean velocity of the elastico-viscous fluid increases as compared to that of the Newtonian fluid. An increase in G or k, when the values P and E are constant, leads to an increase in the mean velocity. But there is a fall in the mean velocity when P is increased. More addition of viscous dissipative heat also leads to a rise in the value of the mean velocity. There is also a rise in the mean temperature of an elastico-viscous fluid as compared to that of a Newtonian fluid of the same Prandtl number and it increases with increasing k. An increase in G leads to a rise in the mean temperature whereas an increase in P leads to a fall in the mean temperature.

Following Soundalgekar (1971, 1972) the numerical values of τ_m are calculated (Table I). A close study of Table I leads us to conclude that an increase in k leads to a decrease in the mean skin friction and for high Prandtl number fluids, there may occur a separation at large values of k. An increase in G leads to an increase in τ_m whereas an increase in P leads to a decrease in τ_m. Also greater viscous dissipative heat causes a rise in the value of τ_m.

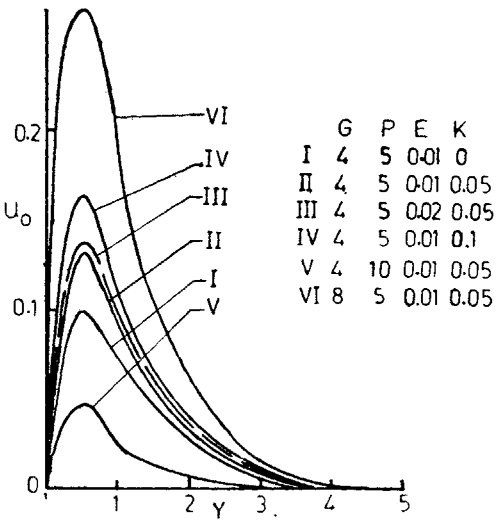

FIG. 1. Mean velocity profiles.

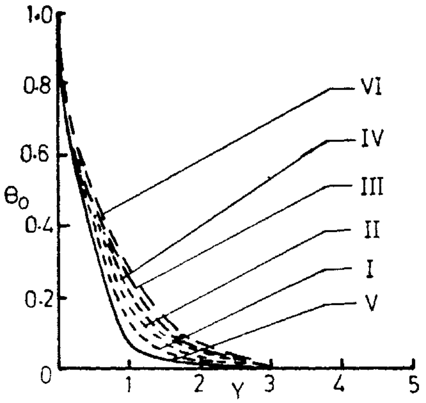

FIG. 2. Mean temperature profiles.

TABLE I

G	P	$E \setminus k$	Values of τ_m			Values of $\{-q_m\}$		
			0.0	0.05	0.1	0.0	0.05	0.1
4	5	0.01	0.8004	0.7265	0.5047	4.997	4.998	4.998
8	5	0.01	1.6030	1.4560	1.0130	4.989	4.991	4.993
4	5	0.02	0.8007	0.7270	0.5053	4.995	4.996	4.997
8	5	0.02	1.6060	1.4600	1.0190	4.979	4.983	4.987
4	10	0.01	0.4000	0.2780	− 0.0879	9.999	10.000	10.000
8	10	0.01	0.8002	0.5564	− 0.1754	9.997	9.998	10.000
4	10	0.02	0.4001	0.2781	− 0.0878	9.999	9.999	10.000
8	10	0.02	0.8004	0.5567	− 0.1748	0.994	0.997	9.999

Following Soundalgekar (1971, 1972) the numerical values of $\{-q_m\}$ are calculated (Table I). It is interesting to note that $\{-q_m\}$ is not significantly affected by the elastic parameter k. It decreases with increasing G. Greater viscous dissipative heat causes a fall in the value of $\{-q_m\}$. However, an increase in P leades to a rise in the value of $\{-q_m\}$.

Following Soundalgekar (1971, 1972), the expressions for transient velocity u and temperature θ are derived. u and θ are shown in Figs. 3 and 4 respectively for $\epsilon = 0.2$. We observe from Fig. 3 that the effects of G, P, E and k being the same as that observed in case of the mean velocity profiles. The transient velocity and the transient temperature have been observed to decrease with increasing ω.

Following Soundalgekar (1971, 1972), the expressions for amplitude $|B|$ and phase of the skin friction $\tan\alpha$ are derived. The numerical values of $|B|$ are shown in Table II. It is interesting to note the behaviour of the amplitude of the skin friction in case of Newtonian and elastico-viscous fluid. For a Newtonian fluid, an increase in ω leads to a decrease in $|B|$. But for an elastico-viscous fluid, an increase in ω leads to an increase in $|B|$. Greater viscous dissipative heat causes a rise in $|B|$ only at small values of ω whereas at large ω, the effects of the greater

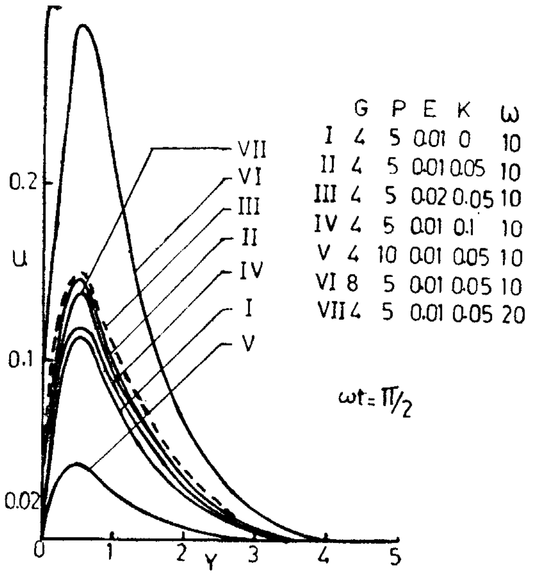

FIG. 3. Transient velocity profiles.

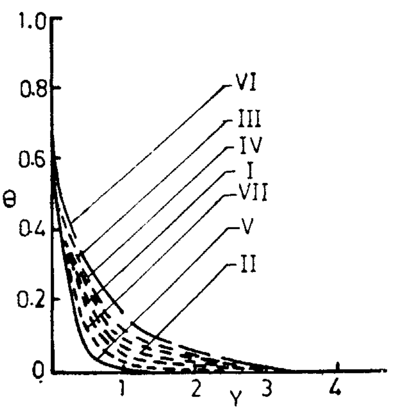

FIG. 4. Transient temperature profiles.

TABLE II

G	P	E	k \ ω	Values of $\|B\|$ 5	10	15	Values of $\|Q\|$ 5	10	15
4	5	0.01	0.0	0.6761	0.5682	0.4972	5.366	6.009	6.636
4	5	0.01	0.05	0.6923	0.6951	0.7764	5.363	6.009	6.636
4	5	0.02	0.05	0.6924	0.6952	0.7764	5.358	6.008	6.636
4	5	0.01	0.1	0.8788	1.2850	1.6950	5.361	6.009	6.637
4	10	0.01	0.05	0.3216	0.4205	0.5298	10.220	10.740	11.330
8	5	0.01	0.05	1.386	1.391	1.553	5.348	6.006	6.636

TABLE III

G	P	E	$k \backslash \omega$	Values of tan α 5	10	15	Values of tan β 5	10	15
4	5	0.01	0.0	0.3303	0.4769	0.5590	0.2174	0.3465	0.4261
4	5	0.01	0.05	0.8838	2.4040	7.1290	0.2177	0.3467	0.4263
4	5	0.02	0.05	0.8849	2.4060	7.1390	0.2185	0.3473	0.4270
4	5	0.01	0.1	3.6750	7.3680	3.6310	0.2180	0.3468	0.4266
4	10	0.01	0.05	1.0910	3.1360	11.4100	0.1198	0.2170	0.2902
8	5	0.01	0.05	0.8870	2.4100	7.1600	0.2202	0.3486	0.4283

viscous dissipative heat are negligible. An increase in G or k leads to an increase in $|B|$. But as Prandtl number of an elastico-viscous fluid is increased, there is a fall in the value of $|B|$. The numerical values of the phase of the skin friction tan α are shown in Table III. We observe that they being all positive, we conclude that there is always a phase lead.

Following (Soundalgekar 1971, 1972), the amplitude $|Q|$ and phase tan β of rate of heat transfer are derived. The numerical values of $|Q|$ are shown in Table II. It is observed from this table that the value of $|Q|$ is affected in a negligible manner by the presence of the elastic property of the fluid at small values of ω. But at large ω, the behaviour of $|Q|$ is the same both in a Newtonian and in an elastico-viscous fluid. Greater viscous dissipative heat causes a fall in the value of $|Q|$. An increase in G leads to a fall in $|Q|$. But an increase in P leads to an increase in the value of $|Q|$.

From the numerical values of tan β (Table III) we observe that there is always a phase lead.

3. CONCLUSIONS

(1) An increase in G or k leads to an increase in the mean velocity. (2) The mean velocity decreases with increasing P. (3) More addition of viscous dissipative heat causes a rise in the mean velocity. (4) The mean temperature increases with increasing G or k but decreases with increasing P. (5) The mean skin friction decreases with increasing k or P, whereas an increase in G or E leads to an increase in τ_m. (6) The mean rate of heat transfer is not significantly affected by the elastic property of the fluid. (7) The mean rate of heat transfer decreases with increasing G or E, whereas it increases with increasing P. (8) $|B|$ increases with increasing ω, E, G or k, but it decreases with increasing P. (9) $|Q|$ is not significantly

affected by the elastic property of the fluid but an increase in G or E leads to a fall in the value of $|Q|$ and an increase in P leads to a rise in $|Q|$.

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