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FLOW OF A CONDUCTING FLUID BETWEEN TWO COAXIAL ROTATING POROUS CYLINDERS BOUNDED BY A PERMEABLE BED

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The steady flow of a viscous incompressible conducting fluid beetween two coaxial rotating porous cylinders with the outer cylinder bounded by a permeable bed is considered. The induced magnetic field is neglected and on the porous bed the boundary condition of Beavers and Joseph is applied. An exact solution of the governing equations is found. The velocity are temperature distribution of the fluid in free region and in porous region are evaluated in dimensionless form. The results are discussed numerically.

1. INTRODUCTION

Flow through and past porous media have wider range of applications in many branches like chemical engineering, petroleum engineering, soil mechanics and bio-medical engineering.

The study of the flow of a viscous incompressible fluid between two coaxial cylinders was first undertaken by Couette[2] with a view to measuring the viscosity of the fluid. Sinha and Choudhary[6] and Jain and Bansal[3] consider the flow of a viscous incompressible fluid betwen two coaxial rotating or non rotating cylinders with the walls of the cylinder being either solid or porous. Jain and Bansal[3] considered the flow of a viscous incompressible fluid between two coaxial rotating porous cylinders. Jain and Mehta[4] obtained exact solution in a closed form of the hydro magnetic equations for an incompressible viscous and electrically conducting fluid flow through an annulus with porous walls in the presence of a transverse radial magnetic field. Syam Babu[7] discussed the flow of a viscous incompressible conducting fluid between two coaxial rotating porous cylinders under the influence of a uniform radial magnetic field.

In this paper we study the flow of viscous conducting incompressible fluid between two coaxial rotating porous cylinders with the outer cylinder bounded by a permeable bed. The cylinders are rotating with an angular velocities ω_1 and ω_2 respectively in the same direction. The fluid is injected/sucked at the inner cylinder with a constant velocity. The flow in the annulus and in the porous medium is governed by the same pressure gradient $-dp/dr$.

2. Nomenclature

$(u, v, 0)$	= dimensionless fluid velocity in the free region
$(u_1, v_1, 0)$	= dimensionless fluid velocity in the porous region
$(Hr, 0, 0)$	= magnetic field strength
$(0, 0, Ez)$	= electric field strength
ρ	= density of the fluid
ν	= kinematic viscosity of the fluid
C_p	= specific heat at constant pressure
κ	= thermal conductivity of the fluid in zone 1
T	= temperature in the fluid region
T_0	= ambient temperature
h_e	= heat transfer coefficient
μ	= coefficient of viscosity
P	= fluid pressure
σ	= porosity parameter
K	= permeability of the porous bed
L	= dimensionless constant
Ω_1, Ω_2	= angular velocities of the inner and outer cylinders respectively
Pr	= prandtl number
Pe	= peclet number
Nu	= nusselt number
μ_e	= magnetic permeability of the fluid
σ_e	= electrical conductivity of the fluid
δ	= dimensionless parameter
M	= magnetic parameter
ϕ	= dissipation function

r_1, r_2 = radii of the inner and outer cylinders respectively

λ = suction/injection parameter

α = slip parameter.

3. Formulation of the Problem

We consider the steady flow of a conducting viscous incompressible fluid between two coaxial rotating porous cylinders composed of an insulated material. The cylinders terminate at perfect electrodes which are connected through a load. The walls of the cylinders being porous with the outer cylinder bounded by a permeable bed. (Fig. 1) The problem is divided into two zones : free zone and porous zone. In zone 1, the fluid is governed by the magneto hydrodynamic equations and in zone 2, the flow is governed by the Darcy's law. The solutions in two zones are matched at the interface

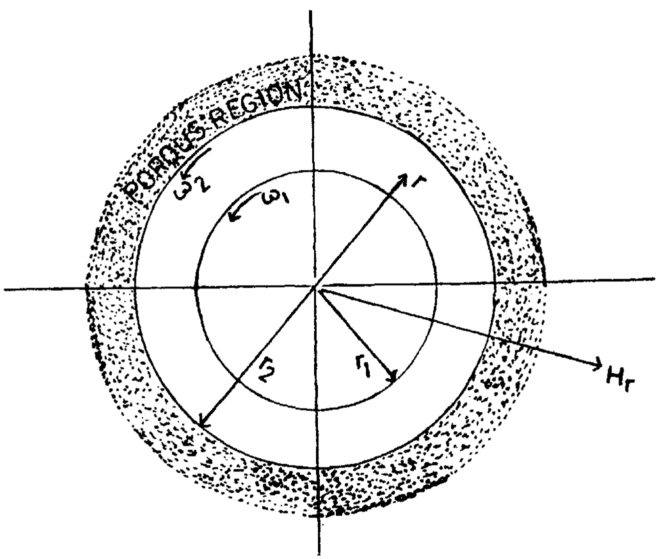

Fig. 1. Physical model.

between zone 1 and zone 2 by assuming the slip velocity boundary condition of Beavers and Joseph[1]. A uniform magnetic field H_0 is applied in the radial direction throughout the flow region. The porous zone is assumed as isothermal and so the bed is kept at constant temperature T_0 which is ambient temperature. The boundary condition of Rudraiah *et al.*[5]. is used for temperature at the interface between the porous zone and free zone.

The governing equations for the steady viscous, incompressible fluid flow in zone 1 and zone 2 are :

Zone 1

$$u' \frac{du'}{dr'} - \frac{v'^2}{r'} = - \frac{1}{\rho} \frac{dp'}{dr'} + \nu \left(\frac{d^2 u'}{dr'^2} + \frac{1}{r'} \frac{du'}{dr'} - \frac{u'}{r'^2} \right) \quad ...(1)$$

$$u' \frac{v'}{dr'} + \frac{u' v'}{r'} = \nu \left(\frac{d^2 v'}{dr'^2} + \frac{1}{r'} \frac{dv'}{dr'} - \frac{v'}{r'^2} \right) + \frac{1}{\rho} (\sigma_e \mu_e H_0 Ez' - \sigma_e \mu_e^2 H_0^2 v') \quad ...(2)$$

$$\frac{d(r' u')}{dr'} = 0 \quad ...(3)$$

$$\frac{d(r' Hr')}{dr'} = 0 \quad ...(4)$$

$$\frac{d(Ez')}{dr'} = 0 \quad ...(5)$$

$$C_p u' \frac{dT'}{dr'} = \frac{\kappa}{\rho} \left(\frac{d^2 T'}{dr'^2} + \frac{1}{r'} \frac{dT'}{dr'} \right) + \phi \quad ...(6)$$

where

$$\phi = \nu \left[2 \left\{ \left(\frac{du'}{dr'} \right)^2 + \left(\frac{u'}{r'} \right)^2 \right\} + \left(\frac{dv'}{dr'} - \frac{v'}{r'} \right)^2 \right].$$

Zone 2

$$u'_1 = - \frac{K}{\mu} \left[\frac{dp'}{dr'} - 2\rho \omega_2 v'_1 \right] \quad ...(7)$$

$$v'_1 = - \frac{K}{\mu} \left[2\rho \omega_2 u'_1 - \sigma_e \mu_e H_0 Ez' + \sigma_e \mu_e^2 H_0^2 v'_1 \right] \quad ...(8)$$

$$\frac{d(r' u'_1)}{dr'} = 0. \quad ...(9)$$

The boundary conditions are

$$u' = u_1, v' = r_1 \omega_1, T' = T_1 \text{ at } r' = r_1 \quad ...(10)$$

$$\left. \begin{aligned} & u' = u'_{b1}, \quad \frac{du'}{dr'} = \frac{\alpha}{\sqrt{K}} \left(u'_{b1} - u'_1 \right) \\ & v' = v'_{b1} + r_2 \omega_2, \quad \frac{dv'}{dr'} = \frac{\alpha}{\sqrt{K}} \left(v'_{b1} - v'_1 \right) \\ & \frac{dT'}{dr'} = \frac{h_e}{\kappa} \left(T'_B - T_0 \right). \end{aligned} \right\} \text{at } r' = r_2 \quad ...(11)$$

The following non-dimensional quantities are used :

$$r = \frac{r'}{r_1}, u = \frac{u' r_1}{\nu}, v = \frac{v' r_1}{\nu}, u_1 = \frac{u_1' r_2}{\nu}, v_1 = \frac{v_1' r_2}{\nu},$$

$$u_{b1} = \frac{u'_{b1} r_2}{\nu}, v_{bl} = \frac{v'_{b1} r_2}{\nu}, P = \frac{P' r_1^2}{\rho \nu^2}, \delta = \frac{r_2}{r_1}, \Omega_1 = \frac{r_1^2 \omega_1}{\nu},$$

$$\Omega_2 = \frac{r_2^2 \omega_2}{\nu}, T_B = \frac{T'_B - T_0}{T_2 - T_0}, \lambda = \frac{r_1 u}{\nu} = \frac{r_2 u_1}{\nu}, Pr = \frac{\mu C_p}{\kappa},$$

$$Pe = \lambda Pr, L = \frac{\mu \nu^2}{r_1^2 \kappa (T_2 - T_0)}, Ez = \frac{Ez'}{E_0}, \text{ where } E_0 = \frac{\mu_e H_0 \nu}{r_1}$$

$$Hr = \frac{Hr'}{H_0}, \sigma = \frac{r_2}{\sqrt{K}}, Nu = \frac{h_e r_1}{\kappa}, M^2 = \frac{\sigma_e \mu_e^2 H_0^2 r_1^2}{\mu}.$$

Using the above non-dimensional quantities, eqns. (1) — (9) reduce to :

Zone 1

$$u \frac{du}{dr} - \frac{v^2}{r} = -\frac{dp}{dr} + \frac{d^2 u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} \qquad \ldots(12)$$

$$u \frac{dv}{dr} + \frac{uv}{r} = \frac{d^2 v}{dr^2} + \frac{1}{r}\frac{dv}{dr} - \left(\frac{1}{r^2} + \frac{M^2 Hr^2}{r^2}\right) v + \frac{M^2 Hr\, Ez}{r} \qquad \ldots(13)$$

$$\frac{d(ru)}{dr} = 0, \quad \frac{d(rHr)}{dr} = 0, \quad \frac{d(Ez)}{dr} = 0 \qquad \ldots(14)$$

$$Pr\, u \frac{dT}{dr} = \frac{d^2 T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + L\left[2\left\{\left(\frac{du}{dr}\right)^2 + \left(\frac{u}{r}\right)^2\right\} + \left(\frac{dv}{dr} - \frac{v}{r}\right)^2\right]. \qquad \ldots(15)$$

Zone 2

$$u_1 = -\frac{1}{\sigma^2}\left[\delta^3 \frac{dp}{dr} - 2 v_1 \Omega_2\right] \qquad \ldots(16)$$

$$v_1 = \frac{M^2 \delta^3 Hr\, Ez - 2\Omega_2 u_1}{\sigma^2 + \delta^2 M^2 Hr^2} \qquad \ldots(17)$$

$$\frac{d(ru_1)}{dr} = 0. \qquad \ldots(18)$$

The boundary conditions (10) – (11) reduce to

$$u = \lambda,\ v = \Omega_1,\ T = 0 \qquad \text{at } r = 1 \qquad \ldots(19)$$

$$\left.\begin{aligned} u &= u_{b1},\ \frac{du}{dr} = \frac{\alpha\sigma}{\delta^2}(u_{b1} - u_1) \\ v &= v_{b1} + \Omega_2\,\delta,\ \frac{dv}{dr} = \frac{\alpha\sigma}{\delta^2}(v_{bl} - v_1) \\ T &= T_B,\ \frac{dT}{dr} = Nu\ T_B. \end{aligned}\right\} \quad \text{at } r = \delta \qquad \ldots(20)$$

4. Velocity Distribution

Solving equations (12) – (14) and (16) – (18) with the help of the boundary conditions (19) and (20), we get

$$u = \lambda/r$$

$$v = C\,r^{N_1} + D\,r^{N_2} + C_1\,r \qquad \ldots(22)$$

$$u_1 = \frac{B}{r} \qquad \ldots(23)$$

$$v_1 = \frac{M^2\,r\,\delta^3\,Ez - 2B\,r\,\Omega_2}{r^2\,\sigma^2 + M^2\,\delta^2} \qquad \ldots(24)$$

where

$$B = \lambda\,[1 + \delta/\alpha\sigma],\ C_1 = \frac{M^2\,Ez}{M^2 + 2\lambda}$$

$$N_{1,2} = \tfrac{1}{2}\,[\lambda \pm g\,\{\lambda^2 + 4\,(1 + M^2 + \lambda\}^{1/2}].$$

The constants C and D are functions of the physical parameters involving in the problem.

5. Temperature Distribution

Using the velocity of the fluid obtained in zone 1, equation (15) reduces to

$$r^2\,\frac{d^2\,T}{dr^2} + (1 - Pe)\,r\,\frac{dT}{dr} = -\frac{C_0}{r^2} - C_2\,r^{2N_1} - C_3\,r^{2N_2} - C_4\,r^{\lambda}$$

where

$$C_0 = 4\,L\,\lambda^2,\ C_2 = LC^2\,(N_1 - 1)^2,\ C_3 = LD^2\,(N_2 - 1)^2$$

$$C_4 = 2\,LCD\,(N_1 - 1)\,(N_2 - 1).$$

This is a second order ordinary differential equation whose general solution will posses a number of singular points. In order to avoid these singularities the solutions are obtained for separate cases by using the boundary conditions (19) – (20).

(i) *No singularities* : $[Pe \neq -2, 2N_1, 2N_2, \lambda, \lambda \neq -(1+M^2)]$

$$T = \frac{C_0(1-r^{-2})}{2(2+Pe)} + \frac{C_2(1-r^{2N_1})}{2N_1(2N_1-Pe)} + \frac{C_3(1-r^{2N_1})}{2N_2(2N_2-Pe)} + \frac{C_4(1-r^{\lambda})}{\lambda(\lambda-Pe)} + B_2(1-r^{Pe}); \qquad \ldots(26)$$

(ii) *Singularities* : $[Pe = -2, Pe \neq 2N_1, 2N_2\lambda, \lambda \neq -(1+M^2)]$

$$T = \frac{C_0 r^{-2}\log r}{2} + \frac{C_2(1-r^{2N_1})}{2N_1(2N_1+2)} + \frac{C_3(1-r^{2N_2})}{2N_2(2N_2+2)} + \frac{C_4(1-r^{\lambda})}{\lambda(\lambda+2)} + B_3(1-r^{-2}); \qquad \ldots(27)$$

(iii) $[Pe = -2 = \lambda, Pe \neq 2N_1, 2N_2, M \neq 1]$

$$T = \frac{C_2(1-r^{2N_1})}{2N_1(2N_1+2)} + \frac{C_3(1-r^{2N_2})}{2N_2(2N_2+2)} + \frac{C' r^{-2}\log r}{2} + B_4(r^{-2}-1); \qquad \ldots(28)$$

(iv) $[Pe = -2 = \lambda, Pe \neq 2N_2, 2N_1 = 0, M = 1, Pe \neq 2N_1]$

$$T = \frac{C' r^{-2}\log r}{2} - \frac{C_2 \log r}{2} + \frac{C_3(1-r^{2N_2})}{2N_2(2N_2+2)} + B_5(r^{-2}-1); \qquad \ldots(29)$$

(v) $[Pe \neq \lambda, \lambda = -(1+M^2), 2N_1 = 0, Pe \neq 2N_2, 2N_1]$

$$T = \frac{C_0(1-r^{-2})}{2(2+Pe)} + \frac{C_2 \log r}{Pe} + \frac{C_3(1-r^{2N_2})}{2N_2(2N_2-Pe)} + \frac{C_4(1-r^{\lambda})}{\lambda(\lambda-Pe)} + B_6(r^{Pe}-1); \qquad \ldots(30)$$

(vi) $[Pe = \lambda = -(1+M^2), Pe \neq 2N_1, 2N_2; 2N_1 = 0, M \neq 1 \text{ or } \lambda \neq -2]$

$$T = \frac{C_0(1-r^{-2})}{2(1-M^2)} - \frac{C_2 \log r}{1+M^2} + \frac{C_3(1-r^{2N_2})}{2N_2(2N_2+1+M^2)} + \frac{C_4 \log r\, r^{-(1+M^2)}}{1+M^2} + B_7(r^{-(1+M^2)}-1); \qquad \ldots(31)$$

(vii) $[Pe \neq -2, 2N_2, \lambda; Pe = 2N_1, 2N_1, 2N_1 \neq 0]$

$$T = \frac{C_0(1-r^{-2})}{2(2+Pe)} - \frac{C_2 r^{Pe}\log r}{Pe} + \frac{C_3(1-r^{2N_2})}{2N_2(2N_2-Pe)}$$

(*equation continued on p.* 533)

$$+ \frac{C_4 (1 - r^\lambda)}{\lambda (\lambda - Pe)} + B_8 (r^{Pe} - 1); \quad ...(32)$$

where B_2 ... B_8 are known constants which are functions of the physical parameters involving in the problem.

6. Discussion

The velocity distribution obtained from eqn. (22) and temperature distribution obtained from eqns. (26) — (32) in the presence of porous media are evaluated numerically and the results are shown graphically. Figures 2 - 5 gives the velocity

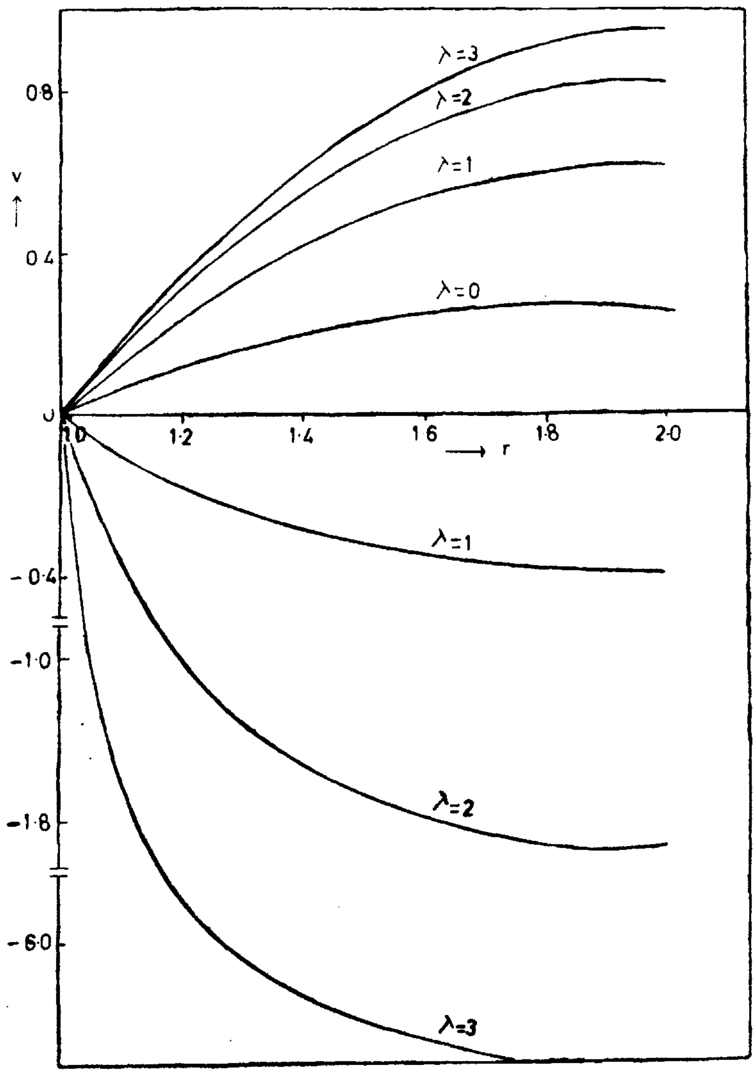

Fig. 2. Velocity profiles for $C_1 = 1$, $\Omega_1 = \Omega_2 = 0$, $\delta = 2$ $M = 1$ and $\sigma = 0.05$.

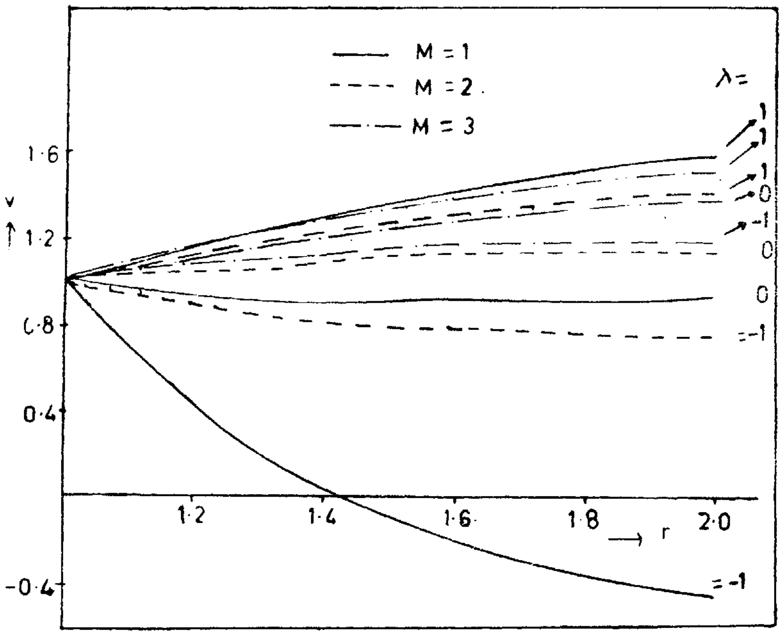

FIG. 3. Velocity profiles for $C_1 = 1$, $\sigma = 0.05$, $\Omega_1 = \Omega_2 = 1$, $\delta = 2$.

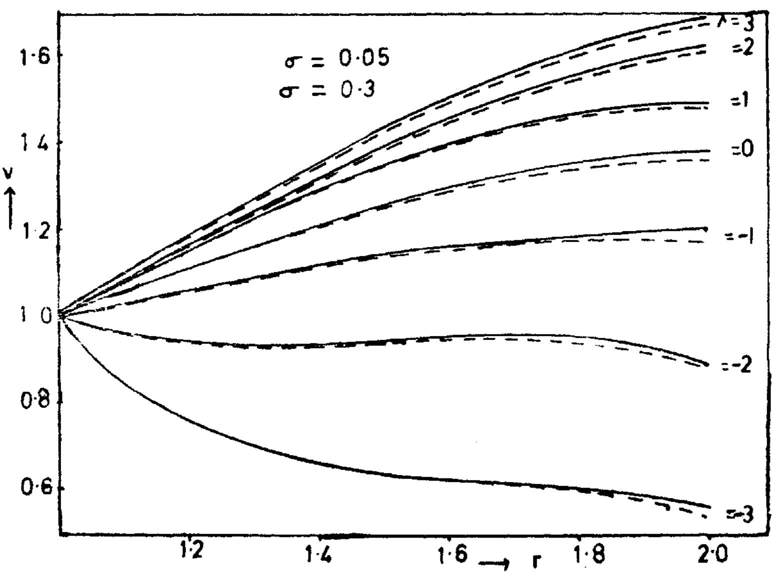

FIG. 4. Velocity profiles for $C_1 = 1$, $\Omega_1 = \Omega_2 = 1$, $M = 3$, $\delta = 2$.

distribution for various parameters involving in the problem and Fig. 6 gives the temperature distribution for fixed values of Peclet number and Nusselt number.

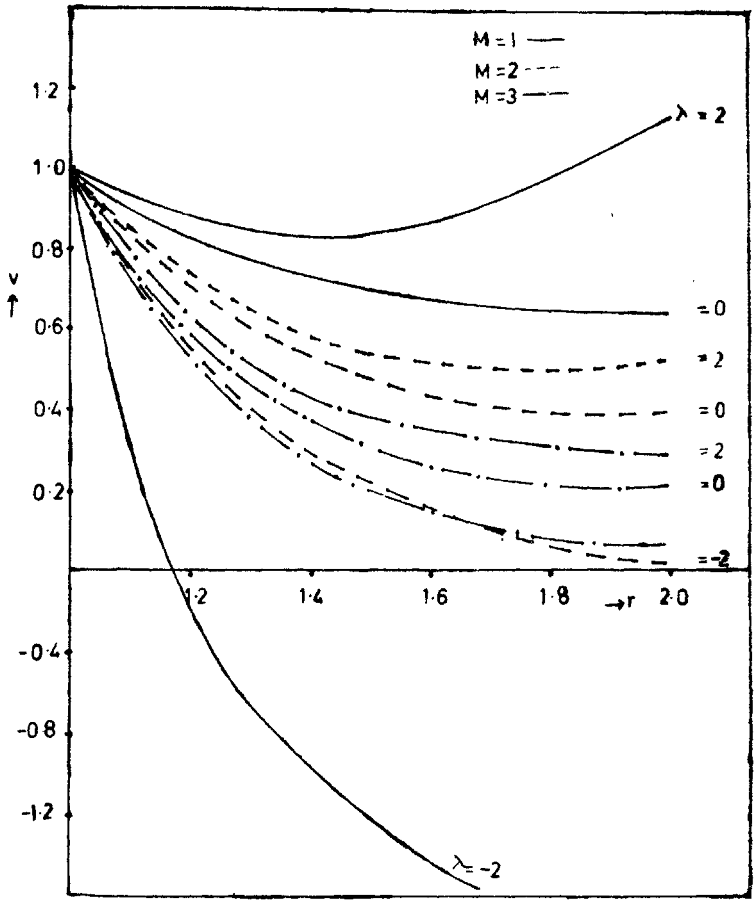

FIG. 5. Velocity profiles for $\sigma = 0.05$, $\Omega_1 = \Omega_2 = 1$, $C_1 = 0$.

Figures 2–4 give the velocity profiles in the presence of an electric field for various values of suction/injection parameter λ. When both the cylinders are stationary or rotating the velocity increases as the suction/injection parameter increases. The velocity is negative for suction and positive for injection. When the porosity parameter 'σ' increases the velocity increases for suction and decreases for injection for the fixed values of the magnetic parameter. When both the cylinders are rotating, the velocity increases as magnetic parameter M increases and for $\lambda = -1$ the velocity is negative. For small values of M the velocity is negative ($M = 1$) and for large values of M the velocity is positive. To remove the back flow, the magnetic parameter is to be increased. For $M = 3$, the velocity decreases as porosity parameter increases.

Figure 5 gives the velocity profiles in the absence of electric field. When both the cylinders are rotating the velocity decreases with increasing M and increases as λ increases. For $M = 1$, and $\lambda = -2$ the velocity is negative and for all other values it is positive.

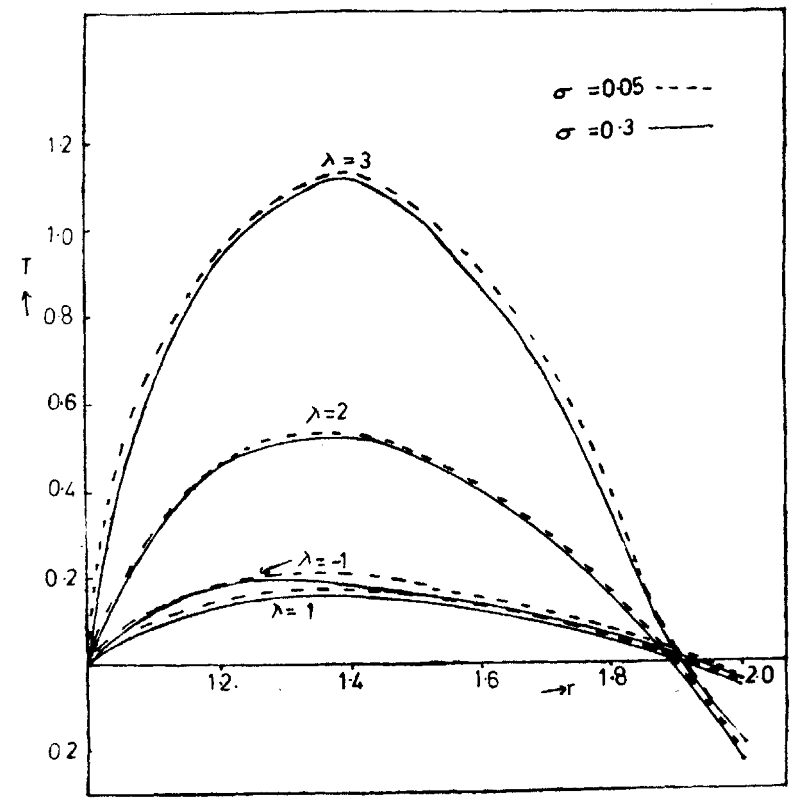

FIG. 6. Temperature profiles for $C_1 = 1$, $M = 1, \Omega_1 = 0, \Omega_2 = 1$.

Figure 6 gives the temperature distribution against 'r' in the presence of electric field. When the inner cylinder is stationary and the outer cylinder is rotating the temperature is maximum for $\lambda = -3$ and minimum for $\lambda = -2$ and $\lambda = 0$. The temperature increases with the injection parameter. When the porosity parameter increases the temperature decreases.

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