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NUMERICAL SOLUTION OF UNSTEADY FLOW AND HEAT TRANSFER IN A MICROPOLAR FLUID PAST A POROUS FLAT PLATE

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The problem of unsteady laminar flow and heat transfer in an incompressible micropolar fluid past an infinite porous flat plate has been examined. The flat plate is subjected initially to a constant suction velocity followed by a step function change for time $t' > 0$. An explicit finite difference scheme has been used to solve the governing equations of motion and energy. The velocity, microrotation and temperature distribution have been displayed through graphs for various values of micropolar parameter R at different time levels.

1. INTRODUCTION

Considerable attention has been paid by the researchers on the phenomenon of boundary layer flow over a permeable surface through which the fluid is either sucked or injected, because of its practical applications to boundary layer control and thermal protection in high energy flow by means of mass transfer. A class of unsteady solution of Navier-Stokes equations which possesses boundary layer character, is obtained when the velocity components are independent of longitudinal co-ordinate with free stream velocity either a function of time or constant[1].

According to Stuart[2], this special consideration leads to the exact solution of the flow equations for the arbitrary free stream velocity.

$$U(t) = U_0 [1 + f(t)] \qquad ...(1)$$

at a very large distance from the solid boundary. Watson[3] investigated the plane flows with special forms of function $f(t)$ in the arbitrary external velocity. Further $f(t) = 0$ in the free stream velocity leads to a simple solution to the plane flows. Using this consideration, the problem of unsteady flow with step function change in suction velocity have been analysed in Newtonian fluid[4].

The general theory of micropolar fluid which has been formulated and presented by Eringen[5,6] is deviating from that of Newtonian fluid by accomodating two new variables viz. microrotation i.e. spin and micro inertia describing the distributions of atoms and molecules inside the fluid elements. Peddieson and McNitt[7] and Wilson[8] first developed the boundary layer concept for such a fluid and investigated the flow past a flat plate. Dey and Nath[9] studied micropolar fluid flow over a semi-infinite plate using parabolic co-ordinates to consider the flow regime including the leading edge. Gorla *et al.*[10] solved the steady state heat transfer in a micropolar fluid flow over a semi-infinite plate using similarity variables. Chawla[11] obtained the solution for unsteady micropolar fluid flow past an infinite plate.

In the present study, the effect of step function change in suction velocity on the flow and heat transfer in an incompressible micropolar fluid past a flat plate has been studied by considering the free stream velocity to be constant for all time $t' \geq 0$. At time $t' = 0$, we assume that there is a steady flow over the plate with constant suction $v_1' < 0$. For time $t' > 0$, a step function change is made in suction velocity which is responsible for unsteadyness of the flow and heat transfer. The velocity and temperature are assumed to be functions of transverse co-ordinate y' and time t only. An explicit finite difference scheme is employed to solve the governing equations.

2. Mathematical Formulation

The unsteady two-dimensional flow of an incompressible micropolar fluid past a semi-infinite porous flat plate is considered. The X'-axis is chosen along the plate with leading edge as the origin and Y'-axis, at right angles to it. Let u' and v' be the velocity components parallel and normal to the plate respectively. All the fluid properties are assumed to be constant. At time $t' = 0$, the flow of the fluid is assumed to be steady with constant suction velocity $v_1' < 0$, normal to the plate. At time $t' > 0$, the suction velocity is suddenly changed into v_2' ($v_2' < 0$ for suction, $v_2' > 0$ for injection) which causes the flow unsteady. At all times, the fluid free stream velocity is assumed to be constant and paralled to the plate. Since the resulting flow is superimposed and weak, and the plate is of infinite extent, concerned dependent variables may be considered as function of y' and t' only.

The governing equations of momentum and energy for heat conducting micropolar fluid[6,12] are

$$\frac{\partial v'}{\partial y'} = 0 \qquad \ldots(2)$$

$$\rho\left[\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right] = (\mu + K)\frac{\partial^2 u'}{\partial y'^2} + K\frac{\partial N'}{\partial y'} \qquad \ldots(3)$$

$$0 = \frac{\partial P'}{\partial y'} \qquad \ldots(4)$$

$$\rho j \left[\frac{\partial N'}{\partial t'} + v' \frac{\partial N'}{\partial y'} \right] = \gamma \frac{\partial^2 N'}{\partial y'^2} - K \left[\frac{\partial u'}{\partial y'} + 2N' \right] \quad \ldots(5)$$

$$U(t) = U_\infty \text{ (constant)} \quad \ldots(6)$$

$$\rho c_p \left[\frac{\partial T}{\partial t'} + v' \frac{\partial T}{\partial y'} \right] = k_f \frac{\partial^2 T}{\partial y'^2} + \left(\mu + \frac{K}{2} \right) \left(\frac{\partial u'}{\partial y'} \right)^2 + \frac{K}{2} \left[\frac{\partial u'}{\partial y'} + 2N' \right]^2 + \gamma \left[\frac{\partial N'}{\partial y'} \right]^2 \quad \ldots(7)$$

where ρ is the density, μ the coefficient of viscosity, c_p the specific heat at constant pressure, T the temperature, γ and K are the micropolar material constant and k_f the thermal conductivity. In view of (1), we take

$$\frac{v_2'}{| v_1' |} = -1, \text{ for } t' = 0$$

$$= -\lambda, \text{ for } t' > 0 \quad \ldots(8)$$

where λ is the suction parameter.

Introducing the following non-dimensional variables

$$y = \frac{| v_1' | y'}{(\mu + K)}, \qquad t = \frac{| v_1' |^2 t'}{(\mu + K)}, \quad u = \frac{u'}{u_\infty}$$

$$N' = \frac{N (\mu + K)}{\rho u_\infty | v_1' |}, \theta = (T - T_\infty)/(T_w - T_\infty)$$

where T_w is the temperature at the isothermal wall and T_∞ the free stream temperature. Equations (3), (5) and (7) can be written as

$$\frac{\partial u}{\partial t} - \lambda \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \frac{R}{1+R} \frac{\partial N}{\partial y} \quad \ldots(9)$$

$$\frac{\partial N}{\partial t} - \lambda \frac{\partial N}{\partial y} = \frac{A}{1+R} \frac{\partial^2 N}{\partial y^2} - \frac{2R(1+R)}{R_e^2} \left(N + \frac{1}{2} \frac{\partial u}{\partial y} \right) \quad \ldots(10)$$

$$\frac{\partial \theta}{\partial t} - \lambda \frac{\partial \theta}{\partial y} = \frac{1}{Pr(1+R)} \frac{\partial^2 \theta}{\partial y^2} + E \left[\frac{A R_e^2}{(1+R)^3} \left(\frac{\partial N}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 + \frac{2R}{1+R} \left(N^2 + N \frac{\partial u}{\partial y} \right) \right] \quad \ldots(11)$$

where R $(= K/\mu)$, A $(= \gamma/\mu j)$ are the micropolar parameters, Pr $(= \mu c_p/k_f)$ the Prandtl number, E $(= u_\infty^2/c_p (T_w - T_\infty))$ the Eckert number and R_e $(=\rho|v_1'| j^{1/2})/\mu$ is a dimensionless constant. The initial conditicns correspond to the solution of the steady state problem.

The boundary conditions can be written as

$$u(0, t) = 0, \; N(0, t) = 0.0, \; \theta(0, t) = 1, \text{ at } y = 0,$$

$$u(\infty, t) = 1, \; N(\infty, t) = 0.0, \; \theta(\infty, t) = 0, \text{ at } y \to \infty. \quad \text{.. (12)}$$

3. MBTHOD OF SOLUTION

The solve the eqns. (9—11) subject to (12), the explicit finite difference scheme has been employed. As in Soundalgekar[14], the boundary conditions at $y = \infty$, have been taken to be satisfied approximately at $y = 6.0$. The flow regime defined as a semi-infinite strip in time, bounded by $y = 0$ and $y = 6.0$ is divided into a grid by lines parallel to y and t axes (Fig. 1). The length step Δy is chosen as 0.1 while the time step Δt is taken to be 0.0025 so as to satisfy the stability condition as per Ralston[15].

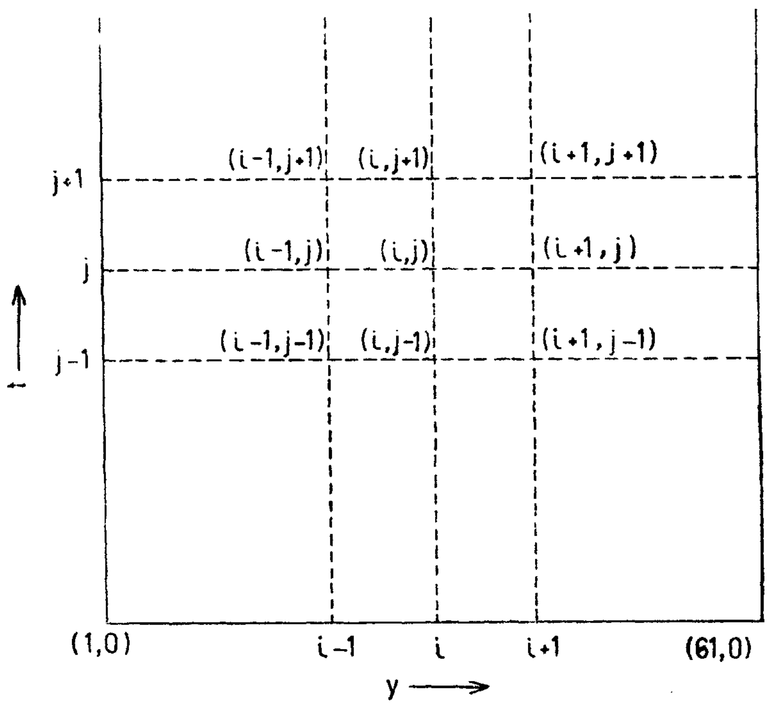

FIG. 1.

Replacing the time derivatives by forward difference and the space derivatives by their corresponding central differences, eqns. (9) – (11) can be re-written as

$$u_i^* = u_i + \Delta t \left[\left(\frac{u_{i+1} - 2_i + u_{i-1}}{(\Delta y)^2} \right) + \lambda \frac{u_{i+1} - u_{i-1})}{2\Delta y} + \frac{R}{(1+R)} \frac{(N_{i+1} - N_{i-1})}{2\Delta y} \right] \quad ...(13)$$

$$N_i^* = N_i + \Delta t \left[\frac{A}{(1+R)} \frac{(N_{i+1} - 2N_i + N_{i-1})}{(\Delta y)^2} + \lambda \frac{(N_{i+1} - N_{i-1})}{2\Delta y} - \frac{2R(1+R)}{R_e^2} \left(N_i + \frac{1}{2} \frac{u_{i+1} - u_{i-1}}{2\Delta y} \right) \right] \quad ...(14)$$

$$\theta_i^* = \theta_i + \Delta t \left\{ \frac{1}{Pr(1+R)} \frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{(\Delta y)^2} + \lambda \frac{\theta_{i+1} - \theta_{i-1}}{2\Delta y} + E \left[\frac{AR_e^2}{(1+R)^3} \left(\frac{N_{i+1} - N_{i-1}}{2\Delta y} \right)^2 + \left(\frac{u_{i+1} - u_{i-1}}{2\Delta y} \right)^2 + \frac{2R}{(1+R)} \left(N_i^2 + N_i \frac{u_{i+1} - u_{i-1}}{2\Delta y} \right) \right] \right\} \quad ...(15)$$

where $2 \leqslant i \leqslant 60$.

The initial condition is taken as the steady state solution obtained by solving the corresponding steady state finite difference equations through Gauss Seidal method, while the boundary conditions (12) transform as

$$u_1 = 0.0, \; u_{61} = 1.0$$

$$N_1 = 0.0, \; N_{61} = 0.0$$

$$\theta_1 = 1.0, \; \theta_{61} = 0.0. \quad ...(16)$$

The solution of the difference equations is obtained at the intersection of the grid lines, called nodes. In Fig. 1, the nodes are specified by double subscripts (i, j) with the origin located at the intersection of the lines $y = 0$ and $t = 0$. The value of the dependent variables u, N and θ at the nodal points along the lines $y = 0$ and $y = 6.0$ are known for all time t, while unknown values of u_i^* $(= u_i^{j+1})$, N_i^* $(= N_i^{j+1})$ and θ_i^* $(= \theta_i^{j+1})$, at internal nodes between $y = 0$ and $y = 6.0$ for any time $t > 0$ are to be determined.

The non-dimensional shear stress, couple stress and the Nusselt number on the wall are respectively given by

$$C_f = \frac{t_{x'y'}}{\rho\, u_\infty \;|\, v_1' \,|} = \left(\frac{\partial u}{\partial y}\right)_{y=0} \qquad \ldots(17)$$

$$C_m = \frac{m_{y'z'}}{(\rho^2\, u_\infty \;|\, v_1' \,|^2\, \gamma)\, k^2} = \frac{R^2}{(1+R)^2}\left(\frac{\partial N}{\partial y}\right)_{y=0} \qquad \ldots(18)$$

and

$$Nu^* = \frac{\mu\, Nu}{(L \;|\, v_1' \,|\, \rho)} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \qquad \ldots(19)$$

It has been observed that by increasing the micropolar effects in the fluid, more and more heat is transferred from the plate at any time level.

4. Numerical Results and Discussion

Fixing $A = 1.5$, $Re = 0.5$, $Pr = 1.0$, $E = 0.1$ and $\lambda = 2.0$, the discussion is limited to the variation of micropolar parameter R for different time levels viz. $t=0.1$, 0.5 and 1.0.

In Fig. 2, as compared to the Newtonian fluid $R = 0.0$, the presence of micropolar additives in the fluid increases the velocity throughout. Also as time grows, the velocity profiles are further accelerated. The microrotation N is found to have negative

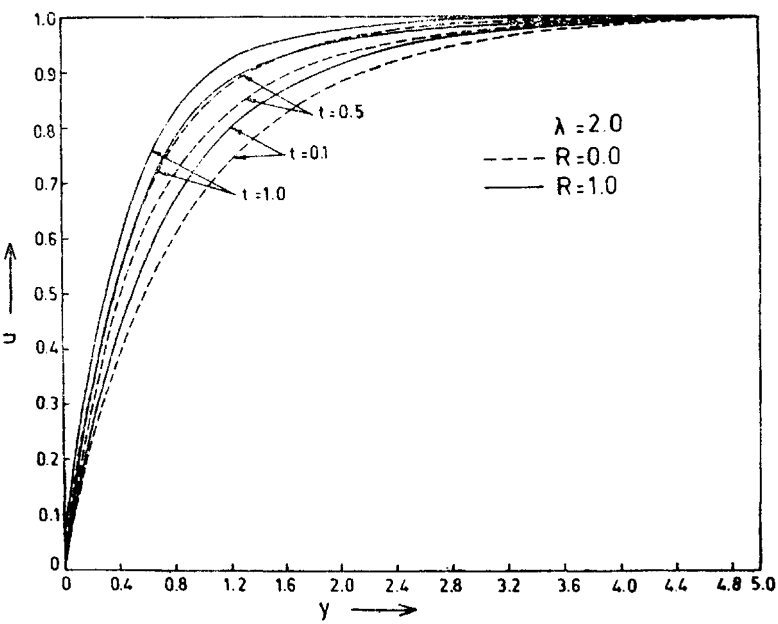

Fig. 2. Velocity distribution for different R with t.

values for all the parameters under study. Numerically the microrotation increases in a region near the plate reaching a maxima with increase in R as well as t (see Fig. 3). In Fig. 4, the temperature profiles are drawn. The temperature decreases with increase in R at all time levels having a square shape profile for $R = 5$.

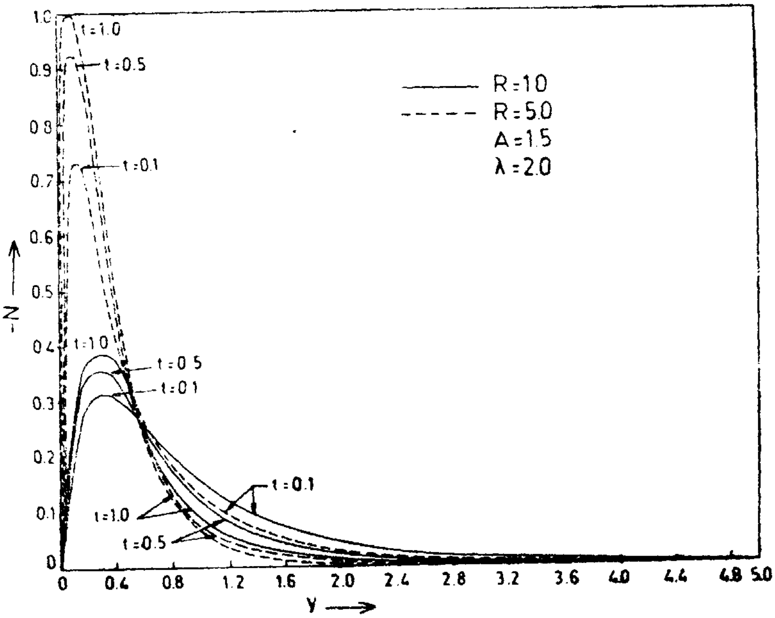

FIG. 3. Microrotation for different R and t.

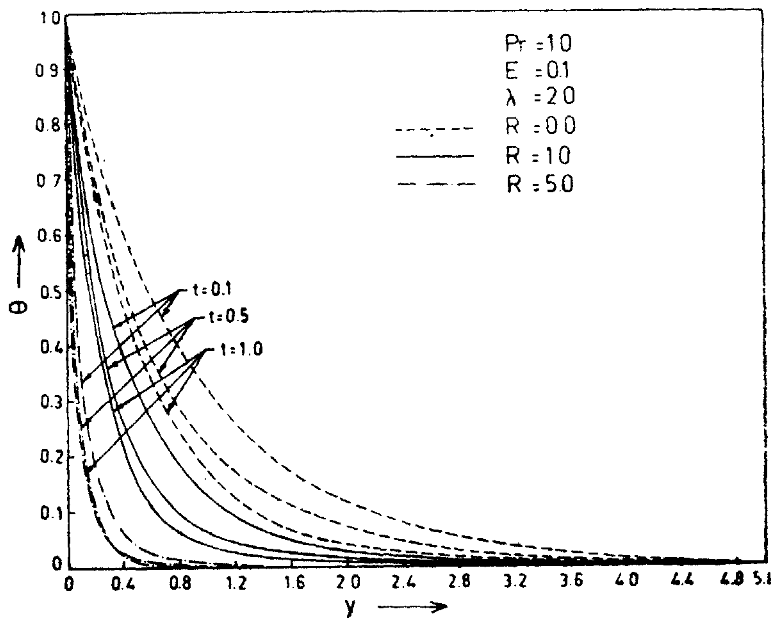

FIG. 4. Temperature distribution for various values of R with time t.

The difference scheme (13) – (15) is consistent as per condition stated in Ralston[15] and Rosenberg[16] with the differential equations (9) – (11), which in turn implies that the difference equations actually do approach the governing differential equations. The truncation error for the approximation in the velocity, microrotation and temperature is $[O(\Delta t) + O(\Delta y)^2]$ which tends to become zero as Δy and Δt tend to zero. The programme has also been executed for smaller values of Δt, viz. $\Delta t = 0.002$, 0.001 and 0.0005 and the results compared with those for $\Delta t = 0.0025$, reveal no sinfiicant change, thus ensuring the convergence of finite difference scheme employed here.

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