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LAMINAR FREE CONVECTION FLOW WITH AND WITHOUT HEAT SOURCES THROUGH COAXIAL CIRCULAR PIPES

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The problem of laminar free convection flow with and without heat sources through coaxial circular pipes has been investigated. The density is assumed to vary as the square of the temperature difference. The results are compared with the corresponding case of linear density temperature variations. A perturbation solution is sought for obtaining the expressions of velocity and temperature.

1. INTRODUCTION

Due to its wide application to the field of chemical engineering, electronics, atomic power and aeronautics the process of natural convection flow has drawn the attention of several authors. Ostrach (1952, 1954) has studied the laminar natural convection flow between vertical heated plates when the walls are kept at a constant temperature and also when the temperature varies linearly along the plate. The corresponding problem with porous walls has been treated by Rao (1962). Nanda and Sharma (1962) have studied this to the case of flow in a circular pipe. Recently, Gupta and Sharma (1978) have extended this to the case of flow in a coaxial circular pipes. In all these investigations, a linear density temperature variation has been taken into account for the natural convection flows. Recently, Goren (1966) has shown that the rate of heat transfer by free convection in water at 4°C may be much reduced from that at other temperatures and this lead to an important consideration in some chemical processes.

The density variation suggested by Goren (1966) for water near 4°C is of the form

$$\Delta\rho = -\rho\gamma(\Delta T)^2 \qquad ...(1)$$

whereas in previous analysis it has been assumed as :

$$\Delta\rho = -\rho\beta(\Delta T). \qquad ...(2)$$

Working with eqn. (1) Sinha (1969) has analysed fully developed free convection flow between vertical parallel plates. Agarwal and Upmanyu (1976) have discussed the laminar free convection flow with and without heat sources in a circular pipe following the density variation proposed by Goren (1966). Recently, Gupta (1978) gives a note on the paper of Agarwal and Upmanyu (1976).

In the present paper we have discussed the laminar free convection flow with and without heat sources through coaxial circular pipes following the density variation proposed by Goren (1966).

2. Formulation of the Problem

Let us consider the fully developed steady, laminar free convection flow of a viscous incompressible fluid in a circular pipe. In a cylindrical system of coordinate let (u, v, w) be the velocity components in the direction of r, ϕ, z. Since the motion is rotationally symmetric and the pipes are along enough, all physical quantities will be independent of ϕ, z and $v \equiv 0$.

The equation of continuity, motion and energy are respectively :

$$0 = \frac{\partial u}{\partial r} + \frac{u}{r} \qquad \text{...(3)}$$

$$\rho u \frac{\partial u}{\partial r} = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right] \qquad \text{...(4)}$$

$$\rho u \frac{\partial w}{\partial r} = -\frac{\partial p}{\partial z} + \rho f_z + \mu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right] \qquad \text{...(5)}$$

and

$$\rho c_p u \frac{\partial T}{\partial r} = k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \mu\left[2\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{u}{r}\right)^2 + \left(\frac{\partial w}{\partial r}\right)^2\right] + Q \qquad \text{...(6)}$$

where Q, a constant, denotes the heat added due to heat source; f_z is the generating body force per unit mass; c_p the specific heat at constant pressure; k the coefficient of thermal conductivity; and p the pressure.

The boundary conditions are

$$\left.\begin{array}{ll} \text{at} & r = a,\quad u = w = 0;\quad T = T_{w_1} \\ \text{at} & r = b,\quad u = w = 0;\quad T = T_{w_2}. \end{array}\right\} \qquad \text{...(7)}$$

Now following Ostrach (1952), the body force term in (5) can be expressed as a bouyancy term. If subscript s, denotes the hydrostatic condition, then (5) gives

$$\rho_s f_z - \frac{\partial p_s}{\partial z} = 0$$

and hence

$$\rho f_z - \frac{\partial p}{\partial z} = -\rho \gamma f_z \theta^2 - \frac{\partial p_D}{\partial z} \qquad \text{...(8)}$$

here

$$p_D = p - p_s \quad \text{and} \quad \theta = T - T_s.$$

Also

$$\gamma = -\frac{\rho - \rho_s}{\rho(T - T_s)^2}$$

is the coefficient of volumetric expansion.

In view of boundary conditions (7), eqn. (3) suggests that

$$u = 0. \qquad \ldots(9)$$

We further introduce,

$$R = \frac{r}{a}, \quad \theta = \frac{T_s T^*}{K}, \quad w = \frac{W w^*}{K} \qquad \ldots(10)$$

where

$$W = \frac{f_z \gamma a^2 T_s^2}{\nu} \quad \text{and} \quad K = \frac{\sigma G_r \gamma f_z a T_s}{c_p},$$

$G_r(= \gamma f_z a^3 T_s^2/\nu^2)$ being the Grashoff number and $\sigma(= \mu c_p/k)$ the Prandtl number. Also the pipes being long enough, the pressure gradient inside it can be taken as equal to the hydrostatic pressure gradient.

Equations (5) to (7) reduce to the following form :

$$w^{*''} + \frac{1}{R} w^{*'} - \frac{1}{K} T^{*2} = 0 \qquad \ldots(11)$$

$$T^{*''} + \frac{1}{R} T^{*'} + (w^{*'})^2 + \alpha K = 0 \qquad \ldots(12)$$

$$\left.\begin{array}{llll} \text{at} & R = 1, & w^* = 0, & T^* = KN_1 \\ \text{at} & R = \dfrac{b}{a}, & w^* = 0, & T^* = KN_2 \end{array}\right\} \qquad \ldots(13)$$

where prime denotes the differentiation with respect to R, $\alpha(= Qa^2/kT_s)$ is the heat source parameter, $N_1 = (T_{w_1} - T_s)/T_s$ and $N_2 = (T_{w_2} - T_s)/T_s$.

3. Solution of the Problem

Let us assume w^* and T^* as

$$w^* = Kw_0^* + K^2 w_1^* + K^3 w_2^* + \ldots \qquad \ldots(14)$$

$$T^* = KT_0^* + K^2 T_1^* + K^3 T_2^* + \ldots . \qquad \ldots(15)$$

Substituting (14) and (15) in eqns. (11) to (13) and equating the coefficients of K and K^2, we get the following perturbed equations :

$$T_0^{*''} + \frac{1}{R} T_0^{*'} + \alpha = 0 \quad \text{...(16)}$$

$$T_1^{*''} + \frac{1}{R} T_1^{*'} + (w_0^{*'})^2 = 0 \quad \text{...(17)}$$

$$w_0^{*''} + \frac{1}{R} w_0^{*'} - T_0^{*2} = 0 \quad \text{...(18)}$$

$$w_1^{*''} + \frac{1}{R} w_1^{*'} - 2T_0^* T_1^* = 0 \quad \text{...(19)}$$

and

$$\left.\begin{array}{lll} \text{at} & R = 1, & w_0^* = w_1^* = 0; \quad T_0^* = N_1, \quad T_1^* = 0 \\ \text{at} & R = \dfrac{b}{a}, & w_0^* = w_1^* = 0; \quad T_0^* = N_2, \quad T_1^* = 0. \end{array}\right\} \quad \text{...(20)}$$

The values of T_0^* and w_0^* are obtained as

$$T_0^* = B_0 - \tfrac{1}{4}\alpha R^2 + A_0 \log R \quad \text{...(21)}$$

$$w_0^* = F_0 + \frac{C_0}{4} R^2 + \frac{\alpha D_0}{32} R^4 + \frac{\alpha^2}{576} R^6 + \left(H_0 + \frac{E_0}{2} R^2 - \frac{\alpha A_0}{32} R^4 + \frac{A_0^2}{4} R^2 \log R\right) \log R. \quad \text{...(22)}$$

Similar polynomial in R can be obtained for T_1^* and w_1^* , where

$$A_0 = \frac{N_2 - B_0 + (\alpha(b/a)^2/4)}{\log (b/a)}, \quad B_0 = N_1 + (\alpha/4)$$

$$C_0 = B_0^2 - 2A_0 B_0 + (3A_0^2)/2, \quad D_0 = \frac{A_0}{2} - B_0$$

$$E_0 = A_0 B_0 - A_0^2, \quad F_0 = -\frac{C_0}{4} - \frac{\alpha D_0}{32} - \frac{\alpha^2}{576}$$

$$G_0 = \frac{C_0}{4}\left(\frac{b}{a}\right)^2 + \frac{\alpha D_0}{32}\left(\frac{b}{a}\right)^4 + \frac{\alpha^2}{576}\left(\frac{b}{a}\right)^6$$

$$H_0 = -\frac{F_0 + G_0}{\log \frac{b}{a}} - \frac{E_0}{2}\left(\frac{b}{a}\right)^2 + \frac{\alpha A_0}{32}\left(\frac{b}{a}\right)^4 - \frac{A_0^2}{4}\left(\frac{b}{a}\right)^2 \log \frac{b}{a}.$$

It can be verified that when $b \to 0$ and $N_1 \to 1$, all the expressions for T_0^* and w_0^* given by (21) and (22) respectively reduce to the expressions as obtained by Gupta (1978) for a circular tube of radius a.

The heat transfer through the pipe walls to the flow per unit area of the pipe surfaces are given by

$$q = \frac{kT_s}{aK}\left(\frac{\partial T^*}{\partial R}\right)_{R=b/a,1}. \qquad \ldots(23)$$

The Nusselt number on the walls are

at the outer wall,

$$Nu_1 = \frac{qa}{kT_s} = \frac{1}{K}\left(\frac{\partial T^*}{\partial R}\right)_{R=1} \qquad \ldots(24)$$

at the inner wall,

$$Nu_2 = \frac{qa}{kT_s} = \frac{1}{K}\left(\frac{\partial T^*}{\partial R}\right)_{R=b/a}. \qquad \ldots(25)$$

4. Free Convection with Linear Density Variation

For the purpose of comparison, the same problem has been solved for the case of linearly varying density given by eqn. (2). Thus expressing the body force term as a bouyancy term and substituting

$$R = \frac{r}{a},\ w = \frac{\bar{w}^* \bar{W}}{K},\ = \frac{T_s \bar{T}^*}{K} \qquad \ldots(26)$$

the resulting equations of velocity and temperature are

$$\bar{w}^{*''} + \frac{1}{R}\bar{w}^{*'} - \bar{T}^* = 0 \qquad \ldots(27)$$

$$\bar{T}^{*''} + \frac{1}{R}\bar{T}^{*'} + (w^{*'})^2 + \alpha K = 0 \qquad \ldots(28)$$

where

$$K = \sigma \bar{G}_r \frac{\beta f_z a}{c_p},\ \ \bar{W} = \frac{f_z \beta a^2 T_s}{\nu},\ \ \bar{G}_r = \frac{\beta f_z a^3 T_s}{\nu^2}$$

and β is the coefficient of thermal expansion.

The set of eqns. (27) and (28) again under the same boundary conditions (13) have been solved by Gupta and Sharma (1978) to yield $\bar{w}_0^*$, $\bar{T}_0^*$, etc.

5. Discussion of the Results

For the sake of convenience the two cases corresponding to the two types of density variations given in eqns. (1) and (2) will henceforth be termed as case I and case II respectively. The velocity (w_0^*) and temperature (T_0^*) have been calculated and plotted for various values of heat source parameter (α) versus R in Figs. 1 and 2 respectively.

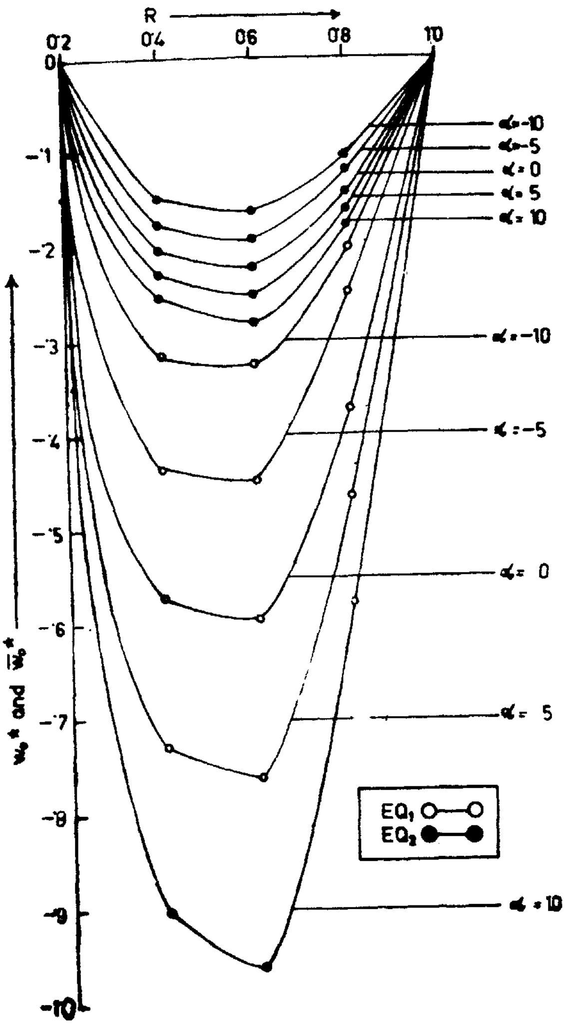

FIG. 1. Dimensionless velocity distribution for various values of α ($N_1 = 2$, $N_2 = 4$, $b/a = 0.20$).

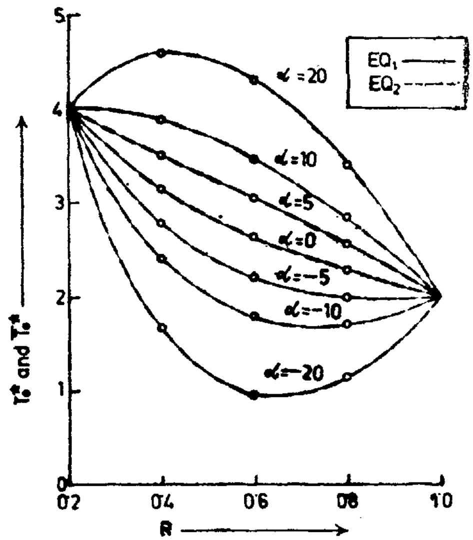

FIG. 2. Dimensionless temperature distribution for various values of α ($N_1 = 2$, $N_2 = 4$, $b/a = 0.20$).

Figure 1 reveals that the velocity (w_0^* , $\bar{w}_0^*$) in both the cases is negative for both the positive (source) and negative (sink) values of α and it decreases to a minima near the middle of the space ($R = 0.6$). Also it decreases more rapidly throughout with increase in α for the case I as compared to case II.

It is clear from Fig. 2 that the temperature field (T_0^* , $\bar{T}_0^*$) for both the cases is same and decreases with the decreasing values of source parameter (α) and increasing values of sink parameter.

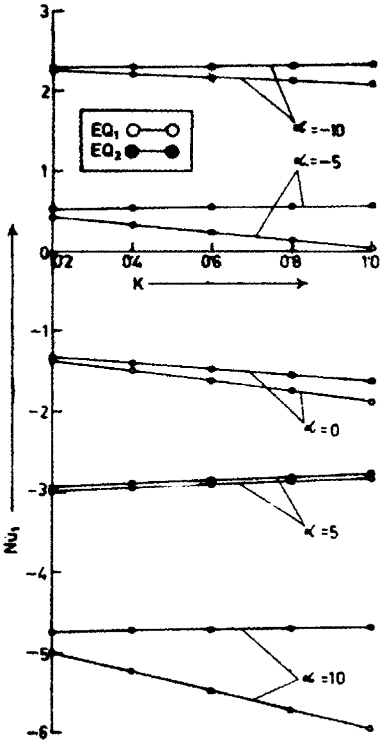

FIG. 3. Nusselt number at the outer wall for various values of α ($N_1 = 2$, $N_2 = 4$, $b/a = 0.2$).

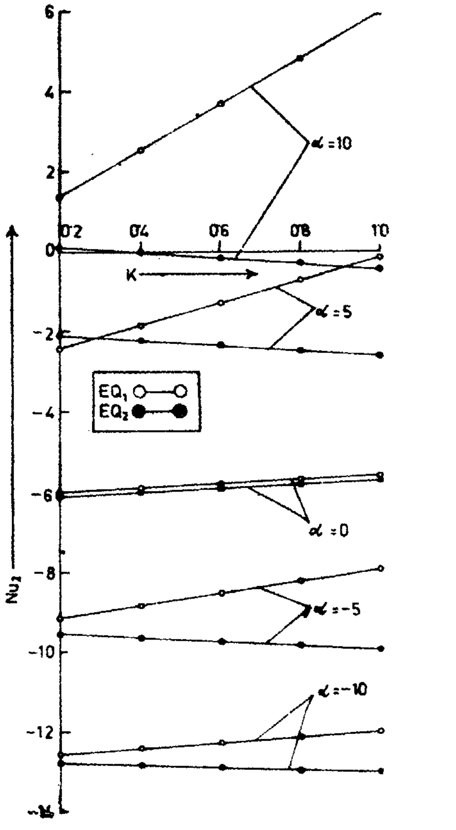

FIG. 4. Nusselt number at the inner wall for various values of α ($N_1 = 2$, $N_2 = 4$, $b/a = 0.20$).

The Nusselt number on the walls [$R = 1.0$ and $R = (b/a) = 0.2$] versus K have been drawn in Figs. 3 and 4 respectively. Figures 3 and 4 show that the walls experience cooling for α negative but for α positive the walls get more and more heated in both the cases. For $\alpha = 0$, inner and outer walls of the channel experience cooling and heating respectively as the temperature currents flow from inner wall to outer wall for both the cases.

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