

An analogue of Hoffman-Wermer theorem for a real function algebra by S. H. KULKARNI AND N. SRINIVASAN *Indian J. Pure Appl. Math.* 19 (2) 1988, 154-66.

Theorem 3.7 reads as follows :—

Let X be a compact Hausdorff space and A a uniformly closed real subalgebra of $C(X)$, which contains real constants and which separates the points of X . If $\text{Re } A$ is uniformly closed in $C(X)$, then there exists a closed subset Z of X , such that $A = \{f \in C(X) : f|Z \text{ is real}\}$.

The above theorem is incorrect as it stands. Counter-example : Let $X = [0, 1]$.

$$A = \{f \in C(X) : f(1-t) = \overline{f(t)} \text{ for all } t \text{ in } X\}.$$

Then $\text{Re } A$ is uniformly closed, but, for no closed subset Z of X , the above conclusion holds.

The 'Proof' of the above theorem breaks down at the following point : In the course of the proof, a compact Hausdorff space \tilde{X} , an involutory homeomorphism $\tilde{\tau}$ on \tilde{X} and an algebra \tilde{A} have been defined. Then it is claimed that \tilde{A} is a real function algebra on $(\tilde{X}, \tilde{\tau})$. This claim is incorrect because \tilde{A} may not separate the points of \tilde{X} .

However, this defect can be rectified by adding the following hypothesis :

For $x \neq y$ in X there exists f in A , such that $f(x) \neq \overline{f(y)}$.

(Note that this hypothesis is satisfied, if $\text{Re } A$ separates the points of X). With this additional hypothesis, \tilde{A} separates the points of \tilde{X} and the proof remains valid. As a conclusion, we do have—

$$A = \{f \in C(X) : f|Z \text{ is real}\}.$$

In particular, this implies that

$$\text{Re } A = C_R(X) \subset A.$$

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